Towards Federated Learning on Fresh Datasets

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***Abstract*—Federated Learning (FL) is an emerging privacy-preserving distributed computing paradigm that enables numerous clients to collaboratively train machine learning models without the need for transmitting the private datasets of clients to the FL server. Unlike most existing research where the local datasets of clients are considered to be unchanging over time during the whole FL process, we consider such scenarios in this paper where the local datasets of clients need to be updated periodically, and the server can stimulate clients to use as fresh as possible datasets to train their local models. Our objective is to determine a client selection strategy to minimize the loss of global model for FL with a limited budget. To this end, we leverage the concept of Age of Information (AoI) to quantify the freshness of local datasets and theoretically analyze the convergence bound of our AoI-aware FL system. Based on the convergence bound, we formalize our problem as a restless multi-armed bandit problem. Then, we devise a Whittle's-Index-based Client Selection algorithm, called WICS, to tackle the client selection problem. Extensive simulations show that the proposed algorithm can reduce the training loss and improve learning accuracy compared to state-of-the-art algorithms.**

***Index Terms*—Federated Learning, Age of Information, Restless Multi-Armed Bandit, Whittle's Index.**

I. Introduction

Federated Learning (FL) [1] is an emerging and promising distributed machine learning paradigm, which enables a potentially large number of clients to collaboratively train a global model under the coordination of a central server. A standard FL procedure usually consists of a certain number of rounds until a satisfactory global model is obtained. On one hand, FL can efficiently preserve clients' privacy by allowing their training datasets to remain local. On the other hand, since only local model parameters rather than local datasets are sent to the server, FL can greatly reduce communication costs. Due to these advantages, there have been various industrial applications of FL, e.g., WeBank for data analysis in finance and insurance [2], Owkin for biomedical data analysis [3], MELLODDY for drug discovery [4], etc. Meanwhile, much effort has also been devoted to investigating different FL issues [5], such as the convergence rate [6]–[8], accuracy [9]–[12], security [13], [14], and resource allocation [15]–[18].

In most existing works, each client is assumed to hold a dataset in advance and will always use the same dataset to train its local model during the whole FL process. However, in many real-world applications, especially in streaming data scenarios, data are continuously generated along with the time. When participating in FL, clients are encouraged to use as fresh datasets as possible to train local models, since fresh data can more accurately characterize the model parameters. For example, a server coordinates some clients to jointly train an object identification model through FL, e.g., recognizing formulas on literature, identifying traffic signs on photos, etc., where clients can adopt the crowdsourcing technique to periodically recruit mobile users to generate labeled datasets. Intuitively, the fresher the labeled datasets, the more effort needs to be devoted to the data labelling, and thus the labeled datasets will be more precise. In such FL scenarios, clients will inevitably spend some extra costs in providing fresh datasets, but the total budget from the server is generally limited. Thus, an important problem that needs to be dealt with is how to select clients in each round of FL under the limited budget, and the server can minimize the loss of global model.

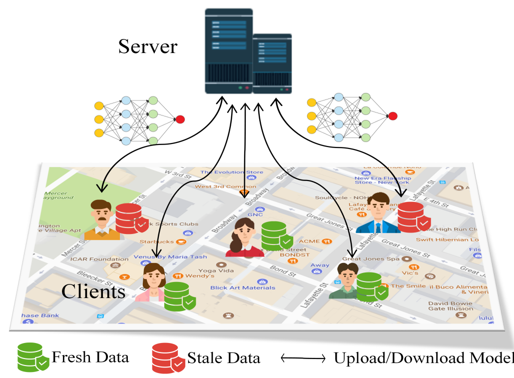

Fig. 1. The architecture of FL training with fresh/stale local data

In this paper, we use the well-known "Age-of-Information" (AoI [19]) metric to indicate the freshness of datasets, which is defined as the elapsed time of data from being collected to being trained for updating local models by clients. Accordingly, the above-mentioned problem is actually instantiated as determining a client selection strategy to minimize the loss of the global model under a given budget, while considering the AoI values of the datasets. Unlike most traditional optimal selection issues with budget constraints, such a problem has two special challenges as follows. Firstly, although the server can reduce the loss of a global model by selecting some clients in each round to update their local datasets and reduce the AoI

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values, there is no obvious quantitative relationship between the loss of the global model and the decrease of the AoI values of clients' datasets. Secondly, the AoI value of each client's dataset will increase along with the rounds of local training and will return to zero until it is selected to update the dataset. Both make the client selection problem much more challenging, especially under the budget constraint.

To address the above challenges, we first derive a convergence upper bound for the novel AoI-aware FL system. On this basis, we transform the optimal client selection problem to minimize the loss of global model to the problem of optimal client selection with minimum average AoI value. Furthermore, we model the problem as a Restless Multi-Armed Bandit (RMAB) problem, where each client is seen as an arm and the AoI values of clients' local datasets are regarded as the corresponding rewards. By solving the RMAB problem, we propose a Whittle's-Index-based Client Selection algorithm, called WICS, in which we calculate the Whittle's Index for each client in each round of FL and then adopt a greedy strategy based on Whittle's Index to select clients, while ensuring the budget is no larger than the given threshold. More specifically, the major contributions are summarized as:

- We introduce an AoI-aware FL system, where the server can select some clients to provide fresh datasets for local model training so as to minimize the loss of the global model under a budget constraint. To the best of our knowledge, this is the first FL work that takes into account the freshness of local datasets for client selection.
- We derive a convergence upper bound for the novel AoI-aware FL system, whereby we analyze the relationship between the training loss of the global model and the AoI values of clients' local datasets. Based on the analysis, we model the client selection problem as a restless multi-armed bandit problem to be solved.
- We deduct the RMAB problem into a decoupled model and theoretically derive the corresponding optimal strategy, based on which we propose the WICS algorithm by applying Whittle's index methodology. In addition, we prove that the WICS algorithm can achieve nearly optimal client selection performance.
- We conduct extensive simulations to verify the performance of our proposed algorithm using two datasets: MNIST and FMNIST. The results show that the performance of WICS is better than comparison algorithms.

The remainder of the paper is organized as follows. In Sec. II, we introduce our model and problem. We carry on the convergence analysis in Sec. III. The WICS algorithm is elaborated in Sec. IV. Then, we evaluation WICS in Sec. V. After reviewing related works in Sec. VI, we conclude the paper. Proofs of theorems are moved to the Appendix.

II. System Overview And Problem Formulation

A. Federated Learning with Data Collection

We consider an AoI-aware FL system, as shown in Fig.1, which is composed of a central server and a set of clients, denoted by $\mathcal{N} = \{1, 2, \cdots, N\}$. In conventional FL systems, the local dataset of each client is generally given in advance and will remain unchanged during the FL process. Unlike these systems, the clients in our system can update their local datasets by spending some costs and can use fresh data to train local models. The fresher the local datasets provided by clients, the more accurate global model will be obtained by the FL system. Besides, the time is divided into T equivalent-length time slots, in each of which the server will conduct a round of federated learning under a budget. For simplicity, we assume the server has the same budget in each round, denoted by B, which can be extended to the case of heterogeneous budgets. Specifically, the whole FL system works as follows.

