# Coalition Formation Game in the Cross-Silo Federated Learning System

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# Roadmap

1. Federated Learning
2. Coalition Game
3. Game Formation
4. Properties
5. Coalition Algorithm
6. Fair Cost Sharing
7. Simulation
8. Conclusions

# 1. Federated Learning (FL)

## Federated Learning (FL)

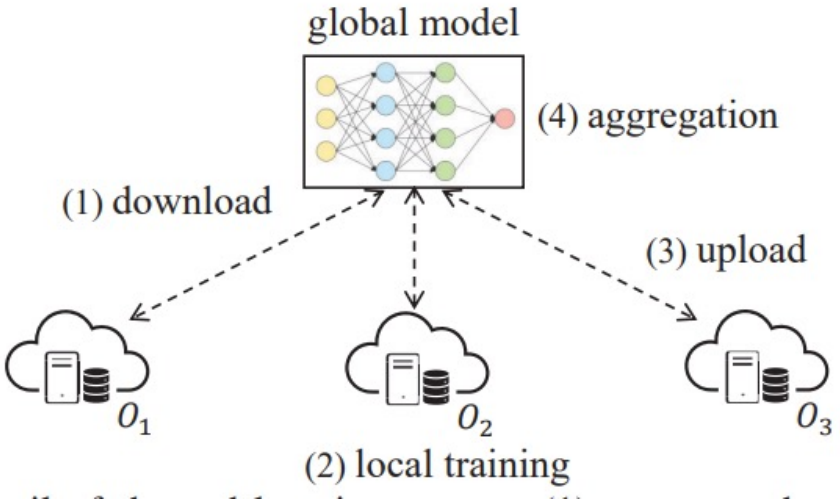
(1) Download global model

(2) Perform local training

(3) Upload local updates

(4) Aggregate local updates

to a new global model (FedAvg)




## Cross-silo FL

A relatively small number, but reliable, of organizations

# 2. Coalition Game

- Utility

  Utility = Benefit – Cost

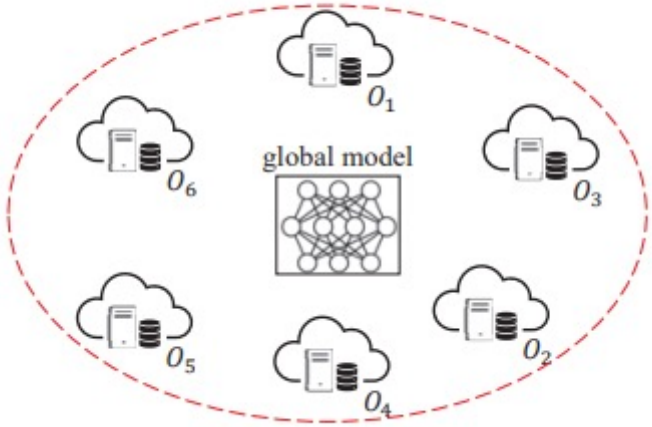
- Cooperative Game

  Joint actions that groups take and the resulting collective payoffs

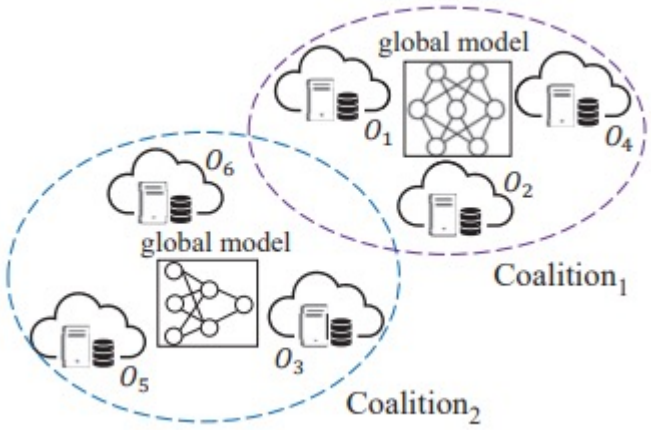
- Coalition

  Stability

- Inspired by
  - Collaborative spectrum sensing
  - Federated MAB (multi-armed bandits)




