# Corrections and Comments on "Efficient Algorithms for $p$-Self-Protection Problem in Static Wireless Sensor Networks"

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***Abstract*— Our paper [1] gave several efficient approximation algorithms for $p$-self-protection problem in wireless sensor networks. It generated many interest among researchers in this area. Recently, via directly contacting to the first author and their paper [2], Islam and Akl pointed out that the approximation algorithms in [1] may fail to find a $p$-self-protection subset in the network by showing a counterexample. This correspondence provides simple corrections of the approximation algorithms so that they can still guarantee the constant approximation ratios for minimum $p$-self-protection problem.**

***Index Terms*— Self Protection, Independent Set, Distributed Algorithms, Wireless Sensor Networks.**

## I. Introduction

*Self protection problem* [3] in wireless sensor networks focuses on using sensor nodes to provide protection to themselves instead of the objects or the area, so that they can resist the attacks targeting on them directly. A wireless sensor network is $p$-self-protected, if at any moment, for any wireless sensor (active or non-active), there are at least $p$ active sensors that can monitor it. Given a sensor network with $n$ sensors, it can be modeled by a sensing graph $G(V, E)$ where $V$ is the set of sensor nodes (both active and non-active) and $E$ is the set of directed links $\overrightarrow{uv}$ between any two sensor $u$ and $v$ if $v$ is inside the sensing range of $u$. Then the *Minimum $p$-Self-Protection problem* is to find a subset (denoted by $MSP_p$) of $V$ to be set as active sensors such that the sensor network is $p$-self-protected and the number of active nodes ($|MSP_p|$) is minimized.

The minimum self-protection problem was first introduced by Wang *et al.* [3], [4] where they proved the NP-hardness of the problem and gave a centralized method with $2(1 + \log n)$ approximation ratio and two randomized distributed algorithms for the minimum 1-self protection problem. In [1], we proposed the first set of centralized and distributed algorithms with *constant approximation ratios* for the minimum $p$-self-protection problem. These algorithms are based on *maximum independent set* (MIS). A subset of vertices in a graph $G$ is an *independent set* if for any pair of vertices, there is no edge between them. It is a *maximum independent set* if no other independent set has more vertices.

Recently, Islam and Akl [2] pointed out that our approximation algorithms may fail to find a $p$-self-protection subset in the network by constructing a counterexample. This correspondence provides simple corrections of our approximation algorithms which can still guarantee the constant approximation ratios for $p$-self-protection problem.

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## II. Counterexample from Islam and Akl [2]

We first review the counterexample constructed by Islam and Akl [2]. Figure 1 shows a sensor network with 13 sensors. Consider the $p$-self-protection problem where $p = 4$. Assume that our algorithm (such as Algorithm 1 in [1]) finds the first MIS $M_1$ which includes three nodes shown with label $M_1$. In the second round, there is no way to find a second MIS $M_2$ for the whole network from the remaining nodes since all MIS needs use one of the node in $M_1$. If we only find the second MIS $M_2$ for the remaining nodes, as shown in the figure with nodes labeled with $M_2$, the process can be continued (generating $M_3$ and $M_4$). However, after the fourth round, the middle node with label $M_1$ is only protected by 2 MIS nodes. Thus, after adding one neighboring node to protect it in the last step of the algorithm, this node will be protected by only 3 active nodes, not as the requirement of 4-self protection. Therefore, Islam and Akl claimed that the constant approximation for $p$-self-protection problem is still open.

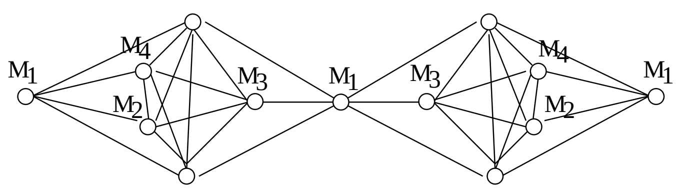


Fig. 1. Illustrations of counterexample from Islam and Akl [2].