Firstly, the server selects a subset of clients $\mathcal{N}_t$ $(\subseteq \mathcal{N})$ to update their local datasets at the beginning of each time slot $t \in \mathcal{T} = \{1, \cdots, T\}$. For each client $i \in \mathcal{N}_t$, we denote its local dataset as $\mathcal{D}_t^i$, which can be regarded as the data collected from some fixed Point of Interests (PoIs) or purchased from some preferred data owners by client i. The dataset might be updated on-demand by the client, so that it might vary over different time slots. For simplicity, we assume that the datasets across different time slots remain the same size (otherwise, we can randomly sample the same number of data items from different sizes of datasets), i.e., $|\mathcal{D}_t^i| = |\mathcal{D}_{t'}^i| = n_i$, for any two time slots $t, t' \in \mathcal{T}$. Since each client might spend some costs in obtaining its local dataset, the server will pay a reward, denoted by p_i, to client i as the compensation. Meanwhile, the server publicizes global model parameters, denoted by ω_{t-1}, to all clients for their local trainings. Here, ω_{t-1} is the result of the $(t-1)$-th round of federated learning. Specifically, ω_0 represents the initial global model parameter.

Secondly, each client i $(\in \mathcal{N})$ performs local training after receiving the global model parameter ω_{t-1} from the server. Denote the loss function of local training as

$$F_{t,i}(\omega; \mathcal{D}_t^i) = \frac{1}{|\mathcal{D}_t^i|} \sum_{x \in \mathcal{D}_t^i} f(\omega; x), \quad (1)$$

where ω is the model parameter, $\mathcal{D}_t^i$ is the local training dataset, and $f(\cdot)$ is a server-specified loss function, e.g., mean absolute loss, mean squared loss, or cross entropy loss. Then, based on the received global model parameter ω_{t-1}, client i performs τ steps of mini-batch stochastic gradient descent to compute its local model parameter ω_t^i as follows:

$$\omega_t^{i,k+1} = \omega_t^{i,k} - \eta_t \nabla F_{t,i}(\omega_t^{i,k}; \xi_t^{i,k}), \quad (2)$$

where $k = 0, \cdots, \tau - 1$, $\xi_t^{i,k}$ is the k-th mini-batch sampled from $\mathcal{D}_t^i$, η_t is the learning rate in the t-th round, $\omega_t^{i,\tau} = \omega_t^i$ and $\omega_t^{i,0} = \omega_{t-1}$. Finally, client i uploads ω_t^i to the server.

Thirdly, the server aggregates the received local model parameters to obtain the global model parameter ω_t as follows:

$$\omega_t = \sum_{i=1}^N \frac{n_i}{n} \omega_t^i, \quad (3)$$

where $n = \sum_{i=1}^N n_i$ is the total quantity of training data in each time slot. Then, the server sends the updated global model ω_t back to each client for the next round of local training.

Overall, the global loss function is defined as follows:

$$F(\omega) \triangleq \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^N \frac{n_i}{n} F_{t,i}(\omega; \mathcal{D}_t^i). \quad (4)$$

The goal of the whole FL system is to obtain the optimal model parameter vector ω^* so as to minimize $F(\omega)$, i.e.,

$$\omega^* = \arg\min_{\omega} F(\omega). \tag{5}$$

B. Problem Formulation

In this paper, we focus on the data freshness in FL systems. Inspired by sensing systems, we utilize the concept of AoI to indicate the freshness of the local dataset, which is defined by the elapsed time since the local data was generated. Specifically, let the current round of FL be in the t-th time slot and $u_i(t)$ be the latest update time slot of client i's local dataset $\mathcal{D}_t^i$. Then, the AoI value of $\mathcal{D}_t^i$ (hereafter called client i's AoI, for simplicity of presentation) is represented as

$$\Delta_i(t) = t - u_i(t). \tag{6}$$

Especially, $\Delta_i(0) = 0$ for all clients. Further, the dynamics of client i's AoI can be described as follows:

$$\Delta_i(t) = \begin{cases} \Delta_i(t-1) + 1 \ , & i \notin \mathcal{N}_t, \\ 0 \ , & otherwise. \end{cases} \tag{7}$$

It is worth noting that different client selection strategies will lead to various local data freshness even for the same client. Furthermore, different client selection strategies will also influence the loss of the global model, since the local data freshness affects the quality of local training. Our goal is to minimize the loss of the global model after the whole FL process by selecting the optimal client set $\mathcal{N}_t$ for each time slot $t \in \mathcal{T}$ under the limited budget B. The client selection strategies considered in this paper are non-anticipative, i.e., these are strategies that do not use future knowledge in selecting clients. Let Π be the class of non-anticipative strategies and $\pi \in \Pi$ be an arbitrary admissible strategy. More specifically, we utilize $A^\pi(t) = [a_1^\pi(t), \cdots, a_N^\pi(t)]$ $(t \in \mathcal{T})$ to indicate whether each client is selected in the t-th time slot, i.e., $a_i(t) = 1$ means $i \in \mathcal{N}_t$; otherwise, $a_i(t) = 0$ means $i \notin \mathcal{N}_t$. Then, we can formalize the problem as follows:

$$\mathbf{P1}: \quad \min_{\pi \in \Pi} \ \mathbb{E}[F(\omega_T)] - F^*, \tag{8}$$

$$\text{s.t.} \quad a_i^\pi(t) \in \{0,1\}, \forall i \in \mathcal{N}, \forall t \in \mathcal{T}, \tag{8a}$$

$$\Delta_i(t) = \mathbb{1}_{\{a_i^\pi(t)=0\}} \left[\Delta_i(t-1) + 1\right], \tag{8b}$$

$$\textstyle\sum_{i=1}^N a_i^\pi(t) p_i \le B, \ \forall t \in \mathcal{T}. \tag{8c}$$

Here, ω_T in Eq. (8) is the aggregated global model after T rounds, $F^* = F(\omega^*)$ is the optimal global loss, and $\mathbb{E}[F(\omega_T)] - F^*$ is the gap between the expected global loss after T rounds and F^*. Naturally, the closer $\mathbb{E}[F(\omega_T)] - F^*$ is to zero, the better is the performance of ω_T, Eq. (8a) represents that each client can only be selected at most once by the server for updating its local dataset in each time slot, Eq. (8b) is the reformulation of Eq. (7), i.e., the dynamics of each client's AoI, where $\mathbb{1}_{\{\cdot\}}$ is an indicator function, and Eq. (8c) indicates that the budget constraint in each round of FL. For ease of reference, we list major notations in Table I.

