

Cooperative Energy Spanners: Energy-Efficient Topology Control in Cooperative Ad Hoc Networks

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Abstract—Cooperative communication (CC) allows multiple nodes to simultaneously transmit the same packet to the receiver so that the combined signal at the receiver can be correctly decoded. Since CC can reduce the transmission power and extend the transmission coverage, it has been considered in topology control protocols [1], [2]. However, prior research on topology control with CC only focuses on maintaining the network connectivity, minimizing the transmission power of each node, whereas ignores the energy-efficiency of paths in constructed topologies. This may cause inefficient routes and hurt the overall network performance. In this paper, to address this problem, we introduce a new topology control problem: *energy-efficient topology control problem with cooperative communication*, and propose two topology control algorithms to build *cooperative energy spanners* in which the energy efficiency of individual paths are guaranteed. Simulation results confirm the nice performance of the proposed algorithms.

I. INTRODUCTION

Topology control have been widely studied and applied in wireless ad hoc networks as one of the key energy saving techniques. In order to save energy and extend lifetime of networks topology control lets each wireless node to select certain subset of neighbors or adjust its transmission power meanwhile maintain network connectivity. Chen and Huang [3] first studied the strongly connected topology control problem, which aims to find a connected topology such that the total energy consumption is minimized. They proved such problem is NP-complete. Several following works [4]–[6] have focused on finding the minimum power assignment so that the induced communication graph has some “good” properties such as disjoint paths, connectivity or fault-tolerance. On the other hand, several localized geometrical structures [7]–[9] have been proposed to be used as underlying network topologies. These geometrical structures are usually kept as few link as possible from the original communication graph and can be easily constructed using location information.

Recently, a new class of communication techniques, cooperative communication (CC) [10], [11], has been introduced to allow single antenna devices to take the advantage of the multiple-input-multiple-output (MIMO) systems. This cooperative communication explores the broadcast nature of the wireless medium and allows nodes that have received the transmitted signal to cooperatively help relaying data for other nodes. Recent study has shown significant performance gain

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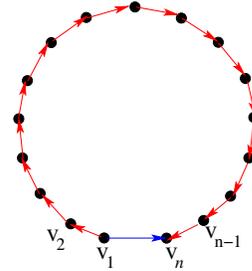


Fig. 1. Inefficiency of Current Topology Control Methods with CC: Solutions in [1], [2] will remove link v_1v_n to minimize the total energy consumption, thus will lead to an inefficient path from v_1 to v_n .

of cooperative communication in various wireless network applications: energy efficient routing [12]–[14] and connectivity improvement [15].

The cooperative communication techniques can also be used in topology control. In [1], Cardei *et al.* first studied the topology control problem under cooperative model (denote by TCC) which aims to obtain a strongly-connected topology with minimum total energy consumption. They first showed that this problem is NP-complete and then proposed two algorithms that start from a connected topology assumed to be the output of a traditional (without using CC) topology control algorithm and reduce the energy consumption using CC model. The first algorithm (DTCC) uses 2-hop neighborhood information of each node to reduce the overall energy consumption within its 2-hop neighborhood without hurting the connectivity under CC model. The second algorithm (ITCC) starts from a minimum transmission power, and iteratively increases its power until all nodes within its 1-hop neighborhood are connected under CC model. Observing that the CC technique can also extend the transmission range and thus link disconnected components. In [2], Yu *et al.* applied CC model in topology control to improve the network connectivity as well as reduce transmission power. Their algorithm first constructs all candidates of bidirectional links using CC model (called *cooperative bridges*) which can connect different disconnected components in the communication graph with maximum transmission power. Then they apply a 2-layer MST structure (one MST over the CC links to connect the components, the other is inside each component) to further reduce the energy consumption.

Even though the proposed solutions in [1], [2] can guarantee the network connectivity and reduce the energy consumption by constructing a sparse structure under CC model, they do not consider the energy efficiency of paths among nodes in

