Minimizing Epidemic Viral Total Exposure under the Droplet and Aerosol Models

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***Abstract*—In recent years, especially after the Coronavirus pandemic, extensive research has been conducted to propose models for the spread of viruses in social networks, and to come up with viable techniques to reduce the propagation of viruses. In this paper, we propose a new general time-evolving graph model that is suitable to be applied to viral spread propagation studies. Furthermore, with a focus on a rare type of infecting mode called the aerosol model, which turns out to be one of COVID's transmission types, we study the simple problem of minimizing the total exposure of a virus within a group of people that visit a place or set of places successively. An extensive simulation is conducted to examine the efficiency of our viral-minimizing spread technique and to compare it to other possible scenarios of the behavior of the population.**

***Index Terms*—Aerosol transmission, droplet transmission, time-evolving graphs, virus propagation.**

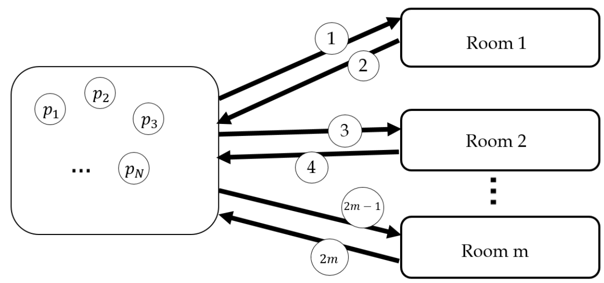

Fig. 1: An illustration for the toy example of the problem, the order of the rooms can changed, and each person can be assigned a specific appointment to visit the rooms. The numbers in the circles represent the order of the step.

I. Introduction

The problem of minimizing the spread of viral infections in daily life has been studied widely in the recent years [1–4]. Models have been proposed to imitate the propagation of viruses within a society. However, the direct droplet infection, which happens by direct contact, is the one mainly considered with little attention to the indirect contact infection that happens via aerosol spreading.

Usually, a person may infect another person by the direct droplet model if they have a direct contact during a period of time (say, within 6 feet during a duration of 15 or more minutes). It was found that SARS-COV-2, the virus that causes the Coronavirus disease COVID-19, can be transmitted by the aerosol model. A person who carries the virus visits a closed room and leaves behind a particle enclosed under pressure suspended in the air, this particle carries an active virus that may infect another person if exposed to it within a short period time (i.e. within 5 minutes) with a high probability, even if the main infecting person is not present in the closed room anymore. It has been speculated that the indirect aerosol particle can stay in the air for as long as one hour [16].

The aerosol model implies that after an infected person leaves the room, it is still possible to infect another person even if they do not have contact directly. In general viral spreading research, the aerosol model was not studied extensively due to the fact that it is not a dominating mode of transmission among other types of viruses [16, 17]. This means that after the main infected person leaves the room, it is still likely that they infect another person who visits the empty room within the next 5 minutes after the first visit. The fact that this type of infection does not dominate in usual viral propagation schemes in social networks and its negligible effect in general leads to the disregard of this way of spread in general cases.

A toy example of the problem is having the case of managing m rooms that a set of N people will visit at different steps as illustrated in Figure 1, each person has an an initial probability of holding the virus. Given that your responsibility as the room manager is to minimize the total final viral exposure within the served population after visiting the rooms, we investigate the optimal time assignment of the population to give them appointments to visit the rooms within the specified periods of time.

We consider multiple cases of having the people visit the room for a relatively short period of time, say instantaneously or for 3 minutes so that there are no contacts among the people inside a room, and for a relatively longer period of time, say 20 minutes, where people have direct contacts. We consider the multiple possibilities to the visitation time for each person and the order of the rooms. Both the aerosol model, which will be dominant in the first case of short-period visiting, and the direct droplet infecting model, which dominates in the second case of long-period visiting, are considered within our model.

In our study here, we consider a simple and extreme case to focus on the aerosol model in viral propagation between several people, as well as considering the droplet model. We start by studying the problem of having multiple people with an initial virus exposure probability aiming to visit a closed room (e.g. a closed restaurant applying COVID restrictions for

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preventing indoor dining) instantaneously or for a short period of time to pick up something (e.g. a food order). Hence, the main way of being exposed to the virus is by catching the aerosol from a previously infected person who has visited the room. The room has its own starting and finishing time.

After that, we consider a more general problem of having those people visit a sequence of rooms, with each room having a specific available period of time to be visited in. The order of the availability of the room and the times at which each person visits each room are the concern of the study to try to minimize the total virus exposure within the population.

Our results in this research study are summarized as follows:

- We propose a general time-evolving graphs model that is appropriate to represent the viral propagation scheme under both the direct droplets and indirect aerosol ways of infection.
- We evaluate the optimal schedule for the population to instantaneously visit a specific room so that the total viral exposure is minimized.
- We evaluate the optimal schedule for both the population and rooms available to be instantaneously visited under the objective of minimizing the total viral exposure.
- Simple polynomial-time algorithms are presented to compute the optimal solutions for the problems.

The remainder of the paper is organized as follows. In Section II, some related works are reviewed. In Section III, the novel model of time-evolving graphs that are appropriate to describe the aerosol and droplet models in the same time is proposed with a focus on our concerned model. In Section IV, various versions of the problem are explored from the simplest one to the more general one. In Section V, the more general problem of having multiple rooms is explored and a corresponding algorithm is provided. In Section VI, simulation results are presented to evaluate the efficiency of our solution. Finally, Section VII gives the conclusion.