Grand coalition



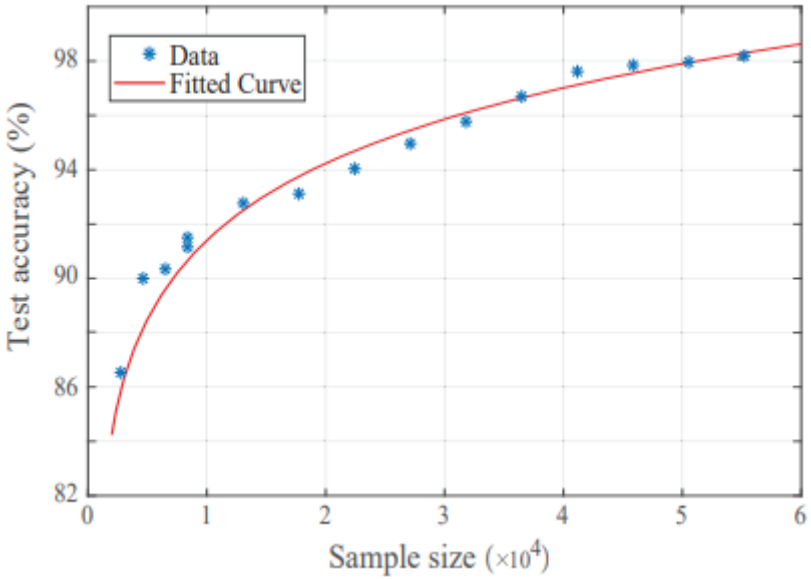

Small coalition

# 3. Game Formation

- Benefit of coalition S

  accuracy vs. data size

$$l(S) = \theta \log(1 + \lambda \cdot q(S))$$




- Cost

  secure aggregation

$$c(S) = c_{srv}(S) + c_{org}(S) = \alpha|S|^2 + \beta|S|.$$

K. Bonawitz et al, "Practical secure aggregation for privacy-preserving machine learning," CCS 2017

# 4. Properties

- Utility

$$u(S) = l(S) - c(S)$$
$$= \theta \log(1 + \lambda \cdot q(S)) - \left(\alpha |S|^2 + \beta |S|\right).$$

- $(O, u)$ is a coalition game, with $O$ being a set of organizations

**Definition 1.** *A coalition game $(O, u)$ with a transferable utility is said to be superadditive if for any two disjoint coalitions $S_i, S_j \subset O$, $u(S_i \cup S_j) \geq u(S_i) + u(S_j)$.*

- Theorem 1. *The proposed organization cooperation game (O, u) with cost is, in general, non-superadditive.*

- Theorem 2. *The grand coalition is not among cooperating organizations.*

# 5. Coalition Algorithm

- Optimal solution
  - NP-complete
- Distributed coalition
  - Merge and split operations
-

**Definition 6.** *Merge Rule - Merge any set of coalitions* $\{S_1, \cdots, S_k\}$ *where* $\sum_{j=1}^{k} u(S_j) < u\left(\cup_{j=1}^{k} S_j\right)$ *so that* $\{S_1, \cdots, S_k\} \rightarrow \cup_{j=1}^{k} S_j$.

**Definition 7.** *Split Rule - Split any set of coalitions* $\cup_{j=1}^{k} S_j$ *where* $\sum_{j=1}^{k} u(S_j) > u\left(\cup_{j=1}^{k} S_j\right)$ *so that* $\cup_{j=1}^{k} S_j \rightarrow \{S_1, \cdots, S_k\}$.

# Algorithm: Merge-and-Split

- Pareto order
  - At least one organization's utility is increased without decreasing other organizations' utilities.

**Algorithm 1** Adaptive Coalition Formation: merge-and-split

**Initial:** The coalition structure of the network is $\mathcal{P} = \{S_1, \cdots, S_N\}$, where $S_i = \{o_i\}$, *i.e.*, all organizations are non-cooperative in the beginning.

**Output:** an updated coalition structure $\mathcal{P} = \{S_1, \cdots, S_k\}$

1: **repeat**
2: **for** $S_i \in \mathcal{W}$ **do**
3: Randomly connect to another coalition $S_j$
4: Perform Merge Rule
5: Perform Split Rule
6: **until** merge-and-split terminates
7: Return updated $\mathcal{P}$

# Algorithm: Properties

- A partition is *stable* if no coalition has an incentive to split or merge.

- Theorem 3. *The partition resulting from our proposed coalition formation algorithm is stable.*

- Complexity and termination of merge and split

W Saad et al, Coalition Formation Games for Collaborative Spectrum Sensing, IEEE VTC 2010

# 6. Fair Cost Sharing

- General rules
  - Individual utility is aligned with its coalition's utility
  - More contributions means less payment
- Strategy-proof
  - Each participant fares the best by being truthful
- Individual contribution
  - Size-based measurement $a_i = d_i.$
  - Accuracy-based measurement $a_i = \theta \log(1 + \lambda d_i).$
  - Sharpley-Value-based measurement