## III. Correction on Centralized Algorithm

In this section, we provide a simple modification of Algorithm 1 in [1] to make it still generating a valid constant approximation. The modification can be summarized as two changes: (1) in each round the MIS $M_k$ is the MIS for all remaining nodes not the entire network; (2) in the last step (line 7) for each node $u$ in $M_i$ with only $q$-protection ($q < p$) we find $p - q$ neighboring non-MIS nodes to protect $u$. Notice that the existence of $p - q$ neighboring non-MIS nodes is guaranteed by Fact 1 in [1]. The new centralized algorithm is given in Algorithm 1.

The basic idea of the algorithm is still the same as in [1]. The algorithm first generates $k$ MISs in $k$ rounds. In each round, a MIS $M_k$ is generated for all non-MIS nodes and all MIS nodes selected in the early rounds is not used again in later rounds. After $k$ MISs are generated, all nodes in these MISs will be in the active set $M$. For each node $u$ inside these MISs, if it has less than $p$ neighbors in MISs (saying only $q$ MIS neighbors) the algorithm adds another $p - q$ neighbors into $M$. Since we assume that each node has at least $p$ neighboring nodes, in Step

**Algorithm 1** General Method for Minimum $p$-Self-Protection

1: Assign each node $v$ a unique rank $r(v) \in [1, n]$ and let $k = 1$.
2: **while** $k \leq p$ **do**
3: Generate a MIS $M_k$ for all nodes with non-zero rank (i.e., all non-MIS nodes) based on the rank of all nodes: a node is selected to the MIS if it has the largest rank among all its non-MIS neighboring nodes.
4: Set the rank of each selected node in $M_k$ to $0$.
5: $k = k + 1$.
6: **end while**
7: For each node $u$ that is selected in $M_i$, $1 \leq i \leq p$, if node $u$ has only $q$-protection ($q < p$) from $\bigcup_{i=1}^{p} M_i$, we find $p - q$ neighboring non-MIS nodes to protect $u$.
8: Let $M$ be the union of all $M_i$ and all selected nodes from last step to protect nodes in $M_i$.

7 there always exists $p-q$ neighboring nodes that are not selected when $u$ has less than $p$ neighboring nodes in $\bigcup_{i=1}^{p} M_i$. For the example shown in Figure 1, the middle node with label $M_1$ is only protected by $2$ MIS nodes after $4$ rounds of MISs. Thus, in Step 7, it will select two additional nodes to provide protection to itself. This satisfies the requirement of $4$-self protection.

Obviously, the time complexity of this algorithm is still $O(n)$. We now prove that this algorithm is still a constant approximation for the minimum $p$-self-protection problem.

*Theorem 1:* The set $M$ by Algorithm 1 is a valid $p$-self-protection, and has size at most $5p + 5$ times of the optimum solution $MSP_p$ when sensing radius of all nodes are the same.

*Proof:* First, the validation of the $p$-self-protection is obvious. For every node $u \notin \bigcup_{i=1}^{p} M_i$, it is protected by at least $p$ MIS nodes since each round of MIS $M_i$ has one node protecting it. Notice that during the process, the nodes already in the MIS selected before will *not* be selected to produce new MIS. For all node $u \in \bigcup_{i=1}^{p} M_i$, it will have at least $p$ protectors after Step 7. Thus all nodes are perfectly protected by at least $p$ active sensor nodes.

Then, we prove the approximation ratio. Remember that for each node there are at most $5$ neighboring nodes chosen in each round MIS $M_i$, thus for each node, there are at most $5 \cdot p$ nodes selected in $\bigcup_{i=1}^{p} M_i$. For the optimal solution $MSP_p$ of the minimum $p$-self-protection, there is at least $p$ neighboring nodes active for protection. Thus, the number of the selected MIS nodes in $\bigcup_{i=1}^{p} M_i$ is at most $5$ times of the size of the optimal solution $MSP_p$. In addition, the additional nodes added in Step 7 for each MIS node with less than $p$ protectors is at most $p$. Therefore, the total number of nodes selected by this method is at most $5(p+1)$ times of the optimal. ■

## IV. Correction on Distributed Algorithm

The distributed version (Algorithm 2 in [1]) needs following corrections: (1) Line 6: $k(u) = p + 1$; (2) Line 12: randomly select $p - p(u)$ neighbors $v$ whose status $s(v) = Nonactive$; (3) Last part of Line 31: $k(x) = p + 1$. Here (1) and (3) make sure that selected nodes in $M_i$ will not participate later rounds. The approximation of this distributed algorithm then is the same as the centralized one (i.e., $5p + 5$). This correction will not affect the message complexity which is still $O(n)$.