II. Related Work

The significant effect of huge social disruption and economic impact of various different epidemic types that propagated worldwide has been widely studied [1–4]. The current COVID19 pandemic has specifically had one of the most disastrous impacts on the world [2, 5–8]. Hence, several studies have been conducted in order to try to understand the main epidemic outbreak models as well as to come up with good strategies to halt the eruption and spread of epidemics [9–11].

Even though there has been extensive research done in the last decades in order to determine the exact way influenza-like viruses spread, it remains not completely understood how those viruses, including the Sars-Cov-2, propagate among people. Typically, the transmission of those viruses has been assumed to happen mainly by the air, direct physical contact, and being exposed to contaminated surfaces [12, 13]. The traditional airborne transmission happens mainly in two different ways: either by droplets, which are large particles of respiratory fluid, or by smaller aerosolized particles that remain held in the air.

Tellier [14, 15] has made a study that implied that direct droplet transmission requires close physical proximity between infected and susceptible individuals since the gravity quickly pulls down larger droplets to the ground, whereas aerosol transmission can happen over larger distances and does not necessarily need to have the infected and susceptible individuals at the same location at the same time.

Until recently, all of the research done considers the close contact transmission as the dominant infecting way, because largely the evidence to support the importance of transmission through aerosols was minimized. However, the question of the importance of the multiple transmission routes has recently received greater attention. Multiple research studies have provided good evidence for the importance of aerosol transmission specifically in the past few years [16–18]. Fruthermore, among various optimization problems related to epidemic control, Zheng *et al* studied a way to open up different sectors of the society to maximize the social welfare subject to a given threshold of epidemic spreading [19].

Noti [16] has conducted an experimental laboratory study using a patient examination room containing a coughing manikin which provided further support for aerosol transmission. Lindsley [20, 21] has conducted a study with outpatients who tested positive for influenza A virus and demonstrated that 53% and 42% produced aerosol particles containing viable influenza A virus during coughing and exhalation, respectively.

III. Time-evolving networks

In this section, we introduce a new model for time-evolving networks. We will construct a novel enhanced time-evolving graph that shows information about both frequency and duration of interactions along both time and space. Our timestamped contact network tracks the frequency and duration of all interactions within a specified radius. Therefore, we propose two additional measures for time-evolving graphs to represent two transmission modes: time-sensitive connectivity/reachability and enhanced time-evolving graphs.

A. The construction of time-evolving graphs

First, we will construct a time-evolving graph to represent our contact network. Let $G(V, E)$ be a graph where V is the set of vertices (i.e., nodes) and E is the set of edges (i.e., links). $G_1, G_2, \ldots, G_T$ is an ordered sequence of spanning subgraphs for time sequence (for simplicity, we use the time sequence $1, 2, \ldots, T$). G_i is a subgraph during the time unit i, where all edges are labelled with timestamp i.

The time-evolving graph is the union of these subgraphs. Figure 2 shows an example of a direct contact (say within 6 feet) based on a given transmission range on a campus with a focus on venues (i.e., a classroom, club, dormroom) where people interact. Figure 2 (a) and (b) show snapshots at two different times, t_1 and t_2, respectively, where (c) shows the

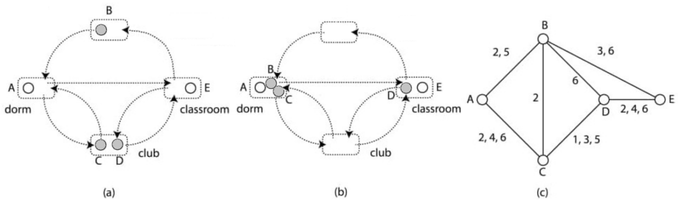

Fig. 2: (a) node distribution at t_1 (b) node distribution at t_2 (c) time-evolving graph over the time period $[t_1, t_6]$. White circles denote static nodes. Gray nodes denote moving nodes.

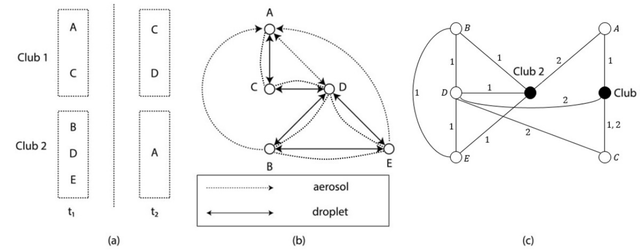

Fig. 3: (a) Club room occupancies at t_1 and t_2 (b) droplet and aerosol transmission (c) enhanced time-evolving graph with the venue node (in bold) where the clique of nodes B, D, and E can be replaced by a hyperlink to further simplify the graph.

corresponding time-evolving graph during the period between t and t_6. For example, as seen in (a), at t_1, node A is in a dorm, nodes C and D interact with one another in a club, and E is in a classroom. In (a), nodes B, C, and D are moving in different cycles and nodes A and E are stationary.

As seen in (b) at t_2, node B moves to the dorm (shown in the dotted arrow line) as a cart of a circular movement between three locations. That is, at t_3 (not shown in the figure), B moves from the dorm to the classroom and at t_4, back to the original place at t_1. At t_2, node C moves from the club to the dorm and node D moves from the club to the classroom. All moves are made within one unit of time, followed by the next move at the beginning of the next time unit.