the constructed structure. Figure 1 shows an example network with n nodes. Assume that the link v_1v_n needs exactly the maximal transmission power at node v_1 , and all other links need slightly smaller transmission power. In addition, we assume that there is no CC transmission possible due to the value of maximal transmission power and the SNR threshold. Then if we apply the topology control algorithms in [1], [2], all of them will remove link v_1v_n to minimize the energy consumption. However, such operation will hurt the route between v_1 and v_n . The least energy path between them now (v_1 sends via $v_2 \cdots v_{n-1}$ to v_n) could be arbitrarily large compared with the optimal one (v_1 directly sends to v_n). It is also easy to construct a similar example where v_1v_n is a link via cooperative communication. Therefore, we hope that the sparseness of the constructed topology under CC model should not compromise too much on energy consumption of communication paths. Such problem has been studied in traditional topology control algorithms (without CC) [7]–[9], [16], [17]. In [7], Li *et al.* first introduced the concept of *energy spanner* into topology control. Here, a subgraph G' is called an *energy spanner* of a graph G if there is a positive real constant t such that for any two nodes, the energy consumption of the least energy cost path in G' is at most t times of the energy consumption of the least energy cost path in G . Several geometric structures [7]–[9] have been proved to be energy spanners. Recently, [16], [17] also studied how to assign the transmission power for each node such that the induced communication graph is an energy spanner of the original communication graph meanwhile the total power level of all nodes is minimized. However, so far there is no energy spanner proposed under CC model yet.

In this paper, we study the energy efficient topology control problem with CC model by taking the energy efficiency of routes into consideration. Taking advantage of physical layer design that allows combining partial signals containing the same information to obtain the complete data, we formally define *cooperative energy spanner* in which the least energy path between any two nodes is guaranteed to be energy efficient compared with the optimal one in the original cooperative communication graph. We then introduce the *energy-efficient topology control problem with CC* (ETCC) in Section II, which aims to obtain a cooperative energy spanner with minimum total energy consumption, and prove its NP-completeness. Therefore, as solutions for ETCC, we propose two topology control algorithms in Section III to build energy-efficient cooperative energy spanners. Both algorithms can guarantee the bounded energy stretch factor. Simulation results in Section IV confirm the nice performance of proposed algorithms.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. Preliminaries

We consider a wireless ad hoc network with n nodes which are capable of receiving and combining partial received packets in accordance with the CC model. Every node v_i can adjust its transmission power P_i which is limited by a maximum value P_{MAX} . The network topology is modeled

as a 2-dimensional directed graph: $G = (V, E)$, where $V = (v_1, \dots, v_n)$ denotes the set of wireless nodes and E denotes a set of directed communication links. We denote $N(v_i)$ as the set of *direct neighbor nodes* of v_i within its maximum transmission range. We assume that each node has a unique ID and knows its own location information. Node ID and location information are exchanged among all nodes. Notice that a directed link $v_iv_j \in E$ denotes node v_i can transmit data to node v_j either directly or using CC, thus E is the unions of all direct links \overline{E} and CC-links \widetilde{E} . Here a *direct link* $\overline{v_iv_j} \in \overline{E}$ is a link representing that node v_i can transit the information to node v_j directly (i.e., without CC). A *CC-link* $\widetilde{v_iv_j} \in \widetilde{E}$ is a link representing node v_i can transit the information to node v_j with its *helper node set* H_{ij} in CC model, where H_{ij} includes all helper nodes of node v_i for its CC transmission to v_j . In this paper, we assume all elements in $N(v_i)$ are the candidates of helper nodes of v_i . We define a direct link $\overline{v_iv_k}$ between a source node v_i and its helper node v_k as a *helper link*. Then we can define the connectivity of the cooperative network $G = (V, E)$ as follows:

Definition 1 (Connectivity of Cooperative Network): The cooperative network G is *strongly connected under CC model* if and only if for any two nodes v_i and v_j there exists a directed path, which consists of directed links in G , to connect them in G . In other words, by using the combination of direct links and CC-links the packet from any source node can reach any other node in the network.

Our cooperative communication (CC) model is similar to those of [1], [2], [10], [11]. It takes advantage of the physical layer design [10] that combines partial signals containing the same information to obtain the complete information. Thus, a complete communication from node v_i to node v_j can be achieved with cooperative communication if v_i transmits the same signal with a set of helper nodes H_{ij} and their transmission power satisfy $\sum_{v_k \in v_i \cup H_{ij}} P_k \cdot (d_{kj})^{-\alpha} \geq \tau$ ($P_k \leq P_{MAX}$). Here, α is the path loss exponent (usually between 2 and 4), τ is the minimum average signal-to-noise ratio (SNR) for decoding received data, and d_{ij} is the distance between node v_i and node v_j . To address the energy efficient problem, we formally define the energy consumption of each link and each path in cooperative ad hoc networks.