TABLE I
DESCRIPTION OF MAJOR NOTATIONS

Variable	Description
i, t	the index of client and time slot, respectively.
$\mathcal{N}, \mathcal{T}$	the set of clients and time slots, respectively.
$\mathcal{N}_t$	the set of selected clients to in time slot t.
$\mathcal{D}_t^i, \lvert\mathcal{D}_t^i\rvert$	the dataset of client i in time slot t and its size.
$F(\omega)$	the global loss function with parameter vector ω.
$F_{t,i}(\omega)$	the loss function of client i in time slot t.
ω_0, ω^*	the initial and optimal model parameter vector.
ω_t, ω_t^i	the global model parameter vector and the local model parameter vector of client i in time slot t.
n_i, n	the size of client i's local dataset and the total size of all clients' local datasets, respectively.
k, τ	the index and total number of local iterations.
$\xi_t^{i,k}$	the k-th mini-batch sampled from $\mathcal{D}_t^i$.
η_t	the learning rate in time slot t.
$\bar{\eta}, \tilde{\eta}$	the minimum and maximum of the learning rate.
p_i	the payment of client i for obtaining fresh data.
B	the budget of the server per time slot.
$u_i(t)$	the latest update time slot of the dataset $\mathcal{D}_t^i$.
$\Delta_i(t)$	the AoI value of client i in time slot t.

III. Convergence Analysis

To identify how each client's AoI affects the global model, we conduct a rigorous convergence analysis of our AoI-aware FL system. We start with several significant assumptions on the local loss function $F_{t,i}(\omega)$.

Assumption 1 : For all $t \in \{1, 2, \cdots, T\}$, $i \in \{1, 2, \cdots, N\}$, $F_{t,i}$ is β-smooth, that is, for $\forall$ ω_1, ω_2, $F_{t,i}(\omega_2) - F_{t,i}(\omega_1) \le \langle \nabla F_{t,i}(\omega_1), \omega_2 - \omega_1 \rangle + \frac{\beta}{2}\|\omega_2 - \omega_1\|^2$.

Assumption 2 : For all $t \in \{1, 2, \cdots, T\}$, $i \in \{1, 2, \cdots, N\}$, $F_{t,i}$ is μ-strongly convex, i.e., for $\forall$ ω_1, ω_2, $F_{t,i}(\omega_2) - F_{t,i}(\omega_1) \ge \langle \nabla F_{t,i}(\omega_1), \omega_2 - \omega_1 \rangle + \frac{\mu}{2}\|\omega_2 - \omega_1\|^2$.

Assumption 3 : For all $t \in \{1, 2, \cdots, T\}$, $i \in \{1, 2, \cdots, N\}$, the stochastic gradients of the loss function is unbiased, i.e., $\mathbb{E}_\xi[\nabla F_{t,i}(\omega;\xi)] = \nabla F_{t,i}(\omega)$.

Assumption 4 : For all $t \in \{1, 2, \cdots, T\}$, $i \in \{1, 2, \cdots, N\}$, the expected squared norm of stochastic gradients is bounded, i.e. $\mathbb{E}_\xi\|\nabla F_{t,i}(\omega;\xi)\|^2 \le G_i^2 + \Delta_i(t)\sigma_i^2$.

Assumptions 1-3 are widely-used assumptions in many existing convex FL works [8], [20], [21], which ensure that the gradient of $F_{t,i}(\omega)$ does not change arbitrarily quickly or slowly with respect to ω and the stochastic gradients sampled from local datasets are unbiased. It is noteworthy that models with convex loss functions, such as Logistic Regression (LR [22]) and Support Vector Machines (SVM [23]), satisfy Assumption 2. The evaluation results in Section VI show that our algorithm can also work well for the models (e.g., CNN [24]) whose loss functions are non-convex.

Assumption 4, however, is a novel assumption we made for our AoI-aware FL systems. Distinctly from the assumptions made in other FL systems, where those works have assumed that $\mathbb{E}_\xi\|\nabla F_{t,i}(\omega;\xi)\|^2$ is bounded by an inherent bound G_i^2 of client i, we take into account the impact of the clients's AoI on model training. Specifically, we assume the upper bound of $\mathbb{E}_\xi\|\nabla F_{t,i}(\omega;\xi)\|^2$ is positively correlated to $\Delta_i(t)$, and the coefficient σ_i^2 represents the sensitivity of client i's local dataset to freshness. The potential insight is that a smaller AoI value means a fresher local dataset and that better models can be trained, which is consistent with a smaller gradient norm indicating a better model performance when the loss function is convex. In particular, if client i is selected by the server to update its local dataset in round t, i.e., $\Delta_i(t) = 0$, then Assumption 4 will degrade to $\mathbb{E}_\xi\|\nabla F_{t,i}(\omega;\xi)\|^2 \le G_i^2$, which is the same as the assumptions in [8], [20], [21]. It is worth noting that all three loss functions (mean absolute loss, mean squared loss, or cross entropy loss) satisfy Assumptions 1-4.

Theorem 1 (Convergence Upper Bound)**.** *For ease of expression, we define $\bar{\eta} = \min_t\{\eta_t\}$ and $\widetilde{\eta} = \max_t\{\eta_t\}$. Suppose that Assumptions 1 to 4 hold and the step size meets $\bar{\eta} < \frac{2}{\mu}$. Then, the FL training loss after the initial global model ω_0 is updated by Eq.* (3) *for T rounds satisfies:*

$$\mathbb{E}[F(\omega_T)] - F^* \le \frac{\beta}{2}(1-\frac{\mu\bar{\eta}}{2})^T\|\omega_0-\omega^*\|^2 + \frac{\beta}{2}\sum_{t=1}^{T}\sum_{i=1}^{N}\alpha_i\left[G_i^2 + \Delta_i(t)\sigma_i^2\right], \quad (9)$$

where $\alpha_i = \frac{\widetilde{\eta}n_i}{\mu n} + N\widetilde{\eta}\left(\tau^2\widetilde{\eta} + \frac{2(\tau-1)^2}{\mu}\frac{n_i^2}{n^2}\right)$.

Theorem 1 clearly presents the relationship between various factors and global loss in our AoI-aware FL system.

IV. Problem Deduction And Algorithm Design

In this section, we propose the client selection algorithm, called WICS. First, we harness the convergence upper bound to convert the optimization objective of Problem $\mathbf{P1}$. In order to minimize the average AoI value, we then model the AoI minimization problem as a RMAB problem [25]. Next, we relax the RMAB problem and apply the Lagrangian Dual approach to decouple it into subproblems. Then, we obtain the optimal strategy for each decoupled problem. Finally, we derive the closed-form expression for the Whittle's Index and present the detailed algorithm.

A. Using the Convergence Bound to Convert Problem

According to Theorem 1, we obtain the convergence bound of the global model after T rounds. It is not difficult to observe that we can control the convergence of the FL process by controlling the right side of Eq. (9). Therefore, we can convert Problem $\mathbf{P1}$ by minimizing the right side of Eq. (9). After neglecting the constant term, the objective of Problem $\mathbf{P1}$ can be converted as follows:

$$\min_{\pi\in\Pi}\frac{1}{TN}\sum_{t=1}^{T}\sum_{i=1}^{N}\phi_i\Delta_i(t),\ \phi_i = \frac{\alpha_i\sigma_i^2\beta NT}{2}. \quad (10)$$

Note that ϕ_i is dependent on α_i and σ_i^2, and α_i is closely related to n_i. This indicates that the size of the local dataset and its sensitivity to data freshness will affect the client selection of the FL process.