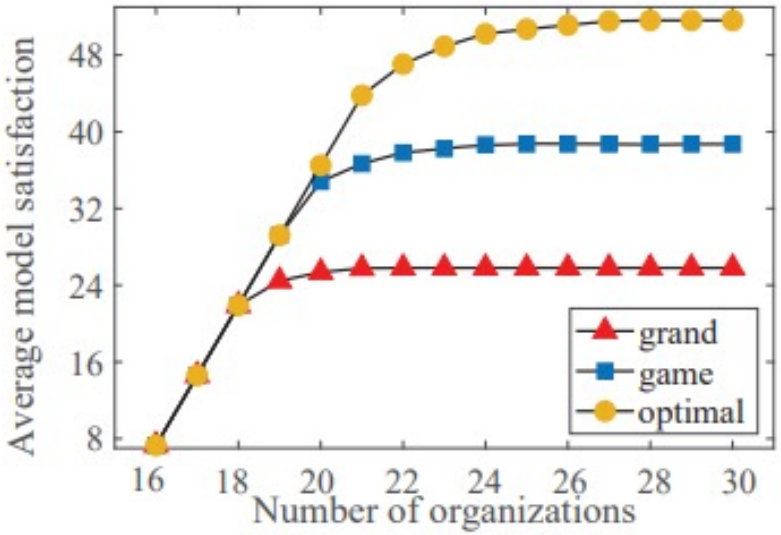
$$\phi_i = \sum_{s \subseteq S - \{i\}} \frac{|s|!\,(|S| - |s| - 1)!}{|S|!} \left[ v(s \cup \{i\}) - v(s) \right]$$

# 7. Simulation

- Dataset and Model
  - Data set: MNIST
  - Local: multinomial logistic regression using SGD
  - Global: FedAvg
- Simulation Parameters
  - Convergence: the loss of two consecutive global rounds ≤ $10^{-5}$
  - Local training: 80 epochs, learning rate of 0.005
  - $(\theta, \lambda, \alpha, \beta)$: $(10, 8 \times 10^{-6}, 0.05, 0.2)$

# Simulation: Satisfaction

- Three approaches
  - Grand: grand coalition
  - Game:  proposed merge-and-split
  - Optimal: central optimal solution



# Simulation: Cost Analysis

| Strategy \ N | 10 | 20 | 30 | 40 |
|---|---|---|---|---|
| optimal | 16 | 32 | 48 | 62 |
| game | 23 | 34.5 | 48.4 | 64 |
| grand | 27 | 38 | 50 | 68.1 |

(a) Average cost under size-based policy.

| Strategy \ N | 10 | 20 | 30 | 40 |
|---|---|---|---|---|
| optimal | 15.8 | 30.2 | 44.3 | 61 |
| game | 17.8 | 31.3 | 46.9 | 63.1 |
| grand | 24.6 | 35.9 | 48.8 | 64.7 |

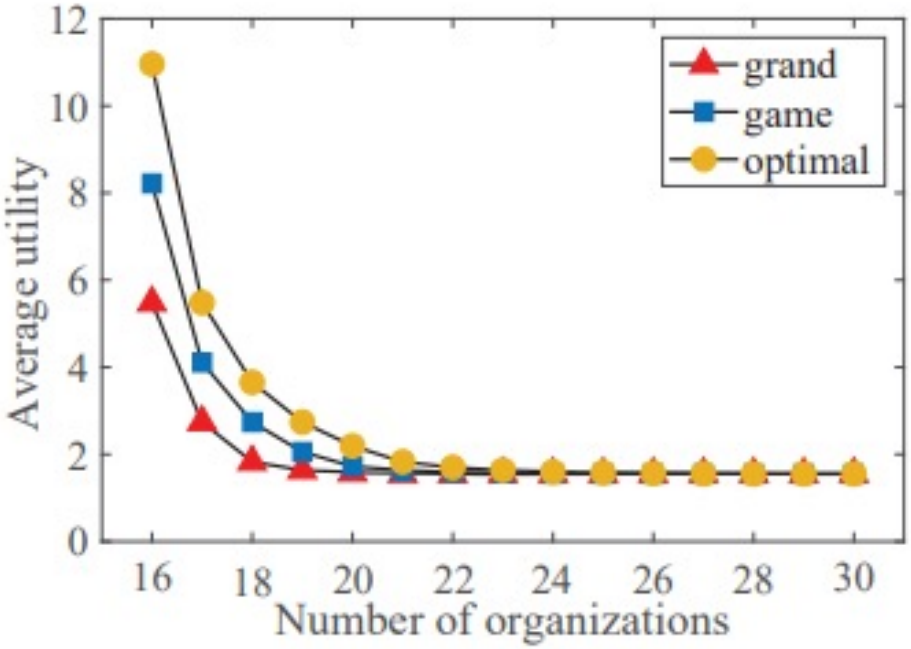
(b) Average cost under accuracy-based policy.

| Strategy \ N | 10 | 20 | 30 | 40 |
|---|---|---|---|---|
| optimal | 15.1 | 28.8 | 41.5 | 51.8 |
| game | 17.7 | 28.9 | 42.7 | 52.8 |
| grand | 23.9 | 34.5 | 43.1 | 53.9 |

(c) Average cost under SV-based policy.

# Simulation: Utility

- The proposed merge-and-split
  - Suitable for FL with a moderate number of organizations



# 8. Conclusions

- Cross-silo federated learning
  - A relatively small number of participants
- Cooperative game
  - A utility model based on benefit minus cost
  - A stable partition into groups
- A distributed solution
  - Merge-and-split
- Future work
  - Non-IID data
  - Realistic benefit/cost measures

# Questions