The time-evolving graph representing all of the interactions over the 6-unit time period is seen in (c). In time-evolving graphs, nodes u and v are said to be connected, if a path $u \rightarrow v$ exists with an alternation of nodes and links, starting from u and ending with v. Links along the path from u to v are associated with labels with increasing timestamps. For example, in (c), A can reach E via B through multiple paths including $A \xrightarrow{5} B \xrightarrow{6} E$ or via C and A can reach D through multiple paths including $A \xrightarrow{4} C \xrightarrow{5} D \xrightarrow{5} E$. As seen in Figure 3 (a), different sets of nodes can occupy the same venue (i.e., club rooms 1 and 2), at different times, t_1 and t_2, in a spatiotemporal pattern.

To simplify our discussion, all individuals in the same room are assumed to be in a proximity close enough to all others in the room to spread the disease through respiratory droplets as shown in solid lines in (b) (figure adapted from Smiezek *et al*). With aerosol transmission, infected nodes can shed viral particles while in the room, which may infect others in the room concurrently or up to three hours later (i.e., $\Delta < 2.5$ hours). Assuming $t_2 \leq t_1 + \Delta$, the unidirectional arrow in (b) shows the additional aerosol transmissions.

We extend the time-evolving graph by adding a *venue node* shown in (c) to represent club 2 (only the subgraph for club 2 is shown). A potential transmission path still requires an increasing timestamp sequence. In addition to that, the time difference between when one node exits a venue and another enters should be within Δ.

We will use a venue node to avoid edge explosion - when there are many people in a club at two sequential timestamps (e.g., if there are two different sets of 25 people in Club 2 at t_1 and t_2, which would generate $25 \times 25 = 625$ aerosol links between unique individual nodes). Using a venue node (c) reduces the number of links significantly to $25+25 = 50$ links (i.e., 50 individuals, each of them connected to one venue).

B. *Enhancement on the time-evolving graph*

The enhanced time-evolving graph has a similar definition to the time-evolving graph, except that each link at t_i is associated with a weight w_i which corresponds to the timestamp. Here, weight w_i is associated with contact duration. We can extract, of *slice*, any subgraph with link timestamps satisfying a particular time or time interval. For example, one can get a subgraph that includes all contacts within a contact duration range. In our analytical extension, we look at the node degree at a particular time slot to calculate the average node degree during that time or time interval. For example, as seen in Figure 2, C's node degree at $t_2 = 2$. C's average node degree during $t_{[1,6]} = \frac{7}{6}$.

We also introduce a new metric, *spread*, for social network analysis. The spread of a given source at t_i corresponds to the connectivity/reachability set which includes all nodes along any path initiated from the source at t_i. We can consider various types of spread by either restricting the final time to t_j, i.e., spreading over the period of $[t_i, t_j]$ or average spread of the source during a given period. Spread measures the reachability of a given node to other nodes. A node can be reached multiple times via different or the same neighbors.

In the example of disease spreading, the spread in a given period measures the disease spread of a particular source across the network. The actual spreading of diseases depends on the compartmental model selected in epidemiology. This indicates that the enhanced time-evolving graph captures both highly dynamic and irregular mobile contact patterns in the real world in a static graph representation. The time-evolving dynamics are captured by labels associated with each link. Given that classes, club meetings, and time spent in one's dorm tend to occur on a similar schedule each week, the label sequence can be condensed. Furthermore, link timestamps can be aggregated to present any time interval (i.e., day, week, month, etc.) depending on the need. Nodes can also be clustered to form a community based on a particular feature (e.g., college freshmen), where link characteristics to outside communities are derived from a particular aggregation function on all members within the community.

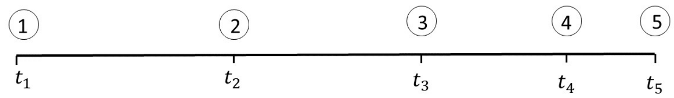

Fig. 4: The optimal distribution of 5 people with equal initial probability under the instant visiting model.

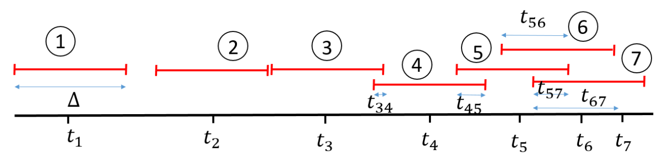

Fig. 5: The optimal distribution of 5 people with equal initial probability under the interval visiting model. τ_2 is high in this example.

IV. Proposed Models

In this section, we introduce our employed aerosol spread models for different settings.

A. Instant Visiting Model

In this subsection, we introduce the model of the problem where N people visit a room within a specific visitation time. Each one of them visits the room in a single different instant of time leaving behind aerosol particles in the air which may infect the person who visits the room afterward. The aerosol effect deteriorates exponentially and aggregates when more than one person contribute in it.