## V. Other Discussions

The part of self-protection and connectivity (Theorem 5) will not be affected by the above correction. For the case for heterogeneous sensing radius, the constant approximation ratio in Theorem 6 should be $6(p+1) \cdot (3\lceil \log_2 \gamma \rceil + 2)$ since the number of selected MIS nodes is at most $6 \cdot (3\lceil \log_2 \gamma \rceil + 2)$ and each MIS node needs at most $p$ additional protections in the end of $p$ rounds.

Finally, the modified method with further improvement (Algorithm 3 in [1]) can be modified as follows.

**Algorithm 2** Modified Method for Minimum $p$-Self-Protection

1: Assign each node $v$ a unique rank $r(v) \in [1, n]$ and let $k = 1$. And assign $p(v) = 0$ for every node $v$.
2: **while** exist non-MIS node $u$ with $p(u) < p$ **do**
3: Let $V_k$ be the set of non-MIS nodes with $p(v) < p$, *i.e.*, nodes in $V_k$ needs additional protections. Let $U_k$ be the set of non-MIS nodes that either is in $V_k$ or that can sense a node from $V_k$, *i.e.*, $U_k$ is the set of non-MIS nodes that can provide protections to nodes in $V_k$.
4: Generate a MIS $M_k$ for $U_k$ based on the rank of all nodes in $U_k$: a node from $U_k$ is selected to the MIS if it has the largest rank among all its neighboring nodes from $V_k$ and it is not marked. Mark all nodes in $M_k$.
5: Assign every node in $M_k$ rank $0$.
6: Update the protection $p(v)$ for every node $v$ in $V_k$ as $p(v) = p(v)+$number of neighboring nodes in $M_k$.
7: $k = k + 1$.
8: **end while**
9: For each node $u$ that is selected in $M_i$, $1 \leq i \leq p$, if node $u$ has only $q$-protection ($q < p$) from $\bigcup_{i=1}^{p} M_i$, we find $p - q$ neighboring non-MIS nodes to protect $u$.
10: Let $M$ be the union of all $M_i$ and all nodes $v$ that are used to protect nodes in $M_i$.

We implement these corrections in our simulations, the results are very similar to the ones in [1]. This indicates (1) the case as the counterexample rarely occurs in random networks; and (2) the corrected algorithms still achieve nice performances in practice.

## VI. Conclusions

This correspondence provided simple corrections of the approximation algorithms in [1] so that they still guarantee the constant approximation ratios for minimum $p$-self-protection problems. The author would like to thank K. Islam for pointing out the problem of our methods and the helpful discussions with him on this correspondence.

## References

[1] Y. Wang, X.-Y. Li, and Qian Zhang, "Efficient Alrogithms for $p$-Self-Protection Problem in Static Wireless Sensor Networks," *IEEE Transactions on Parallel and Distributed Systems*, vol. 19, no. 10, pp. 1426–1438, 2008.

[2] K. Islam and S. G. Akl, "A Distributed Constant Factor Self-Protection Algorithm for Wireless Sensor Networks," in *Proceedings of the 3rd International Workshop on Localized Algorithms and Protocols for Wireless Sensor Networks (LOCALGOS)*, 2009.

[3] D. Wang, Q. Zhang, and J. Liu, "Self-protection for wireless sensor networks," in *Proc. of IEEE International Conference on Distributed Computing Systems (ICDCS)*, 2006.

[4] D. Wang, Q. Zhang, and J. Liu, "The Self-Protection Problem in Wireless Sensor Networks," *ACM Transactions on Sensor Networks (TOSN)*, vol. 3, no. 4, pp.20, 2007.