As a result, we can convert Problem $\mathbf{P1}$ as follows:

$$\begin{aligned}
\mathbf{P2}:\quad & \min_{\pi\in\Pi}\frac{1}{TN}\sum_{t=1}^{T}\sum_{i=1}^{N}\phi_i\Delta_i(t), && (11)\\
\text{s.t.}\quad & a_i^\pi(t)\in\{0,1\},\forall i\in\mathcal{N},\forall t\in\mathcal{T}, && (11b)\\
& \Delta_i(t) = \mathbb{1}_{\{a_i^\pi(t)=0\}}\left[\Delta_i(t-1)+1\right], && (11c)\\
& \textstyle\sum_{i=1}^{N}a_i^\pi(t)p_i \le B,\ \forall t\in\mathcal{T}. && (11d)
\end{aligned}$$

B. RMAB Modeling and Solution

To solve Problem $\mathbf{P2}$, we cast it as a Restless Multi-Armed Bandit (RMAB) problem [25] by means of the stochastic control theory. Distinct from classic MAB problems [26], where the unused arms neither yield rewards nor change states and the states of all arms are known at any time, the arms in RMAB might continue to change states according to different transition rules even if they are not being pulled. In this paper, we regard each client as a restless bandit and the AoI value as its state since the AoI value changes in every time slot even if the client is not selected. However, the RMAB problem is usually PSPACE-hard [25]. To this end, we adopt the Whittle's methodology to solve this problem [27].

Firstly, we relax Problem $\mathbf{P2}$ by replacing the constraint Eq. (11d) with a relaxed version: $\frac{1}{TN}\sum_{t=1}^{T}\sum_{i=1}^{N}a_i^\pi(t)\frac{p_i}{B} \le \frac{1}{N},\ \forall t\in\mathcal{T}$. Then, we apply the Lagrangian Dual approach to transform Problem $\mathbf{P2}$ into a max-min problem, i.e., Problem $\mathbf{P3}$, which is shown as follows:

$$\begin{aligned}
\mathbf{P3}:\quad & \max_\lambda\min_{\pi\in\Pi}\mathcal{L}(\pi,\lambda), && (12)\\
\text{s.t.}\quad & a_i^\pi(t)\in\{0,1\},\forall i\in\mathcal{N},\forall t\in\mathcal{T}, && (12b)\\
& \Delta_i(t) = \mathbb{1}_{\{a_i^\pi(t)=0\}}\left[\Delta_i(t-1)+1\right], && (12c)\\
& \lambda\ge 0. && (12d)
\end{aligned}$$

Here, the lagrange dual function $\mathcal{L}(\pi,\lambda)$ is given by

$$\begin{aligned}
\mathcal{L}(\pi,\lambda) &= \frac{1}{TN}\sum_{t=1}^{T}\sum_{i=1}^{N}\left[\phi_i\Delta_i(t) + \lambda a_i^\pi(t)\tfrac{p_i}{B}\right] - \tfrac{\lambda}{N}\\
&= \frac{1}{N}\sum_{i=1}^{N}\tfrac{1}{T}\left\{\sum_{t=1}^{T}\left[\phi_i\Delta_i(t) + \lambda a_i^\pi(t)\tfrac{p_i}{B}\right]\right\} - \tfrac{\lambda}{N}, \quad (13)
\end{aligned}$$

where λ is the lagrange multiplier.

To address Problem $\mathbf{P3}$, we first solve the problem $\min_{\pi\in\Pi}\mathcal{L}(\pi,\lambda)$, which means that we need to derive the optimal client selection strategy π^* to minimize $\mathcal{L}(\pi,\lambda)$ for any given λ. Notice that we can ignore the constant item $\frac{\lambda}{N}$ and let $\mathcal{L}(\pi,\lambda)$ be solved for each individual client separately. The problem associated with each client is actually a decoupled problem, whose goal is to determine whether or not the client should be selected to update its local dataset in each time slot. Specifically, we formalize the decoupled problem over an infinite time-horizon as:

$$\begin{aligned}
\mathbf{P4}:\quad & \min_{\pi\in\Pi}\left\{\lim_{T\to+\infty}\frac{1}{T}\sum_{t=1}^{T}\left[\tfrac{B\phi_i}{p_i}\Delta_i(t) + \lambda a_i^\pi(t)\right]\right\} && (14)\\
\text{s.t.}\quad & a_i^\pi(t)\in\{0,1\},\forall i\in\mathcal{N},\forall t\in\mathcal{T}, && (14b)\\
& \Delta_i(t) = \mathbb{1}_{\{a_i^\pi(t)=0\}}\left[\Delta_i(t-1)+1\right], && (14c)\\
& \lambda\ge 0. && (14d)
\end{aligned}$$

Then, to address the decoupled problem, we model it as a Markov Decision Process (MDP), which consists of the AoI state $\Delta_i(t)$, the control variable $a_i^\pi(t)$, the state transition functions $\mathbb{P}(\cdot)$, and the cost function $\mathbb{C}_i(\cdot)$. Specifically, the state transition from time slot t to time slot $t+1$ in MDP is deterministic as follows:

$$\begin{aligned}
&\mathbb{P}(\Delta_i(t+1) = \Delta_i(t)+1|a_i^\pi(t)=0) = 1;\\
&\mathbb{P}(\Delta_i(t+1) = 0|a_i^\pi(t)=0) = 0;\\
&\mathbb{P}(\Delta_i(t+1) = \Delta_i(t)+1|a_i^\pi(t)=1) = 0;\\
&\mathbb{P}(\Delta_i(t+1) = 0|a_i^\pi(t)=1) = 1; \quad (15)
\end{aligned}$$

Algorithm 1: Whittle's Index based Client Selection

Input: AoI value of each client $\{\Delta_1(t), \cdots, \Delta_N(t)\}$, weight of each client $\{\phi_1, \cdots, \phi_N\}$, payment of each client $\{p_1, \cdots, p_N\}$, budget B

Output: The index set of selected clients $\mathcal{N}_{t+1}$

1: **for** each client i in $\mathcal{N}$ **do**
2: Calculates its WI value $WI_{i,t}$ according to Eq.(18) and sends it to the server
3: **end for**
4: The server sorts the clients into $(i_1, i_2, \cdots, i_N)$ such that $WI_{i_1,t} \geq WI_{i_2,t} \geq \cdots \geq WI_{i_N,t}$, and initializes an empty set $\mathcal{N}_{t+1}$, an initial index $k = 1$
5: **while** $\sum_{i \in \mathcal{N}_{t+1}} p_i + p_{i_k} < B$ **do**
6: $\mathcal{N}_{t+1} \leftarrow \mathcal{N}_{t+1} \cup \{i_k\}$, $k = k + 1$
7: **end while**

Moreover, we can see the objective of Problem **P4** as the cost function of MDP. The cost function on the transition from time slot t to time slot $t+1$ is defined as

$$C_i(\Delta_i(t), a_i^{\pi}(t)) \triangleq \frac{B\phi_i}{p_i}\Delta_i(t) + \lambda a_i^{\pi}(t), \tag{16}$$

where the first part is associated with the resulting AoI value in time slot t. Hereafter, for better presentation, we regard the Lagrange multiplier λ as a kind of service charge for client i under the MDP model, which is generated only when $a_i^{\pi}(t) = 1$. Note that cost fuction and service charge are not the real charge and cost, which will only be mentioned in MDP.