Definition 2 (Link Weight): For each link $v_iv_j \in E$, we define the link weight $w(v_iv_j)$ represents the minimum total energy consumption (i.e., total transmission power) of all nodes participated in maintaining a directed link from v_i to v_j . For a direct link $\overline{v_iv_j}$, $w(\overline{v_iv_j}) = \tau d_{ij}^\alpha$. For a CC-link $\widetilde{v_iv_j}$ with its selected helper node set H_{ij} , we use an estimated total energy cost¹ as follows,

$$w(\widetilde{v_iv_j}) = \frac{\tau}{\max_{v_k \in H_{ij}} (d_{ik})^{-\alpha}} + \frac{(|H_{ij}| + 1) \cdot \tau}{\sum_{v_k \in v_i \cup H_{ij}} (d_{kj})^{-\alpha}}, \quad (1)$$

¹Notice that for a pair of nodes v_i and v_j to select a set of helper nodes H_{ij} for v_i from its one-hop neighbors $N(v_i)$, such that the total energy consumption of the constructed CC-link is minimized, is a challenging problem itself. Therefore, here we simplify the CC-model by assuming the transmission powers of node v_i and its helper node set H_{ij} during CC transmission are the same.

Here, $|H_{ij}|$ denotes the number of elements in set H_{ij} . The link weight of a CC-link includes two parts: the first one is the energy consumption for node v_i to communicate with its helper node set H_{ij} directly, and the second part is the total energy consumption for v_i and its helpers in H_{ij} to simultaneously communicate with v_j .

Consider a unicast path $\pi(v_i, v_j)$ in a cooperative network G from node v_i to node v_j under CC model, the total energy consumed by this path $\pi(v_i, v_j)$ is $p(\pi(v_i, v_j)) = \sum_{e \in \pi(v_i, v_j)} w(e)$. Let $P_G(v_i, v_j)$ be the path consuming the least energy among all paths connecting v_i and v_j in G . We called $P_G(v_i, v_j)$ the least-energy path in G for v_i and v_j . Let $p(P_G(v_i, v_j))$ be the total energy of the least-energy path. Then we can define the stretch factor as follows:

Definition 3 (Energy Stretch Factor): Let G' be a subgraph of G . The energy stretch factor of a node pair (v_i, v_j) in G' under CC model with respect to G is the defined as $\rho_G^{G'}(v_i, v_j) = \frac{p(P_{G'}(v_i, v_j))}{p(P_G(v_i, v_j))}$. The *energy stretch factor* of G' under CC model with respect to G is the defined as

$$\rho_G(G') = \max_{v_i, v_j \in V} \rho_G^{G'}(v_i, v_j) = \max_{v_i, v_j \in V} \frac{p(P_{G'}(v_i, v_j))}{p(P_G(v_i, v_j))}. \quad (2)$$

Definition 4 (Cooperative Energy t -Spanner): A subgraph G' of the cooperative network G is a *cooperative energy t -spanner* of G if its energy stretch factor under CC model with respect to G is no more than a constant t , i.e., $\rho_G(G') \leq t$. If a topology is a cooperative energy spanner of the original communication graph G , then we guarantee there is a path between each pair of nodes whose energy consumption is similar to the original optimal one when all possible direct and CC links are used. This will benefit energy efficient routing performance on the network topology.

B. Problem Formulation

Now we can define the new topology control problem, **Energy-Efficient Topology Control with CC (ETCC)**: Given a wireless multi-hop network $G = (V, E)$ which is strongly connected under CC model, assign transmission power P_i to every node v_i such that (1) the induced topology G' from this power assignment is a cooperative energy t -spanner of G , i.e., $\rho_G(G') \leq t$; and (2) the sum of transmission power of all nodes, $\sum_{v_i \in V} P_i$, is minimized.

Notice that the spanner property also guarantees that the induced topology G' is strongly connected under CC model. The above topology control problem is equivalent to the following one: given the strongly connected G , construct a substructure G' spanning all nodes such that (1) G' is a cooperative energy t -spanner of G ; and (2) the induced power assignment from G' minimizes the total transmission power $\sum_{v_i \in V} P_i$. Here, the induced transmission power of node v_i from G' is the maximum power to support all of its outgoing links in G' . Notice that TCC problem [1], which only maintains the connectivity, is a special case of ETCC when t is set to an arbitrarily large constant. In such case, the spanner property reduces to the basic connectivity requirement. As TCC has been proved to be NP-complete, ETCC is also NP-complete. Therefore, we

will focus on heuristic algorithms to guarantee the spanner property while minimizing the transmission energy.

