Efficient Symbol-Level Transmission in Error-Prone Wireless Networks

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Agenda

- Introduction
- Motivation
- Setting
- Proposed methods
  - Single packet
  - Multiple packets
  - Multiple packets with network coding
- Simulation results
- Conclusion
Introduction

- Broadcasting in wireless networks
  - Disseminating data and control messages

- Error-prone wireless links
  - Provide reliability
    - ARQ
    - Hybrid-ARQ
    - Erasure codes
    - Fountain codes (rateless codes)
Introduction

- Errors in packets
  - Not binary
- Numeric data
  - Like sensed data by sensor nodes
  - The important of the symbols (bits) are different
    - The importance of the symbols should be considered
- Choices
  - Reliable transmissions
  - Maximizing the expected gain with a fixed given number transmissions
Motivation

\[ w_1 = 2 \quad w_2 = 1 \]

Packet

\[ u = w_1 \times (1 - p^{x_1}) + w_2 \times (1 - p^{x_2}) \]

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>1.28</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1.2</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>0.64</td>
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</tbody>
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<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0</td>
<td>1.568</td>
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<tr>
<td>2</td>
<td>1</td>
<td>1.68</td>
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<td>1</td>
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<td>1.44</td>
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<tr>
<td>0</td>
<td>3</td>
<td>0.78</td>
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</table>

2 transmissions

3 transmissions
Setting and Objective

- One-hop network
- Lossy links
- Transmission window size
  - $t$ slots for a packet

- Objective: maximizing the total weight of the received symbols
The case of a packet size equal to 2 symbols

\[ u = w_1 \times (1 - p^{x_1}) + w_2(1 - p^{x_2}) \]

st. \[ x_1 + x_2 = t \]

\[ w_1 = 5 \]
\[ w_2 = 1 \]
Single Packet (One Destination)

- We consider the problem in rounds of transmissions
- The first time we should increment $x_2$ is when
  \[ p^{x_1} < \frac{w_2}{w_1} \]
- After the saturation point, the distribution of the transmissions has a *round-robin* incrementing pattern
- The proof of optimality is provided in the paper
Single Packet (One Destination)

- Generalizing to $m$ symbols
  - We assign the transmissions to $x_1$ until $p^{x_1} < \frac{w_2}{w_1}$
  - Then, we distribute the transmissions between $x_1$ and $x_2$ until $p^{x_1} < \frac{w_3}{w_1}$ and $p^{x_2} < \frac{w_3}{w_2}$
  - After this point, we continue the round-robin pattern among $x_1$, $x_2$, and $x_3$

In general, we start incrementing when $x_j$

\[ p^{x_i} < \frac{w_j}{w_i}, \forall i, 1 \leq i \leq j, j = 1 \]

The proof of optimality: in the paper
Single Packet (Multiple Destinations)

- In the case of different transmission error rates, the round-robin pattern does not exist
- Iterative algorithm
  - We assign the transmissions to the symbols in $t$ rounds
    \[
    \Delta x_i = w_i \times \sum_{l=1}^{n} \left[ 1 - p_{l}^{x_i+1} - (1 - p_{l}^{x_i}) \right] = w_i \times \sum_{l=1}^{n} \left[ p_{l}^{x_i} - p_{l}^{x_i+1} \right]
    \]
- At each iteration we assign the current transmission to the symbol with maximum $\Delta x_i$
Multiple Packets

- Our model
  - The size of the packets are equal
  - The weights of the $i$-th symbols in different packets are the same

- The problem of sending $k$ independent packets becomes $k$ similar problems with the same solution

- We can solve the problem for a single packet, and repeat it for any packet
Multiple Packets- with Network Coding

- We first find the optimal $\alpha_i$
- We code all of the $i$-th symbols of the $k$ packets together
  - Instead of sending the $i$-th symbols of each packet $\alpha_i$ times, we send $\alpha_i \times k$ coded symbols

![Diagram showing multiple packets and their coded symbols](image_url)
Multiple Packets- with Network Coding

- Using network coding might increase or decrease the gain
  - Since partial decoding is not possible
  - For each set of the $i$-th symbols we compare the gain of coding and non-coding
    \[
    u_i^{NC} = w_i \times k \times \sum_{l=1}^{n} \left[ \sum_{j=k}^{x_i \times k} (k \times x_i) \times (1 - p_l)^j \times p_l^{x_i \times k - j} \right]
    \]
    \[
    u_i = w_i \times k \times (1 - p_l^{x_i})
    \]
  - We turn off coding if it decreases the gain
Simulations Setting

- MATLAB environment
- 1,000 random topologies
  - Different links’ error rates
- Weight of the i-th symbol: $2^{m-i}$
- Compare with simple retransmission method
  - Distribute the transmissions evenly to the different symbols of the packets
Simulations - (Single Destination)

- Single packet - 10 symbols

\[ p = 0.3 \]

\[ p = 0.5 \]
Simulations- (Multiple Destinations)

- Single packet- 10 symbols
- 10 transmissions

\[ p \in [0.2, 0.4] \]

\[ p \in [0.2, 0.6] \]
Simulations

- Packet size: 5 symbols
Simulations

- Packet size: 5 symbols
- 5 destinations
  \[ p \in [0.3, 0.5] \]
Simulations Summary

- Our proposed MPT mechanism can increase the gain up to 22% compared to that of a simple retransmission mechanism.

- Our network coding scheme enhances the expected total gain up to 45% compared to the simple retransmission mechanism.
Summary

- There is much work on reliable transmissions over error-prone wireless channels
- We propose a novel transmission scheme which is based on the importance of the symbols (bits)

- Proposed methods
  - Single packet
  - Multiple packets
  - Multiple packets with network coding
Questions