Now, we construct the objective function to optimize after building the model for the given settings. We have the set of moments at which the N people visit the room $T = [t_1, t_2, \ldots, t_N]$, where people are ordered from person 1 to visit the room to the N^{th} person. The initial probability that the people carry the virus is denoted as $p_1, p_2, \ldots, p_N$ respectively. Equation (1) [14] represents the probability that person i infects person j, depending on this aerosol model under the simple exponential approximation; the infection of person j from person i depends on the final probability that i carries the virus, which is calculated from equation (2) [14]. τ_1 depends on the volume of the room and the climate of its air in addition to its humidity and ability to carry the aerosol particles.

$$p_{ij}(T) = \begin{cases} 0.5p_i^f(T) \times e^{-\frac{t_j - t_i}{\tau_1}} & i \neq j \\ p_i & i = j \end{cases} \tag{1}$$

$$p_i^f(T) = 1 - \prod_{j=1}^{j=i} (1 - p_{ji}(T)) \tag{2}$$

$p_i^f(T)$ is the final exposure for person i given the time assignment of the people in the set T. The construction of this model in this accurate way makes the function recursive in its nature. Hence, the function in equation (2) calls the function in equation (1) which calls the function in equation (2), until the base condition $p_{11}(T) = p_1^f(T) = p_1$ is reached. Now, after computing $p_1^f, p_2^f, \ldots, p_N^f$, we will have the optimization problem in equation (3) given the specific domain of the entries of T.

$$\min_T \sum_{i=1}^{i=N} p_i^f(T) \tag{3}$$

Our objective is to minimize the total exposure to the virus. The recursive nature of $p_i^f(T)$ makes no closed-form solution able to be evaluated directly. Running a software for evaluating the exact solution where the initial probability is the same shows the optimal T has both t_1 and t_N fixed on the two extreme ends of the possible time domain of the room, and has the time instances for the people in between distributed in a way such that the time gap between the people decreases with time. Figure 2 illustrates the optimal distribution of people for a toy example.

B. Interval Visiting Model

Here, the studied problem where N people visit a room such that each person visits it once for a fixed range of time, named Δ. Thus, a second way of infecting has to be considered, this way is by droplets of direct exposure. The direct exposure effect between two people depends on their common time visiting the room in the same time.

Now, we denote the common time between person i and person j by t_{ij} (which equals t_{ji}), and we, for simplicity, may consider the presence of the people to be instant for their aerosol effect. Equation (4) [14] shows the probability that person i infects person j with the direct exposure model under the exponential approximation. R is the set of the direct exposure time between every pair of people, which is given by $\{t_{ij} | 1 \leq i \leq j \leq N\}$. τ_2 is the direct exposure factor that depends on the vicinity of the people visiting the room and the air of it.

$$g_{ij}(R) = p_i(1 - e^{-\frac{t_{ij}}{\tau_2}}) \tag{4}$$

Now, to modify the effect from a person to another person, we add the aerosol and the droplet effects under the standard probability model. The probability that person i infects person j under this model is represented in equation (5).

$$p_{ij}(T, R) = \begin{cases} p_{ij}(T) + g_{ij}(R) - p_{ij}(T)g_{ij}(R) & i \neq j \\ p_i & i = j \end{cases} \tag{5}$$

Having the recursive pair between equations (2) and (5), with the small modification of equation (2) in its arguments to include R too, the optimizing problem in (3) will be the same for this model with the inclusion of R. Figure 3 shows the settings of this model for an example. It is worth mentioning that in Figure 4, the transmission from C to D cab be done via aerosol or droplets (a direct link from C to D or a path from C to D through Club 1).

V. The Optimal Solution of the Problem

In this section, we demonstrate the construction of the optimal solution of the problem of figuring out the arrangement of the people to visit a room instantaneously given their initial probability and the time period for the room, which spans from t_1 to t_N. In order to construct the optimal solution, we are required to determine the optimal order of the people to visit the room under the instant visiting model, and then demonstrate the method to evaluate the exact time assignment for each one of the people.

A. Determining the optimal order of the people

Since our objective is to minimize the total average exposure to the virus, the optimal order of the people to visit a room instantaneously turns out to depend on the initial probability of the people regardless of their number and the allowed period of the room.

Theorem 1. *The optimal order of the people visiting a room instantaneously is the decreasing order of their initial probability of carrying the virus.*

Proof. Given an initial clean room, for which the first-visiting person does not have their initial exposure probability p_1 increased, the only cause to increase the probability of exposure for any person under the instant visiting model is the aerosol aggregated from all of the previous visiting people to the room. Hence, $p_1^f = p_1$. This aerosol effect from the previous people depends exactly on two parameters; their probabilities of having the virus p_2 and the moments at which they had visited the room t_2, given that the first visiting-person is already picked.

Now, we assume that we already have the optimal assignment of the moments at which the people would visit the room without knowing the optimal order for them (i.e, knowing the optimal set T that minimizes the total exposure). Having done that makes the only parameter that would minimize the total exposure to the virus $\sum_{i=1}^{i=N} p_i^f(T)$ to be the initial probability of the people visiting the room p_1.

Now, we narrow down our consideration to the individuals and consider them independently. We start from the second person, to have his probability p_2^f increased as little as possible means that the first person will definitely have to be the one with the least initial exposure probability. Moreover, to have the total exposure for those two people minimized, we have to choose the second person to be the one with the least initial probability of the rest of the people.

Considering the third person, to increase their probability p_3^f as little as possible, we would need to have the people visiting the room before them to have the least possible initial exposure probability. Hence, it would mean including the people with the least initial probability and the least total exposure probability, which exactly implies having the optimal solution of two people (the one with the least exposure probability then the one with the second least exposure probability.)