III. ENERGY-EFFICIENT TOPOLOGY CONTROL WITH COOPERATIVE COMMUNICATION

In this section, we propose two topology control algorithms which build energy-efficient cooperative energy spanners. To keep the proposed algorithms simple and efficient, we only consider its one-hop neighbors as possible helper nodes for each node when CC is used. Thus, the original cooperative communication graph G contains all direct links and CC-links with one hop helpers, instead of all possible direct links and CC-links. In addition, for each pair of nodes v_i and v_j , we only maintain one link with least weight if there are multiple links connecting them. Both proposed algorithms are greedy algorithms. The major difference between them is the processing order of links. The first algorithm deletes links from the original graph G greedily, while the second algorithm adds links into G'' greedily. Here G'' is a basic connected subgraph of G . Both algorithms can guarantee the cooperative energy spanner property of the constructed graph G' .

A. Greedy Algorithm 1 - Deleting Links

We first propose a simple greedy algorithm for energy-efficient topology control, which is inspired by the classical greedy algorithm [18], [19] for low-weight spanner. The general idea is to start with the original cooperative communication graph G as described above. Then, gradually delete the least energy-efficient link (the link with highest weight) from G if doing so does not break the energy stretch factor requirement. Finally, the transmission power of each node is decided from the constructed topology G' . The details of these three steps are as follows.

Step 1: Construction of G . Initially, G is an empty graph. First, add every direct links $\overline{v_i v_j}$ into G , if node v_i can reach node v_j when it operates with P_{MAX} . Then, for every pair of nodes v_i and v_j , we select a set of helper nodes H_{ij} for node v_i from its one-hop neighbors $N(v_i)$, such that the link weight $w(\overline{v_i v_j})$ of the constructed CC-link is minimized. This helper nodes decision problem is challenging even under our assumption that the transmission powers of v_i and its helper node set to maintain CC-link are the same. If we try all combinations of the helper sets, the computational complexity is exponential to the size of $N(v_i)$. It is impractical to do so in case of a large number of neighbors. Thus we directly use the greedy heuristic **Greedy Helper Set Selection** $(v_i, N(v_i), v_j)$ (Algorithm 1 in [2]) to select the helper set H_{ij} . Then we compare $w(\overline{v_i v_j})$ with $p(P_G(v_i, v_j))$ which is the current shortest path from node v_i to node v_j in G . If $w(\overline{v_i v_j}) < p(P_G(v_i, v_j))$ and $\frac{\tau}{\sum_{v_k \in v_i \cup H_{ij}} (d_{kj})^{-\alpha}} \leq P_{MAX}$, add this CC-link $\overline{v_i v_j}$ into G and remove the direct link $\overline{v_i v_j}$ if it already exists.

Step 2: Construction of G' . Copy all links in G to G' , and sort them in the descending order of their weights. Start to process all links one by one and delete the link $v_i v_j$ from G' if $G' - v_i v_j$ is still a cooperative energy t -spanner of G .

In addition, when a CC-link $\widetilde{v_i v_j}$ is kept in G' , all its helper links must be kept in G' too.

Step 3: Power Assignment from G' . For each node v_i , its transmission power is decided by the following equation.

$$P_i = \max\left\{\max_{\widetilde{v_i v_j} \in G'} P_i^d(j), \max_{\widetilde{v_i v_j} \in G'} P_i^{cc}(j)\right\}. \quad (3)$$

Here $P_i^d(j) = \frac{\tau}{d_{ij}^{-\alpha}}$ and $P_i^{cc}(j) = \frac{\tau}{\sum_{v_k \in v_i \cup H_{ij}} (d_{kj})^{-\alpha}}$ are the energy consumption at v_i for a direct link $\widetilde{v_i v_j}$ and a CC-link $\widetilde{v_i v_j}$, respectively.

We call this algorithm *Greedy Algorithm 1* (denoted by GreedyDelLink). The guarantee of the energy t -spanner property is straightforward, since links are deleted from G' only when the deletion does not break the t -spanner requirement.

B. Greedy Algorithm 2 - Adding Links

The second topology control algorithm starts with a sparse topology G'' which is strongly connected under CC model. We can use the output of the algorithm in [2] as the initial topology. Then, we gradually add the most energy-efficient link into G'' . Here, the energy-efficiency of a link is defined as the gain on reducing energy stretch factors by adding this link. Our algorithm will terminate until the constructed graph G' satisfies the energy stretch factor requirement. The detail steps are summarized as follows.