Finally, we derive the optimal strategy of this MDP and prove that it is a special type of deterministic strategy.

Theorem 2 (Optimal Strategy for Problem **P4**)**.** *Consider the decoupled model over an infinite time-horizon. The optimal strategy π^* is selecting client i in each time slot t to update its local dataset only when $\Delta_i(t) > H_i - 1$, where*

$$H_i = \left\lfloor -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2\lambda p_i}{B\phi_i}} \right\rfloor. \tag{17}$$

Note that the threshold H_i is a function of the service charge λ. Intuitively, we expect that the server selects client i when $\Delta_i(t)$ is high to reduce the AoI value and does not select client i when $\Delta_i(t)$ is low so as to avoid the service charge λ.

C. The WICS Algorithm

Our goal is to solve the Lagrange dual function for Problem **P3**, i.e., $\max_{\lambda} \min_{\pi \in \Pi} \mathcal{L}(\pi, \lambda)$. Now, Theorem 2 has provided the solution of $\min_{\pi \in \Pi} \mathcal{L}(\pi, \lambda)$ for any given λ, i.e., the optimal strategy π^*. Then, we need only to focus on the problem of $\max_{\lambda} \mathcal{L}(\pi^*, \lambda)$, i.e., finding an optimal λ to maximize $\mathcal{L}(\pi^*, \lambda)$. However, it is hard to find the optimal solution to solve this problem. We can only approximately deal with the problem by applying the Whittle's Index methodology [27]. More specifically, we still decouple the problem $\max_{\lambda} \mathcal{L}(\pi^*, \lambda)$ to find an individual λ parameter for each client i separately, denoted by λ_i.

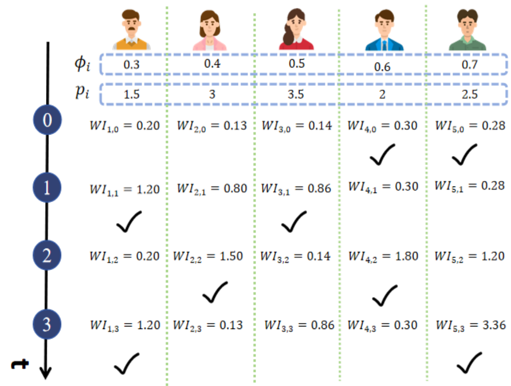


Fig. 2. An example for Algorithm with $B = 5$

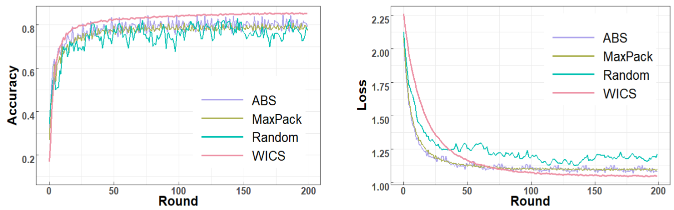
After decoupling the problem, we turn to maximize $\frac{1}{T}\sum_{t=1}^{T}\left[\frac{B\phi_i}{p_i}\Delta_i(t) + \lambda_i a_i^{\pi^*}(t) - \frac{B\lambda_i}{Np_i}\right]$ for each client i separately, where λ_i might not be same for different clients. Actually, it is a monotonic increasing function of λ_i when given initial state $\Delta_i(0) = 0$. In addition, λ_i needs to satisfy the condition of Theorem 2, i.e., $\Delta_i(t) > H_i - 1$, which indicates that λ_i is bounded in each time slot. Therefore, λ_i maximizes the average value of $\frac{1}{T}\sum_{t=1}^{T}\left[\frac{B\phi_i}{p_i}\Delta_i(t) + \lambda_i a_i^{\pi^*}(t) - \frac{B\lambda_i}{Np_i}\right]$ when $\Delta_i(t) = H_i - 1$. Expressing this critical value as $WI_{i,t}$, we can get the closed-form expression of $WI_{i,t}$ as follows:

$$WI_{i,t} \triangleq \lambda_i(\Delta_i(t)) = \frac{(\Delta_i(t)+1)(\Delta_i(t)+2)B\phi_i}{2p_i}, \tag{18}$$

where $WI_{i,t}$ stands for the Whittle's Index. Note that σ_i^2, n_i are included in ϕ_i, so $WI_{i,t}$ is dependent on σ_i^2, n_i, B, and $\Delta_i(t)$. Since σ_i^2, n_i, and B are constants, $WI_{i,t}$ is a function of $\Delta_i(t)$, which means that λ_i can also be seen as a function of $\Delta_i(t)$. In general, the Whittle's Index is not the same for different clients, which indicates that the λ_i that optimizes the different decoupled problem is different.

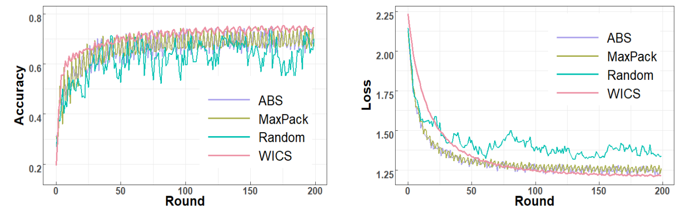
Now, based on the Whittle's Index, we can design the WICS algorithm for Problem **P3** (and also for Problem P2 based on the Lagrange duality). The basic idea is to select the clients with higher WI values in each time slot, while ensuring that the budget is not exceeded. As shown in Algorithm 1, we first calculate the WI value for each client according to Eq. (18) and then sort all clients in $\mathcal{N}$ into the set $(i_1, i_2, \cdots, i_N)$ such that $WI_{i_1,t} \geq WI_{i_2,t} \geq \cdots \geq WI_{i_N,t}$ (Steps 1-3). Next, we greedily select the clients into a winning set $\mathcal{N}_t$ and give the corresponding payments for the winning clients until the remaining budget cannot afford the next client (Steps 4-6). We illustrate an simple example in Fig. 2.

Finally, we analyze the performance of the WICS algorithm. Obviously, the computational complexity of WICS is dominated by the sorting operation on clients' WI values, i.e., $O(N \log N)$. In addition, we define the ratio $\rho^{\pi} \triangleq \frac{U_B^{\pi}}{L_B}$ to indicate the performance of strategy π, where L_B is a lower bound to the optimal performance of Problem **P2** and U_B^{π} is an upper bound to the performance of Problem **P2** under strategy π, and we say that strategy π is ρ^{π}-optimal. Then, the WICS algorithm satisfies the following theorem.