Applying the same argument on the visiting people from the second person to the last one would directly give us the decreasing order of the people's initial probability to be the optimal order of them. □

Having proved the optimal order of the people to be their decreasing order with respect to their initial exposure probability, regardless of their exact probabilities and the time span available for the room, the time complexity to determine this optimal order would trivially be the same for sorting them in decreasing order of their probabilities, which is known to take $O(Nlog(N))$ time where N is the total number of people that will visit the room.

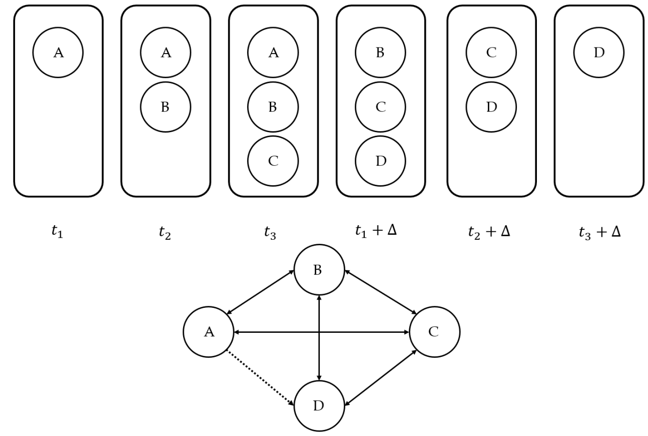

Fig. 6: An example of the interval-visiting settings for the optimal order applied depicted using the proposed evolving graph.

Figure (6) shows an example, people from A to D have initial viral exposures in increasing order. Person A infects person D with the aerosol infecting way; all other interactions between the people are mixed by having them infecting each other using the droplets and the aerosol way together. Δ is the visiting period for each person assuming that they are equal.

B. Determining the optimal time assignment for the people

Since we have already determined the optimal order of the people visiting the room, determining their optimal time assignment would not be more than the optimization mathematical problem shown in equation (3).

Solving the problem for determining T would mean solving the system of partial differential equations of setting the following value to equal the zero $(N-2)$-dimensional vector.

$$\nabla \sum_{i=1}^{i=N} p_i^f(T) = \begin{bmatrix} \frac{\partial \sum_{i=1}^{i=N} p_i^f}{\partial t_2}(t_2, t_3, \ldots, t_{N-1}) \\ \frac{\partial \sum_{i=1}^{i=N} p_i^f}{\partial t_3}(t_2, t_3, \ldots, t_{N-1}) \\ \vdots \\ \frac{\partial \sum_{i=1}^{i=N} p_i^f}{\partial t_{N-1}}(t_2, t_3, \ldots, t_{N-1}) \end{bmatrix} = \mathbf{0} \quad (6)$$

The process of solving this equation can be done analytically or numerically. The T that would result from it is the optimal time assignment for the people to visit the room.

Theorem 2. *The optimal set of visiting times to be assigned for the visiting people T is the one that solves equation (6).*

Proof. Since our objective function to be minimized $\sum_{i=1}^{i=N} p_i^f(T)$ is convex, the closed-form solution that is obtained analytically from directly evaluating the minimum point

Algorithm 1 One-room optimal time assignment

Input: Initial exposure probabilities $\{p_1, p_2, \ldots, p_N\}$ and the availability time of the room $t_N - t_1$.
Output: The optimal time assignment for the visiting people.
1: Sort the probabilities in increasing order and set them to $[p_1, p_2, \ldots, p_N]$.
2: Compute $t_2, t_3, \ldots, t_{N-1}$ from equation (6).
3: **Return** the optimal time assignment T.

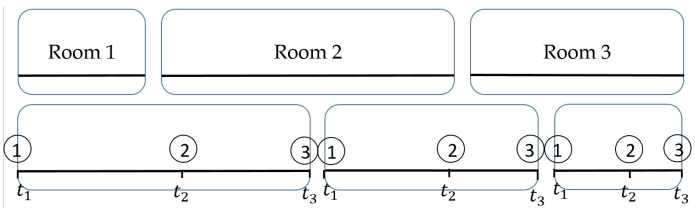

Fig. 7: The optimal distribution of 3 people with equal initial probability under the interval visiting model to visit three rooms with different time periods.

of the function calculating T that sets the gradient of the function to zero directly gives the optimal solution. □

Algorithm 1 is a simple algorithm that evaluates the optimal time assignment for the instant visiting model by which the total exposure to the virus is minimized. The complexity of the algorithm depends on the way chosen to compute the solution of equation (6). Considering the standard $O(N^3)$ solution for the system of linear differential equations gives us a time complexity of $O(Nlog(N) + N^3) = O(N^3)$.

VI. The Problem of Including Multiple Rooms

In this section, we include a more interesting problem of having multiple rooms (or activities) to be visited by each person once. Each room is available for a specific length of time, but no two rooms are available at the same time. The objective is to find the optimal order of activities given a set of people with their initial probability of having the virus and the period of each room's availability.

Figure (9) shows a toy example of the concerned problem where the initial probability of having the virus is the same for all people and nontrivial (i.e., between 0 and 1.) The lengths of the rooms are different as shown. The instant visiting model is considered for this problem.

Theorem 3. *The optimal order of the rooms to be visited instantly is the increasing order of their time period lengths.*

Proof. For a specific number of people with their initial exposure probabilities, visiting a room instantly with a longer time span would result in a less final total exposure to the virus. This is a direct corollary from equation (3) which would minimize the total exposure as $t_N - t_1$ increases.

