FINAL EXAM

CIS 5515 Design and Analysis of Algorithms (Spring 2022)

Note: For answer to each question, please explain your answer **in plain English** first. There are a total of 100 points plus 10 bonus points.

NAME:

(**Problem 1**. 20 points, 4 points each) For each of the statements below, determine whether it is **True** or **False**. Briefly explain your answer.

(a) Are the following graph problems for a pair of source and destination known to be in **P**?

- Find a shortest simple path.
- Find a longest simple path.

(b) Are the following graph problems for a pair of source and destination known to be in **NP**?

- Is the length of the shortest simple path $\geq k$.
- Is the length of the longest simple path $= k$?

(c) The following problems are related to **P**.

- The optimal solution for any given problem in P has been found.
- The tight bound has been found for any given problem in P.

Problem 1 Cont'd

(d) Suppose $Y \in$ NP-Complete, and $X \in$ NP, $Z \in$ NP, $X \leq_P Y \leq_P Z$, then

- Z is NP-complete
- Both X and Z are NP-complete

(e) The following problems are related to lower bounds:

- Finding the median is a harder problem (i.e., needs more comparisons) than finding max and min.
- Finding the largest key and the second largest key is harder than finding the second largest key alone.

(**Problem 2**. 20 points) Consider a *survey design* with three customers (X, Y, Z) who can ask 2 or 3 questions about four products (A, B, C, D), each of which requiring 1 or 2 surveys.

1. Represent the problem as a circulation problem and then as a max-flow problem.
2. Briefly describe how a solution is derived from the max-flow network. You only need to show one final result for case 1.
3. Suppose Z cannot survey C and D and each product has to be questioned by exactly 2 customers, is a feasible design still possible?
4. If there is more than one solution for case 3, show all integer solutions.

Problem 2 Cont'd

(**Problem 3**. 20 points) A *hypergraph* is a special graph in which each edge can be connected to more than two vertices. Hypergraph-Vertex-Cover can be described as the following: Given a hypergraph and an integer k, is there a subset of k (or fewer) vertices such that each edge is incident to at least one vertex in the subset?

1. Show that Hypergraph-Vertex-Cover $\leq_P$ Set-Cover.

2. Show that Set-Cover $\leq_P$ Hypergraph-Vertex-Cover.

3. Using your own words to explain why we cannot derive Set-Cover $\leq_p$ Vertex-Cover for a regular graph.

Problem 3 Cont'd

(**Problem 4**. 20 points) In a *load balancing* problem with only two types of jobs in terms of their duration: one has the double length of the other.

1. What approximation bound can you derive for algorithm **Greedy-Balance**?
2. What approximation bound can you derive for algorithm **Sorted-Balance**?

Your bounds should be as tight as possible. Justify your answer for each case.

Problem 4 Cont'd

(**Problem 5**. 20 points) Show that the bound for finding a *king* is more than 4 games in a tournament of 4 players. You can use facility-allocation-type augment to show that the adversary can always force the programmer to play more than 4 games in order to decide a king. Your augment should cover all cases, under all possible strategies used by the programmer.

Problem 5 Cont'd

(**Bonus problem**, 10 points) The following problems are related king in a tournament.

1. Find an n-player tournament outcome such that all players are kings. Show such an example for $n = 7$.
2. Give an informal augment to show that for a given random and complete n-player tournament graph, there are many kings. By random, we mean that for each edge its orientation is random.