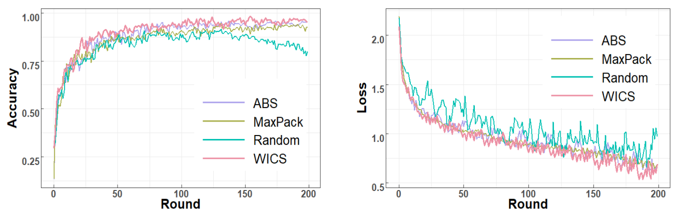
(a) Accuracy of LR on MNIST (b) Loss of LR on MNIST

Fig. 3. Performance of LR on MNIST



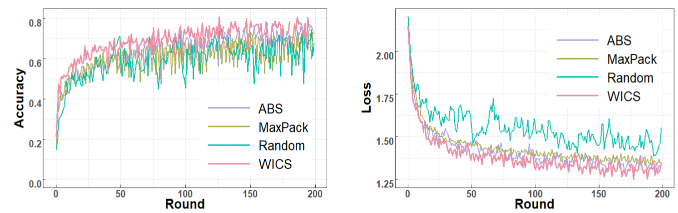
(a) Accuracy of LR on FMNIST (b) Loss of LR on FMNIST

Fig. 4. Performance of LR on FMNIST



(a) Accuracy of CNN on MNIST (b) Loss of CNN on MNIST

Fig. 5. Performance of CNN on MNIST



(a) Accuracy of CNN on FMNIST (b) Loss of CNN on FMNIST

Fig. 6. Performance of CNN on FMNIST

Theorem 3 (Approximate Optimality)**.** *The solution produced by the WICS algorithm to Problem* $\mathbf{P2}$ *over an infinite time-horizon is* ρ^{WI}*-optimal, where*

$$\rho^{WI} < \frac{18N-2}{M-1}, \tag{19}$$

$M = \left\lfloor \frac{B}{p_{max}} \right\rfloor$ *and* $p_{max} = \max_i\{p_i\}$.

Note that the objective of Problem $\mathbf{P2}$ (i.e., Eq. (11)) is derived from the objective of Problem $\mathbf{P1}$ (i.e., Eq. (8)) according to the convergence bound analysis. Thus, the WICS algorithm is at least ρ^{WI}-optimal for Problem $\mathbf{P1}$.

V. PERFORMANCE EVALUATION

A. Evaluation Methodology

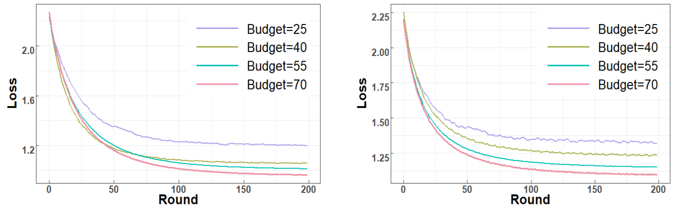
1) Simulation Setup: We conduct extensive simulations on two widely-used real datasets: MNIST [28] and Fashion-MNIST (FMNIST [29]). The MNIST dataset contains 60,000 handwritten digits for training and 10,000 for the test, while the Fashion-MNIST dataset contains 60,000 fashion clothes for training and 10,000 for the test. We adopt both the convex model (i.e., LR) and the non-convex model (i.e., CNN). The CNN consists of two 5×5 convolution layers (32, 64 channels), each of which is followed by 2×2 max pooling, two fully-connected layers with 3136 and 512 units, and a ReLU layer with 10 units. We first let the number of clients $N = 10$ and the number of time slots $T = 200$. Next, we generate the simplified budget in each time slot (i.e., B) from $\{25, 40, 55, 70\}$. Then, we determine the cost p_i. We assume that the cost p_i is proportional to the number of local data and let the cost for each client not exceed $[5, 15]$. For all experiments, we initialize our model with $\omega_0 = \mathbf{0}$ and use an SGD batch size of $b = 16$. Without loss of generality, we set the learning rate of LR to be $\eta_t = 0.005$ and the learning rate of CNN to be $\eta_t = 0.01$ for all time slots and each client performing $\tau = 10$ local iterations. After that, we randomly select the weight $\phi_i \in (0, 1)$ according to Eq. (10), which is similar to the method in [30]. In order to reflect the impact of AoI on the local data, we mislabel some local data of each client according to its AoI value in each time slot. Specifically, we will mislabel more data if the client has a larger AoI value.

2) Algorithms for Comparison: Since WICS considers the freshness of local datasets in FL, there are no existing algorithms that can be directly applied to our problem. To the best of our knowledge, the closest algorithm that can be adapted to our setting is the ABS algorithm proposed by [31], which is also an index-based strategy. However, since the ABS algorithm considers the age-of-update (AoU) rather than AoI, we need to modify it to deal with the AoI in our model. More specifically, the modified ABS index of client i in time slot t is given by $\frac{\Delta_i(t)\phi_i}{p_i}$. Similarly to our WICS strategy, the ABS algorithm greedily selects clients with larger modified ABS index values while ensuring that the budget is not exceeded in each time slot. The difference is that only selected clients can participate in this round of FL training. Moreover, we implement the MaxPack algorithm [32] and the Random algorithm for better comparison. The MaxPack algorithm is the comparison algorithm of the ABS algorithm in [31], which selects clients with larger AoI values while ensuring that the budget is not exceeded in each time slot.

B. Evaluation Results

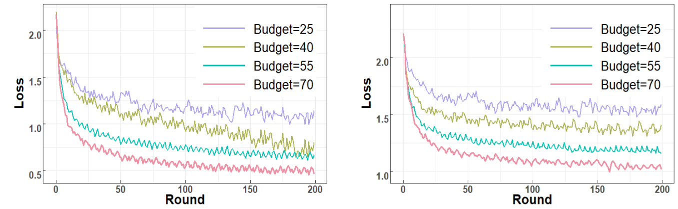
In this section, we train different models (i.e., LR, CNN) on both MNIST and FMNIST to compare the performance of different algorithms. Notably, we conduct experiments with variant budget B, which shows a similar performance. Due to the limited space, we only illustrate the result of $B = 40$.

First, we exhibit the performance of different algorithms for LR on MNIST and FMNIST in terms of both accuracy and loss, as shown in Fig. 3 and Fig. 4, respectively. Accuracy measures the number of correct predictions, and loss measures the difference between the prediction and actual output. In Fig. 3, we can observe that the achieved accuracy of all four algorithms rises along with the increase of rounds, while the achieved loss of all four algorithms decreases with the increasing of rounds. Moreover, the performance of WICS in terms