Now, since we know that the function in equation (3) increases monotonically with the initial total probabilities of exposure for the people visiting a room instantly, it would be apparent that starting with the people with less total initial exposure p_i will never result in a greater total final exposure $\sum_{i=1}^{i=N} p_i^f(T)$ compared to starting with the people with more total initial exposure.

Having determined that the people would leave the room with the longer time period with less total exposure than what they would have if they visit a room with shorter time period, and having determined that starting with a less total initial exposure always results in a smaller total final exposure, we would be able to conclude directly that the optimal order of the rooms is the same order by which the total initial exposure for the people before starting visiting any room is minimized.

Minimizing the initial total exposure for the people before visiting a specific room in this case is equivalent to assuring that the people would visit the rooms with longest period possible before visiting the specific room. Applying that to the rooms from the second to the last one results in having the increasing order of rooms with respect to their time periods to be the optimal order. □

Algorithm 2 Multiple-rooms optimal time assignment

Input: Initial exposure probabilities $\{p_1, p_2, \ldots, p_N\}$ and the availability times of the rooms.
Output: The optimal time assignment for the visiting people and the order of the rooms.
1: Sort the rooms by the decreasing order of their time periods.
2: **For** each one of the rooms **do**
3: Call Algorithm 1 and set the exposure probabilities to $\{p_1^f, p_2^f, \ldots, p_N^f\}$.
4: **Return** the optimal time assignments for all of the rooms.

Having proved the optimal order of the rooms to be their increasing order with respect to their time periods regardless of the people's exposure probabilities and the exact time span available for the room, the time complexity to determine this optimal order would trivially be the same for sorting them in increasing order of their time periods, which is known to take $O(mlog(m))$ time where m is the total number of rooms to be visited instantly.

Algorithm 2 determines the optimal order of the people and the rooms with their time assignments for each room. The total time complexity for this algorithm would again depend on the method of solving equation (6) when calling Algorithm 1, if we opt to solve equation (6) with the simple linear approximation for the involved functions, and taking that approximately solving a system of linear differential equations takes $O(N^3)$ time, the total time complexity for solving the problem in the given equation-solving method would be $O(mN^3 + mlog(m))$.

VII. Simulation

In this section, we conduct simulations to evaluate the effectiveness of the algorithms discussed in this paper.

A. Experimental Settings

In our simulations, we consider multiple scenarios of the possible distribution of the people visiting and different orders

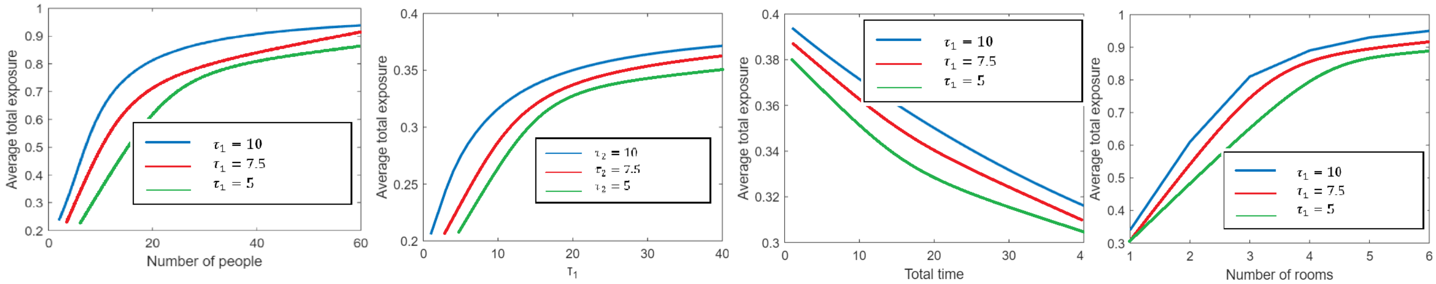

Fig. 8: The average total exposure of the virus under the instant visiting model versus (a) the number of people ($p_i = 0.2$,$t_N - t_1 = 5$) (b) τ_1 ($p_i = 0.2$,$N = 5$,$t_N - t_1 = 5$) (c) total time ($p_i = 0.2$,$N = 5$) (d) the number of rooms ($p_i = 0.2$,$N = 5$,$t_N - t_1 = 5$).

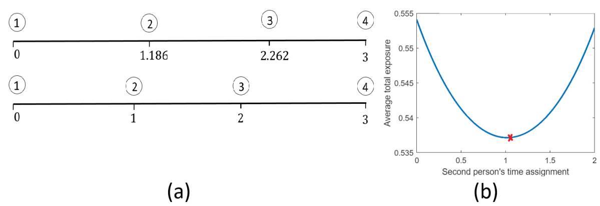

Fig. 9: (a) An optimal and uniform time assignments for four people visiting a room under the instant visiting model, $p_1 = p_2 = p_3 = 0.5$. (b) The average total exposure of three people under the instant-visiting model. $t_1 = 0$, $t_3 = 2$, $p_1 = p_2 = p_3 = 0.5$. The minimum point is pointed out in red.

of them. Studying both the single-room and multiple-rooms scenarios, the main studied orders of the visiting people in terms of their initial exposure probability p_i are:

- Increasing order.
- Decreasing order.
- Uniformly random order
- Clustered random order.