Step 1: Construction of G and G'' . The step of constructing G is the same as the one in GreedyDelLink. Then we construct G'' , a connected sparse subgraph of G by calling the algorithm in [2].

Step 2: Construction of G' . Initialize $G' = G''$, for every link $v_i v_j \in G - G'$, compute its stretch-factor-gain $g_G^{G'}(v_i v_j)$ as follow:

$$g_G^{G'}(v_i v_j) = \sum_{v_p, v_q \in V} (\rho_G^{G'}(v_p, v_q) - \rho_G^{G' + v_i v_j}(v_p, v_q)), \quad (4)$$

Here, $G' + v_i v_j$ denotes the graph generated by adding edge $v_i v_j$ into G' . In other words, the total gain of a link $v_i v_j$ is the summation of the improvement of stretch factors of every pair of nodes in G' after adding this link. In each step, we greedily add the link with the largest stretch-factor-gain into G' . If there is a tie, we use the link weight to break it by adding the link with the least weight. We repeat this procedure until G' meets the stretch factor requirement t .

Step 3: Power Assignment from G' . For every node v_i , assign its power level P_i using Equation (3).

We call this algorithm *Greedy Algorithm 2* (denoted by GreedyAddLink). The guarantee of the energy t -spanner property is also straightforward since the algorithm terminates adding links until the t -spanner requirement is satisfied.

IV. SIMULATIONS

We evaluate our proposed topology control algorithms, namely, GreedyDelLink and GreedyAddLink, by comparing their performances with *Cooperative Bridges based Method* (Coop. Bridges) [2]. We implement all these three algorithms. The underlying networks G are randomly generated in an

area of 100×100 . For convenience, we set $\alpha = 2$, $\tau = 1$, $P_{MAX} = 400$. We take three metrics as the performance measurements of the output G' of topology control algorithms on G : *total link cost* $w(G') = \sum_{e \in G'} w(e)$, *average node transmission power* $\overline{P_i}(G') = \frac{1}{n} \sum_{v_i \in V} P_i$, and *maximum energy strength factor* $\rho_G(G') = \max_{v_i, v_j \in V} \frac{p(P_{G'}(v_i, v_j))}{p(P_G(v_i, v_j))}$. For all the simulations, we repeat the experiment for multiple times and report the average performance. It is clear that a desired topology should have small total link cost and small node transmission power while the energy strength factor should be smaller than the requirement t .

For the first set of simulations, we increase the network density by rising n from 10 to 100, and keep $t = 1.6$. Figure 2(a) shows the ratios between the total link cost of the generated graph G' and that of the original cooperative communication graph G when n increases. This ratio implies how much cost saving achieved by the topology control algorithm, compared with the original network without topology control. Figure 2(b) shows the ratios between average node transmission power assigned by the topology control algorithm and that assigned from G . Here, for each topology control algorithm, we also apply the DTCC algorithm in [1] to further reduce the transmission power with CC. From these results, all topology control algorithms can significantly reduce the total cost of maintaining the connectivity and save the node transmission energy. When the network is denser, the saving of topology control is larger. Compared with Coop Bridges, our two methods generate a denser topology. However, their costs and transmission power are just slightly larger than that of Coop Bridges. DTCC algorithm can indeed further reduce the transmission power. Figure 2(c) shows the energy stretch factor $\rho_G(G')$ of all methods. Clearly, Coop Bridges cannot satisfy the energy stretch factor requirement $t = 1.6$, and its stretch factor increases with the number of nodes. When the network is dense, it could be more than 10, which means some paths could have huge energy consumption.

In the second set of simulations, we fix $n = 50$ and run our algorithms with different energy stretch factor requirement t from 1.0 to 2.0. From Figure 3(a) and (b), we can observe that tighter (i.e., smaller) stretch factor requirement results in higher cost and larger node transmission power of all algorithms. The reason is when the stretch factor requirement becomes larger, the restriction on removing links becomes weaker. Clearly, there is a trade off between stretch factor and link cost or transmission power. We can choose a suitable stretch factor bound to match the particular application needs. From Figure 3(c), we can see that the stretch factor of Coop Bridges does not affect by the stretch factor requirement however both of our methods can perfectly satisfy the requirement.

From all simulation results, we can conclude that both of our proposed methods can achieve similar small energy stretch factors, all of them are bounded by the requirement t , with the costs of very slightly larger total link costs and transmission power than Coop Bridges. This demonstrates the energy efficiency of our energy cooperative spanners.

