Online to Offline Business: Urban Taxi Dispatching with Passenger-Driver Matching Stability

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Road Map

- Introduction & Motivation
- Problem Formulation & Analysis
- Non-sharing taxi dispatching
- Sharing taxi dispatching
- Experiments
- Conclusion
Introduction & Motivation

Traditional taxi business
- Taxi belongs to the company
- Driver’s interest is ignored during taxi dispatch

Online to offline taxi (e.g., uber)
- Taxi does not belong to the company
- Driver’s interest must be considered during taxi dispatch
- Balanced interest between passengers and drivers
Introduction & Motivation

Taxi dispatch (i.e., matching)
- Time-window-based dispatch \( \rightarrow \) match in snapshot
- Non-sharing: one request \( \rightarrow \) one taxi
- Sharing: multiple requests \( \rightarrow \) one taxi

Challenges
- Matching fairness between passengers and drivers
- Complexity and scalability of dispatch
Problem Formulation & Analysis

Taxi dispatch Scenario

- A set of taxis, $T = \{t_i\}$
- A set of passenger request, $R = \{r_j\}$
- $r_j^s$ and $r_j^d$: pick-up and drop-off locations of $r_j$
- Distance function, $D(\cdot, \cdot)$

Formulation by matching

- Stable marriage approach
Problem Formulation & Analysis

Extended stable marriage (taxi and request)
- # of taxis does not equal to # of requests
- Allow empty/dummy match (i.e., taxi not dispatched)
- Preference list includes empty/dummy entry

Classic stable marriage (man and woman)
- # of men does equal to # of woman
- Does not allow empty/dummy match
- Preference list does not include empty/dummy entry
Extended stable marriage (taxi and request)

- Each taxi (request) ranks requests (taxis) as its preference list, including an empty/dummy entry.

Extended stability

- A matching (i.e., taxi dispatch schedule) is stable, if an arbitrary matched passenger and another arbitrary matched driver will not prefer each other over their existing partners (including dummies partners, and dummies always prefer non-dummies over dummies).
Problem Formulation & Analysis

Extended stable marriage (taxi and request)

Theorem 1: A stable matching exists, if exact one dummy entry exists in the preference order of each passenger request and that of each taxi.

- Proof idea: convert extended stable marriage to traditional stable marriage by adding dummies

- Schedulability for taxi dispatch is guaranteed
Non-sharing Taxi Dispatch

One request -> one taxi

- Two sub-algorithms: proposal and refusal

Proposal sub-algorithm

- Request proposes to taxi (terminated at dummy)

Refusal sub-algorithm

- Taxi accepts proposals that are better than current, and refuses current (otherwise refuse proposal)
Non-sharing Taxi Dispatch

Proposal sub-algorithm

- Request proposes to taxi (terminated at dummy)

1: Sub-algorithm Proposal
2: **Input**: passenger request $r_j$.
3: **if** the next entry in $r_j$’s preference order is non-dummy (let $t_i$ denote this entry) **then**
4: Update $r_j$’s current entry to be $t_i$.
5: Call sub-algorithm Refusal for $S^*$, $t_i$, and $r_j$.
6: **else**
7: $r_j$ is unserved (no taxi dispatch), and update $S^*$. 
Non-sharing Taxi Dispatch

Refusal sub-algorithm

- Taxi accepts proposals that are better than current, and
  refuses current (otherwise refuse proposal)

8: Sub-algorithm Refusal
9: Input: schedule $S^*$, taxi $t_i$, and passenger request $r_j$.
10: if $t_i$ is undispatched and prefers $r_j$ over no dispatch then
11: Dispatch $t_i$ to $r_j$, and update $S^*$.
12: else if $t_i$ is dispatched to $r_{j'}$, but prefers $r_j$ over $r_{j'}$ then
13: Dispatch $t_i$ to $r_j$, and update $S^*$.
14: Call sub-algorithm Proposal for $r_{j'}$.
15: else
16: Call sub-algorithm Proposal for $r_j$. 
Non-sharing Taxi Dispatch

Non-sharing taxi dispatch

- Request’s preference: distance to taxi (waiting time)
- Taxi’s preference: distance to taxi (waiting time) and distance from pick-up to drop-off (carry distance)

Non-Sharing Taxi Dispatch

Compute preference orders for $\forall r_i \in R$ and $\forall t_j \in T$ based on $D(t_i, r_j^s)$ and $D(t_i, r_j^s) - \alpha D(r_j^s, r_j^d)$, respectively.

Initialize each taxi in $T$ as undischached.

for each passenger request, $r_j \in R$ do
  Call sub-algorithm Proposal for $r_j$.
return $S^*$ as a passenger-optimal taxi dispatch schedule.
Non-sharing Taxi Dispatch

Algorithmic example

(a) Initialization.

(b) $r_1$’s proposal.

(c) $r_2$’s proposal.

(d) $r_3$’s proposal.
Non-sharing Taxi Dispatch

Property 1: In the stable matching obtained by Algorithm 1, if a taxi prefers no dispatch over all passenger requests, then it will not be dispatched. Similarly, if a passenger request prefers no service over all taxis, then it will not be served.

Property 2: Among all stable matchings, the stable matching obtained by Algorithm 1 satisfies that passenger requests have their best partners, but taxis have their worst partners.

Theorem 2: In the passenger-optimal stable matching obtained by Algorithm 1, if a passenger request is unserved, it is unserved in all possible stable matchings.
Theorem 3: Given a set of request and a taxi, it is NP-hard to compute a directed shortest path that goes through the pick-up location before the drop-off location for each passenger request.

- Reduction from Shortest Hamiltonian Path Problem
- In practice, only pack 2 to 3 requests to a taxi
- Exhaustive search is used to determine route
Sharing Taxi Dispatch

Packing 2 to 3 requests

- Exhaustively compute all possible request sets
- Eliminate bad sets by threshold
- Use minimum set packing to pack requests

\[
\begin{align*}
\text{maximize} & \quad \sum_{k} x_k \\
\text{subject to} & \quad \sum_{k: r_j \in e_k} x_k \leq 1 \text{ for } \forall j \\
& \quad x_k \in \{0, 1\} \text{ for } \forall k
\end{align*}
\]

- Taxi’s and passenger’s interests need to consider sharing
Experiments

Real data-driven

- New York trace: 1,445,285 requests and 700 taxis
- Boston trace: 406,247 requests and 200 taxis
- Taxi speed: set to 20km/h
- Dispatch window: 1 minute

Comparison algorithm

- Nearest, Hungarian, SCRAM (minimize maximum)
Experiments

Non-sharing taxi dispatch (New York trace)

- Passenger’s dissatisfaction slightly increases, but taxi’s dissatisfaction significantly decreases
Experiments

Non-sharing taxi dispatch (Boston trace)

- Passenger's dissatisfaction slightly increases, but taxi's dissatisfaction significantly decreases.
Conclusion

Online to offline taxi (e.g., uber)
- Balanced interest between passengers and drivers
- Extended stable marriage approach (enable dummy)

Non-sharing and sharing taxi dispatches
- Taxi’s / request’s interest (waiting time & carry distance)
- Pack passenger requests to a taxi
- Certain algorithm properties
End

Questions & Answers