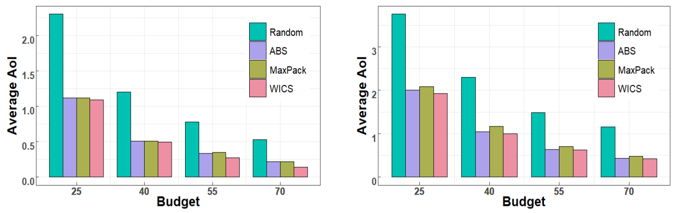
(a) N=10 (b) N=20

Fig. 7. Loss vs. the number of clients N



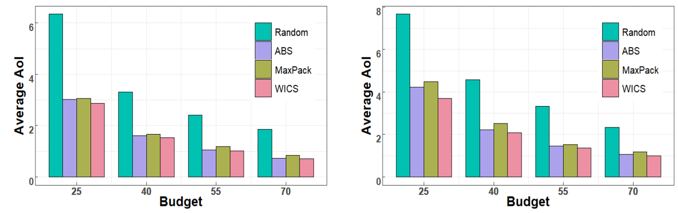
(a) N=30 (b) N=40

Fig. 8. Loss vs. the number of clients N



(a) N=10 (b) N=20

Fig. 9. Average AoI vs. the number of clients N



(a) N=30 (b) N=40

Fig. 10. Average AoI vs. the number of clients N

of both accuracy and loss is better than the three compared algorithms. In Fig. 4, we conduct the same experiments of LR on FMNIST and obtain similar results. We see that WICS can also achieve the best results in all algorithms. This means that WICS is effective for models with a convex loss function, which matches with the theoretical convergence bound. To verify the effectiveness of WICS when the loss function is non-convex, we further train CNN on MNIST and FMNIST. Figs. 5-6 show that the performances of WICS are still better than those of other algorithms when the loss function does not satisfy the convex assumption.

Furthermore, we analyze the influence of different budgets B on all four algorithms in terms of loss, where we set the number of clients $N = 10$. We take WICS as an example, and display the results in Figs. 7-8. The figures indicate that whether the loss function of the model is convex or not, the larger B is, the smaller the achieved loss of the model. This can be explained by the reason that a larger B allows more clients to update their local datasets in each time slot, so as to make the local datasets fresher and achieve better learning performance, which also matches with the convergence upper bound analysis of Section III.

Finally, we evaluate the performance of all four algorithms in terms of average AoI, which is computed by $\Delta = \frac{1}{NT}\sum_{t=1}^{T}\sum_{i=1}^{N}\frac{n_i}{n}\Delta_i(t)$. The evaluation results are depicted in Figs. 9-10, where we scale B from 25 to 70 with an increment of 15 and evaluate the effects of the number of clients N. The figures show that WICS can achieve the lowest weighted average AoI in all four algorithms. More specifically, ABS, MaxPack, and WICS are far better than the Random algorithm, and the performance of ABS is the closest when compared to WICS. In addition, the weighted average AoI exhibits an uptrend with the increasing of N. This is because when we keep the budget fixed, the number of clients who are not selected by the server in each time slot will increase with N, i.e., the increment of AoI values in each time slot will become larger. Hence, the weighted average AoI is also increasing with the increment of N. Moreover, the results in Figs. 9-10 are consistent with those in Figs. 3-6, which verifies the correctness of our Assumption 4.

VI. RELATED WORK

Client Selection for FL: Client selection has been widely investigated in the literature of FL [1], [33], [34], considering various facets of the system, such as statistical heterogeneity and system heterogeneity. Different optimization objectives like importance sampling and resource-aware optimization-based approaches have also been considered. The goal of importance sampling is to reduce the variance in traditional optimization algorithms based on stochastic gradient descent (SGD). For example, most existing works make use of clients' local gradient [33] or local loss [34] information to measure the importance of clients' local data, and then select the clients according to the data importance. In addition, resource-aware optimization-based approaches, such as CPU frequency allocation [35], communication bandwidth allocation [36] and straggler-aware client scheduling [37], select appropriate clients to optimize the different aspects of the federated learning system. However, most of these works assume that the local data of clients do not change during the FL process, while our work focuses on the scenario where clients' local data need to be updated periodically.

Age of Information: AoI is a novel application-layer metric for measuring freshness that was initially conceived by [19]. Since its inception, there has been a lot of studies on AoI optimization (see an online bibliography in [38]), which includes a wide range of problems. An important class of problems that has attracted much attention is how to design schedulers to minimize AoI [19], [38]–[42]. For instance, Kaul et al. [19] develops a new analytical model for mobile crowd-learning, which takes into account the strong couplings between the stochastic arrivals of participating users, PoIs information evolutions, and reward mechanisms. Sun et al. [38] considers how to minimize AoI when scheduling decisions are restricted to both bandwidth and power consumption constraints. Dai et al. [39] studies how to minimize the average AoI of the deployed sensor nodes in data collection by mobile crowdsensing. The authors in [40] studies the problem of minimizing AoI in general single-hop and multi-hop wireless networks. Fang et al. [41] studies how to design a joint preprocessing and transmission policy to minimize the average AoI at the destination and the energy consumption at the IoT device. Tang et al. [42] considers how to minimize AoI when scheduling decisions are restricted to both bandwidth and power consumption constraint. However, none of these existing works consider the problem of minimizing the average AoI value of local datasets in FL systems.

VII. CONCLUSION

In this paper, we introduce a novel AoI-aware FL system, where clients might use fresh datasets to perform local model training and the server tries to select some clients to provide fresh datasets in each time slot but is constrained by a limited budget. We use AoI to indicate the freshness of datasets and theoretically analyze the convergence upper bound of the AoI-aware FL system. On this basis, we model the corresponding client selection issue as a restless multi-armed bandit problem, and propose a Whittle's-Index-based client selection algorithm, i.e., WICS, to solve this problem. Moreover, we prove that the WICS can achieve nearly optimal performance on client selection. Finally, we also conduct extensive simulations on two real datasets and the simulation results demonstrate the effectiveness of our algorithm. Future investigations will consider the fine-grained integration of fresh data and stale data, as well as the discount factor based on time, which gives more weight to fresh information.

APPENDIX

A. Proof of Theorem 1

Proof. First, we analyze how the difference between $\mathbb{E}[F(\omega_t)]$ and F^* (i.e., $F(\omega^*)$) changes in each round. Due to β-smooth and by using the fact that $\nabla F(\omega^*) = 0$, we have

$$\begin{aligned}\Omega_t = \Omega_{t-1} + \mathbb{E}\| \textstyle\sum_{i=1}^N \frac{n_i}{n}(\omega_t^i - \omega_{t-1})\|^2 \\ + 2\mathbb{E}\left\langle \omega_{t-1} - \omega^*, \textstyle\sum_{i=1}^N \frac{n_i}{n}(\omega_t^i - \omega_{t-1})\right\rangle. \quad (20)\end{aligned}$$

We use A_1 and A_2 to denote the second and third terms in (20), respectively. For A_1, we can bound it by using the AM-GM inequality and the Cauchy-Schwarz inequality:

$$\begin{aligned}A_1 &= \| - \textstyle\sum_{i=1}^N \frac{n_i\eta_t}{n} \sum_{k=0}^{\tau-1} \nabla F_{t,i}(\omega_t^{i,k}; \xi_t^i)\|^2 \\ &\le N\tau \textstyle\sum_{i=1}^N \frac{n_i^2\eta_t^2}{n^2} \sum_{k=0}^{\tau-1} \|\nabla F_{t,i}(\omega_t^{i,k}; \xi_t^i)\|^2. \quad (21)\end{aligned}$$

Further, according to Assumption 4, it follows:

$$\mathbb{E}[A_1] \le N\tau^2\eta_t^2 \textstyle\sum_{i=1}^N \frac{n_i^2}{n^2} \left[G_i^2 + \Delta_i(t)\sigma_i^2\right]. \quad (22)$$