The uniformly random order is the order which assigns each person with a specific initial exposure probability p_i to a random index in a uniform way. The clustered random order is the order which assigns each person with a specific initial exposure probability p_i to a random index in a way that makes it more likely that similar initial exposure probabilities will end up ordered in a successive order of indeces. We consider the clustered random order because this is the most realistic model; if we consider a household of n members visiting the concerned room, the initial exposure probabilities of the members of the household would be highly correlated to the extent that we can consider it to be equal.

After considering the possible orders of the visiting people, we will consider three possible different visiting times of the people:

- Optimal visiting times; which is denoted as the solution set T of equation (6), considering the first and last people to visit the room at the first and last moments. Solving the underlying system of nonlinear partial differential equations has been done numerically.
- Uniformly-distributed visiting times; which are simply the times in which the visiting people are spaced uniformly (i.e., the spacing between them is $\frac{t_N - t_1}{N}$).
- Clustered-distributed visiting times; which are the most realistic, in which a random group of people visit the room nearly together. Finally, for each case, we will consider both the instant visiting model and the interval visiting model.

When we say that a person visits the rooms at time t_0, it means that the person is visiting the room in the interval from $[t_0, t_0 + \Delta]$.

B. Experimental Results

In this subsection, we demonstrate the results of the outcome for the above different configurations of the people. We set the parameters of the exposure model to more realistic settings to represent the affects of the direct and the aerosol infecting models. The parameter τ_1 will be set to 10 minutes, which represents the time needed for the aerosol particles to have diminished affect by $\frac{1}{e}$, i.e., the effectiveness of the aerosol particles becomes around 36.8% from its strongest effectiveness factor after 10 minutes from the moment the affecting person visited.

Parameter τ_2 is set to be 5 minutes, which means that a 5 minutes duration of direct exposure between an infected person and an uninfected person is enough to infect the uninfected person $1 - \frac{1}{e}$ of the time, which is 63.2% of the time. Furthermore, for simplicity, we will set Δ to be 10 minutes always for the duration of the visit to the room.

Figure (9) studies closely a simple room with four people and three people under the instant-visiting model, we will study the average of the total. For four people, the decreasing gap in the time assignment in the people is apparent in contrast to the uniform time assignments case. Figure (9) (b) shows the final exposure for three people as a function of the visiting time of the second person; we see that the optimal time assignment is slightly larger the middle point, which indicated that the uniform distribution of the time assignments is not the optimal one. The minimum point in the figure is the one that solves equation (6).

Another good observation for the instant visiting model is the average total exposure versus the number of people visiting the room within the same time period. The time assignment and the order of the people was set to the optimal assignments.

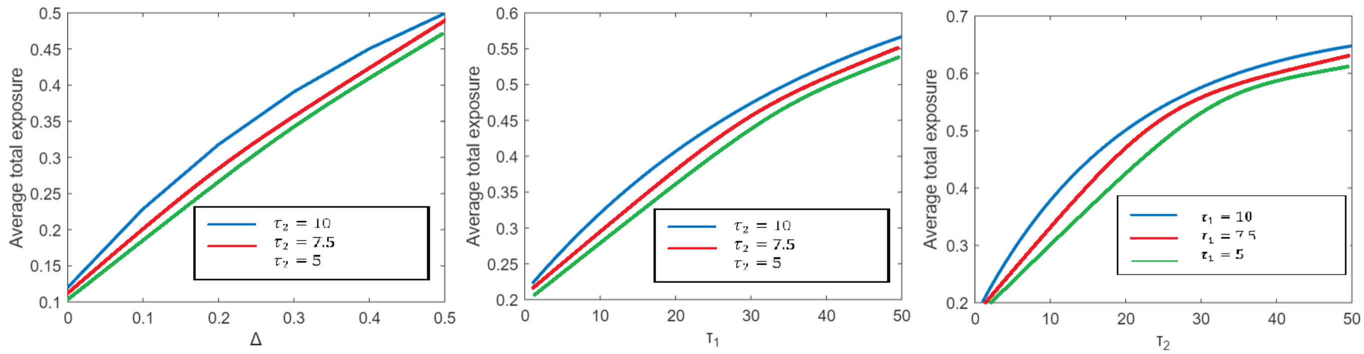

Fig. 10: The average total exposure of the virus under the interval visiting model versus (a) Δ ($p_i = 0.1$,$\tau_1 = 10$,$t_N - t_1 = 5$,$N = 5$) (b) τ_1 ($p_i = 0.2$,$\Delta = 0.3$,$t_N - t_1 = 5$,$N = 5$) (c) τ_2 ($p_i = 0.2$,$\Delta = 0.3$,$\tau_1 = 10$,$t_N - t_1 = 5$,$N = 5$) (d) total time ($p_i = 0.2$,$\Delta = 0.3$,$\tau_1 = 10$,$N = 5$).

	Optimal visiting time			Uniform visiting time			Clustered visiting time		
	$\tau_1 = 1$	$\tau_1 = 5$	$\tau_1 = 9$	$\tau_1 = 1$	$\tau_1 = 5$	$\tau_1 = 9$	$\tau_1 = 1$	$\tau_1 = 5$	$\tau_1 = 9$
Increasing order	0.45	0.51	0.57	0.53	0.59	0.67	0.68	0.70	0.73
Decreasing order	0.65	0.70	0.72	0.69	0.72	0.73	0.71	0.73	0.75
Uniformly random order	0.59	0.62	0.63	0.62	0.64	0.66	0.69	0.71	0.74
Clustered random order	0.62	0.64	0.66	0.74	0.75	0.76	0.76	0.77	0.78

Fig. 11: The average total exposure for 50 people visiting a room under the instant visiting probability within two hours, where their $p_i = 0.35$.

