## Optimizing Carpool Scheduling Algorithm through Partition Merging

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## Carpool scheduling problem

- Target: minimizing carpools needed, each user has distinct src $s$ and dest $d$
- Detour constraint $\alpha$, e.g. no more than $\alpha=20 \%$ of the shortest path
- Capacity constraint $\boldsymbol{k}$, vehicle has capacity limitation, e.g. $k=2$
- NP-hard problem: a special case can be reduced to Hamilton tour



## Motivation



- Another user can re-carpool with drivers after previous user got off
- A 4-people carpool D - A -A' -B B' $-\mathrm{C}-\mathrm{C}$ ' $-\mathrm{D}^{\prime}$ with $k=2, \alpha=20 \%$
- The minimum number of carpools needed is 1


## A Greedy Solution

- Based on component merge
- Initialization, each component contains only one element

$$
\mathrm{A}-\mathrm{A}^{\prime} \mathrm{B}-\mathrm{B}^{\prime} \quad \mathrm{C}-\mathrm{C}^{\prime} \quad \mathrm{D}-\mathrm{D}^{\prime}
$$

- Each component is a local sequence of $s$ and $d$. $d$ always appears after corresponding $s$.

$$
\mathrm{D}-\mathrm{A}-\mathrm{A}^{\prime}-\mathrm{B}-\mathrm{B}^{\prime}-\mathrm{C}-\mathrm{C}^{\prime}-\mathrm{D}^{\prime}
$$

- Merge: Two components can be merged if all elements can be combined that satisfies capacity constraint $k$ and detour constraint $\alpha$.
- Construct a component matching graph,
- vertex: component
- edge: two mergeable components.

- Maximum matching on the component graph to generate new graph
- Repeat maximum matching on the new graph until convergence.


## Merge Methods

- E.g. S: A- B- B'- A'; S': C- D- D'- C'
- Simple merging (SPA) ${ }^{1:} \mathrm{O}\left(\mathrm{n}^{2.5}\right)$
- Full permutation (PMA): $\mathrm{O}(\mathrm{n}!$ )

D-A - B - C-A'-B' - C' - D'
not properly nested

- Driver-alone insertion (PMAD): $\mathrm{O}\left(\mathrm{n}^{2.5}\right)$

properly nested
- General insertion (PMAG): $O\left(\mathrm{n}^{2.5}\right)$ $\xrightarrow[\text { seat }+2]{\mathrm{A}-\mathrm{B}-\mathrm{B}^{\prime}-\mathrm{A}^{\prime}} \xrightarrow{\mathrm{A}-\mathrm{D}-\mathrm{D}^{\prime}-\mathrm{C}^{\prime}}, \stackrel{\mathrm{B}^{\prime}-\mathrm{A}^{\prime}}{\mathrm{C}-\mathrm{D}-\underset{\mathrm{D}^{\prime}-\mathrm{C}^{\prime}}{\text { seat }}+2}$ properly nested
$+1+2+1$
$+1+2+1$
$\mathrm{S}: \mathrm{A} \stackrel{\downarrow}{ } \stackrel{+}{-}-\mathrm{B}^{\prime}-\mathrm{A}^{\prime}$
$S^{\prime}: C-D \stackrel{\downarrow}{-}{ }^{\downarrow} \stackrel{\downarrow}{-}$,
(a) $\mathrm{S}^{\prime}$ inserted to S
(b) S inserted to $\mathrm{S}^{\prime}$


## Improvement

- In Euclidean space, matching eligibility via geometry properties

feasible area: $d_{1}+d_{2}=(1+\alpha)\left|A A^{\prime}\right|$


## Simulation Results

- Synthetic dataset: $s$ and $d$ locations are individual and range from 0-30 miles in 2-D space.
- Real-world dataset: $s$ and $d$ are extracted from traces of NYC cabs

| NYC Yellow Cab Trip Record Data |  |
| :---: | :---: |
| Time span | $01 / 01 / 2017$ to $01 / 31 / 2017$ |
| Avg. requests/min | 21662 |
| Avg. travel time | 14.92 mins |
| Avg. trip distance | 2.831 miles |
| Avg. passenger counts | 1.6 |








- 1. F. Buchholz, "The carpool problem," 1997.