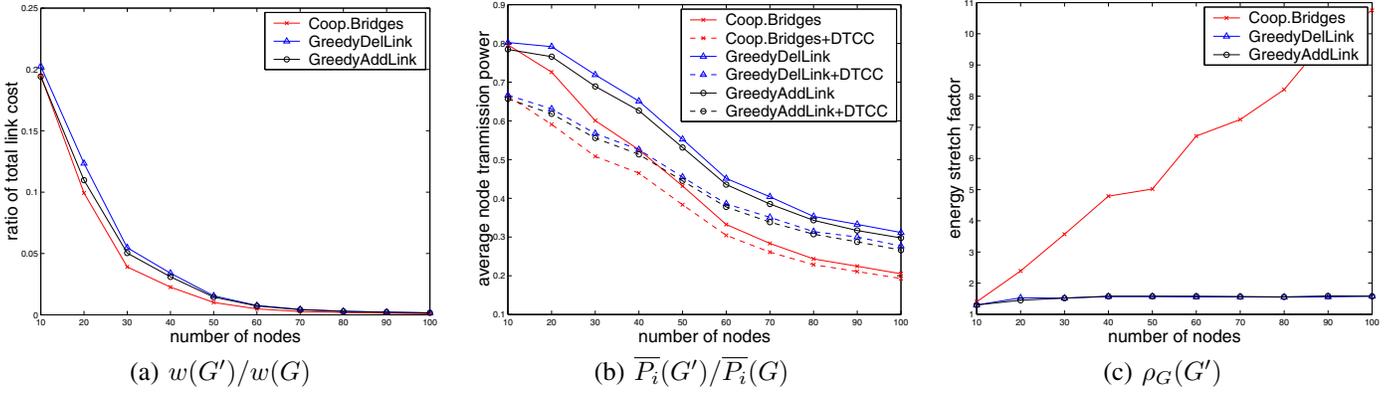


Fig. 2. Simulation results on random networks with fixed stretch factor requirement $t = 1.6$ and varying number of nodes n .

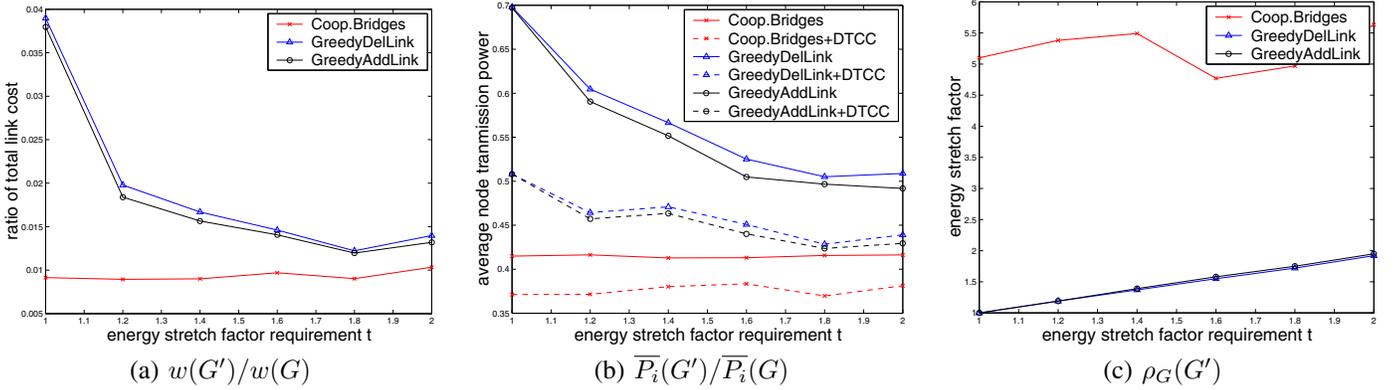


Fig. 3. Simulation results on random networks with fixed number of nodes $n = 50$ and varying stretch factor requirement t .

V. CONCLUSION

In this paper, we introduced a new topology control problem: *energy-efficient topology control problem with cooperative communication*, which aims to keep the energy-efficient paths in the constructed topology. We proved that this problem is NP-complete and proposed two new topology control algorithms using cooperative communications. Both algorithms can build a *cooperative energy spanner* in which the energy efficiency of individual paths are guaranteed. Simulation results confirm the nice performance of both proposed algorithms.

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