	Optimal visiting time			Uniform visiting time			Clustered visiting time		
	$\Delta = 1$	$\Delta = 2$	$\Delta = 3$	$\Delta = 1$	$\Delta = 2$	$\Delta = 3$	$\Delta = 1$	$\Delta = 2$	$\Delta = 3$
Increasing order	0.57	0.52	0.58	0.54	0.62	0.68	0.69	0.71	0.73
Decreasing order	0.66	0.70	0.73	0.70	0.73	0.73	0.73	0.74	0.76
Uniformly random order	0.61	0.63	0.64	0.63	0.65	0.67	0.70	0.71	0.75
Clustered random order	0.63	0.65	0.67	0.76	0.77	0.78	0.77	0.77	0.79

Fig. 12: The average total exposure for 50 people visiting a room under the interval visiting probability within two hours, where their $p_i = 0.35$.

Finally, we consider the average total exposure under different settings of the single room under both the instant visiting model and the interval visiting model, Figure (9) and Figure (10) show the variety of the average total exposure considering the two different models. From the numbers shows, we can conclude, as expected, that the average total exposure $\frac{1}{N}\sum_{i=1}^{i=N} p_i^f(T)$ is less in all cases under the instant visiting model. We may observe that the uniformly-distributed time assignment for the visiting people, especially under the decreasing and increasing orders of them, is very near to the optimal time assignment that minimizes the total exposure. Another very interesting result is the disparities of the values of the average total exposure, whether under the instant visiting or the interval visiting models, between the optimal order, which is the increasing order, and the worst-case order, which is the decreasing order case.

Figure (8) shows the patterns of the change of our objective function of the average total viral exposure with respect to the different parameters of the number of people, τ_1, and $t_N - t_1$ for the single-room instant visiting model, and the change with the respect to the total number of rooms in the multiple-rooms situation. We may conclude that for the number of people, the total average viral exposure increases significantly at first and then starts to saturate rapidly.

Observing the behavior of the objective function at Figure(8) with the change of tau_1, we can see the gradual increase of the value of the objective function there. On the other hand, the direct nearly-linear correlation between the total average viral exposure and the total time shows the significance of this factor, especially in the case of the single-room scenario. Part (d) of the figure shows the more general sequential multiple-room problem.

Figure (9) shows the most simple case of having three people visit a room within 2 time units, setting τ_1 to unity. The intricate exponential function's extremum lies slightly after the middle point. This unevenness extends to multiple people distribution in a way that shifts the optimal time assignments slightly to the right from the uniform distribution of time assignments.

Figure (10) shows the average total exposure under the interval visiting model of the period of delta. Starting from the first parameter of having the visiting period change, we observe the dominance of the droplet transmission model over the aerosol; having the visiting time increase a little bit in a way that guarantees that some kind of overlap may happen between different visits of the people guarantees that the term $g(R)$ in equation (5) prevails.

Furthermore, we can consider that the correlation between the average total exposure and the change of τ_2, which is the direct droplet model parameter, is stronger than its correlation with the change of τ_1, which is the aerosol model parameter. Moreover, the width of the window plays significant role in affecting the average total exposure until it reaches around 130% of the initial probability. At that point, the value of the objective function saturates.

C. Simulation Summary

In this subsection, we discuss the summary of the previous results and their usefulness in our model.

Starting from the instant-visiting model, we observe from the results the intuitive outcomes of the increasing nature of the total viral exposure with the number of people visiting the room in a specific time window. When the visiting people are sorted in the optimal order, which is the increasing order in terms of their initial exposure probability, the total viral exposure would be within 65% of the more realistic order, which is the clustered random order, considering the best time assignment for both settings. Furthermore, in the multiple-rooms settings, we see how visiting successive multiple rooms increases the total exposure of the virus in different ways depending on the order of the rooms to be visited.

Regarding the interval-visiting model, the results show that the total exposure of the virus increases with the number of people steeply at certain thresholds at which the optimal time assignment would involve significant overlapping times between the visits of the different people in a way that the direct exposure infection (i.e. under the droplet model) starts to prevail over the aerosol infection model. The significant effect of the visiting time for each person Δ is apparent too. Furthermore, in this setting, the optimal order of the visiting people, which is the increasing order in terms of their initial viral exposure, does not significantly differ from the clustered random order because of the dominance of the droplet infecting way. The multiple-room settings show how the total viral exposure increase with the number of rooms to be visited.

VIII. Conclusion

In this paper, we propose a new general time-evolving graph model that is appropriate to apply to viral spread propagation studies. In addition to that, with a focus on a rare type of infecting mode called the aerosol model (which is one of COVID main ways of transmission, beside the main direct infecting way done by the droplets) we study the simple problem of minimizing the total exposure of the virus within a group of people that visit a place or set of places successively. Both instant and interval visiting models are considered. An extensive simulation is done to examine the efficiency of our viral-minimizing spread technique. The results show the disparities in the average total exposure for the virus between the increasing order, which is the optimal order, and other orders, as well as between different time assignments of the visiting people. The simulations show that our optimal order and time assignment reduce the average total exposure for the same settings by around 35%.

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