For A_2, we have the following equation:

$$\begin{aligned}A_2 = \left\langle \omega_{t-1} - \omega^*, - \textstyle\sum_{i=1}^N \frac{n_i\eta_t}{n} \nabla F_{t,i}(\omega_{t-1}; \xi_t^i)\right\rangle \\ + \left\langle \omega_{t-1} - \omega^*, - \textstyle\sum_{i=1}^N \frac{n_i\eta_t}{n} \sum_{k=1}^{\tau-1} \nabla F_{t,i}(\omega_t^{i,k}; \xi_t^i)\right\rangle. \quad (23)\end{aligned}$$

Then, we use B_1 and B_2 to denote the first and second terms in (23), respectively. Next, we bound $\mathbb{E}[B_1]$ and $\mathbb{E}[B_2]$, respectively. Using the μ-strongly convex of $F_{t,i}(\cdot)$ and the fact that $F_{t,i}^* \le F_{t,i}(\omega_{t-1})$, we can bound $\mathbb{E}[B_1]$ as follows:

$$\begin{aligned}\mathbb{E}[B_1] &= - \textstyle\sum_{i=1}^N \frac{n_i\eta_t}{n} \langle \omega_{t-1} - \omega^*, \nabla F_{t,i}(\omega_{t-1})\rangle \\ &\le \textstyle\sum_{i=1}^N \frac{n_i\eta_t}{2n\mu}\left[G_i^2 + \Delta_i(t)\sigma_i^2\right] - \frac{\mu\eta_t}{2}\Omega_{t-1}. \quad (24)\end{aligned}$$

For $\mathbb{E}[B_2]$, we utilize the μ-strongly convex in assumption 2 and have the following inequality:

$$\begin{aligned}\mathbb{E}[B_2] &\le \frac{\mu\eta_t}{4}\mathbb{E}\|\omega_{t-1} - \omega^*\|^2 \\ &\quad + \frac{1}{\mu\eta_t}\mathbb{E}\| \textstyle\sum_{i=1}^N \frac{n_i\eta_t}{n} \sum_{k=1}^{\tau-1} \nabla F_{t,i}(\omega_t^{i,k}; \xi_t^i)\|^2 \\ &\le \frac{\mu\eta_t}{4}\Omega_{t-1} + \frac{N\eta_t(\tau-1)^2}{\mu} \textstyle\sum_{i=1}^N \frac{n_i^2}{n^2}\left[G_i^2 + \Delta_i(t)\sigma_i^2\right]. \quad (25)\end{aligned}$$

Combining the above equations, we can obtain that:

$$\begin{aligned}\Omega_t &\le (1 - \frac{\mu\eta_t}{2})\Omega_{t-1} + \frac{\eta_t}{\mu} \textstyle\sum_{i=1}^N \frac{n_i}{n}\left[G_i^2 + \Delta_i(t)\sigma_i^2\right] \\ &\quad + N\eta_t \frac{\tau^2\eta_t\mu + 2(\tau-1)^2}{\mu} \textstyle\sum_{i=1}^N \frac{n_i^2}{n^2}\left[G_i^2 + \Delta_i(t)\sigma_i^2\right] \\ &\le (1 - \frac{\mu\bar{\eta}}{2})\Omega_{t-1} + \sum_{i=1}^N \alpha_i\left[G_i^2 + \Delta_i(t)\sigma_i^2\right]. \quad (26)\end{aligned}$$

Then, by induction, we can prove:

$$\Omega_T \le (1 - \frac{\mu\bar{\eta}}{2})^T\Omega_0 + \textstyle\sum_{t=1}^T \sum_{i=1}^N \alpha_i\left[G_i^2 + \Delta_i(t)\sigma_i^2\right]. \quad (27)$$

Finally, we have the following bound:

$$\begin{aligned}\mathbb{E}[F(\omega_T)] - F^* &\le \frac{\beta}{2}(1 - \frac{\mu\bar{\eta}}{2})^T\|\omega_0 - \omega^*\|^2 \\ &\quad + \textstyle\sum_{t=1}^T \sum_{i=1}^N \frac{\alpha_i\beta}{2}\left[G_i^2 + \Delta_i(t)\sigma_i^2\right]. \quad (28)\end{aligned}$$

Therefore, the theorem holds. ☐

B. Proof of Theorem 2

Proof. Due to limited space, we present our proof sketch of Theorem 2. Consider the decoupled model with AoI state Δ_i and control variable a_i. Then, Bellman equations are given by the following equation:

$$S(\Delta_i) + \zeta = \min_{a_i \in \{0,1\}} \{\frac{\phi_i}{p_i}\Delta_i + S(\Delta_i + 1), \frac{\phi_i}{p_i}\Delta_i + \lambda\}, \quad (29)$$

where $\Delta_i \in \{0, 1, \cdots\}$ and $S(\Delta_i)$ is the differential cost-to-go function. Note that the first part of Eq. (29) corresponds to $a_i = 0$, while the second part is opposite.

In fact, any selection strategy can be regarded as a threshold strategy. Therefore, we start the proof by assuming that the optimal strategy π^* is a threshold strategy that selects client i when $0 \le \Delta_i(t) \le H - 1$, and does not select when $\Delta_i(t) \ge H$, for a given value of $H \in \{1, 2, \cdots\}$.

First, we analyze the case $\Delta_i \ge H$. According to Eq. (29), we can easily get the condition for the strategy π to select client i with state $\Delta_i \ge H$, which is as follows:

$$S(\Delta_i + 1) > \lambda \ , \ S(\Delta_i) = \lambda - \zeta + \frac{\phi_i\Delta_i}{p_i}. \quad (30)$$

Next, we analyze the case $0 \le \Delta_i \le H - 1$. Similarly, the condition for the strategy π that does not select client i is

$$S(\Delta_i + 1) < \lambda \ , \ S(\Delta_i) = S(\Delta_i + 1) - \zeta + \frac{\phi_i\Delta_i}{p_i}. \quad (31)$$

Now, we can derive Theorem 2 according to [43]. ☐

C. Proof Sketch of Theorem 3

Proof. Due to limited space, we borrow the basic idea in [43] to present our proof sketch. We denote the low bound of the performance of Problem $\mathbf{P2}$ as L_B and the upper bound of the performance of Problem $\mathbf{P2}$ under WICS as U_B^{WI}.

First, U_B^{WI} will be smaller than the upper bound in [43] since selecting more clients in each time slot will get a smaller average AoI value, i.e., we have:

$$U_B \leq (9 - 1/N) \sum_{i=1}^{N} \phi_i. \tag{32}$$

For L_B, we use the same method in [43], which will lead to a smaller optimal performance of Problem $\mathbf{P2}$. Similarly to [43], we employ Fatou's lemma and have

$$L_B \geq (M - 1) \sum_{i=1}^{N} \phi_i / 2N. \tag{33}$$

Next, we can derive the following bound $\rho^{WI} = U_B^{WI}/L_B < (18N - 2)/(M - 1)$. Thus, the theorem holds. □

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