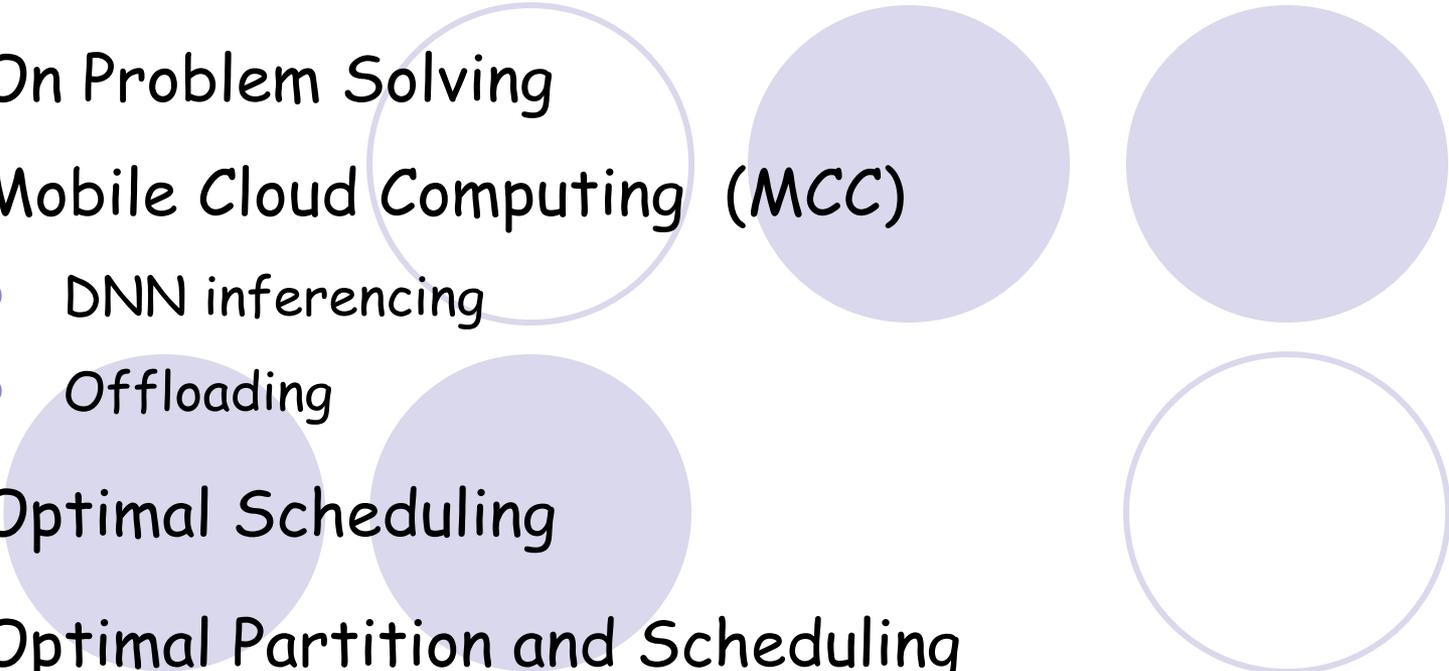


# On Optimal Partitioning and Scheduling of DNNs in Mobile Edge/Cloud Computing

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# Outline

1. On Problem Solving
  2. Mobile Cloud Computing (MCC)
    - DNN inferencing
    - Offloading
  3. Optimal Scheduling
  4. Optimal Partition and Scheduling
  5. Conclusions and Future Work
  6. Some Reflections
- 

# 1. On Problem Solving

How to Solve It (Poyla, 1945)

If you can't solve a problem, then there is an easier problem you can solve: find it.



Is Computing An Experimental Science ? (Milner, 1986)

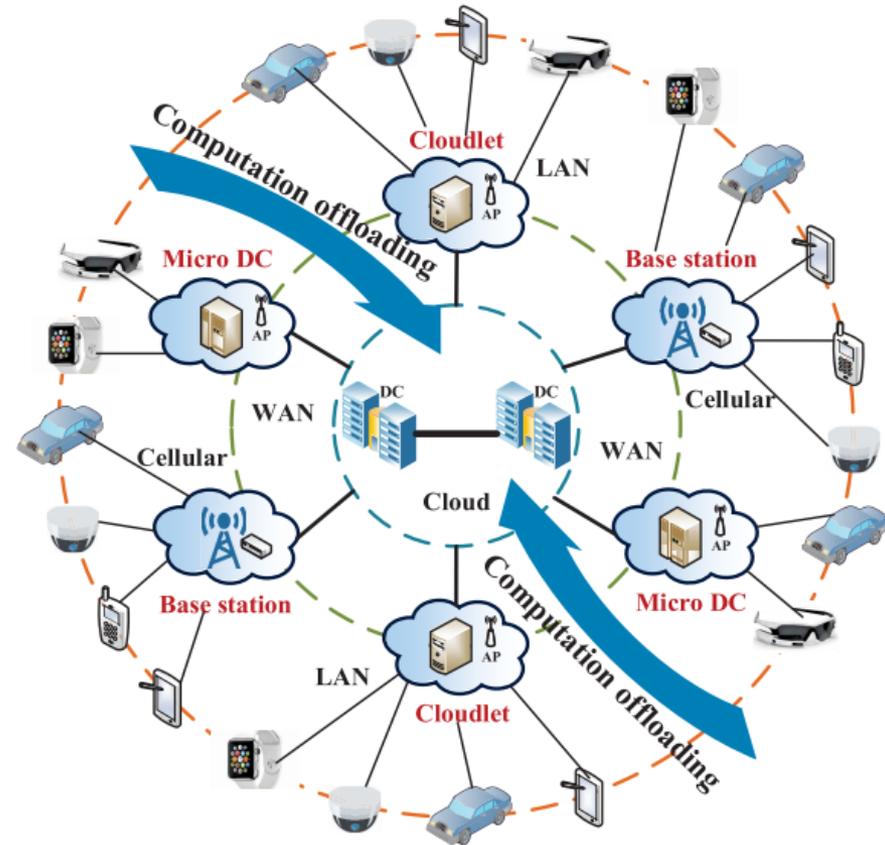
A theory can only emerge through protracted exposure to application.



Ideas and applications developed side-by-side

## 2. Mobile Cloud Computing (MCC)

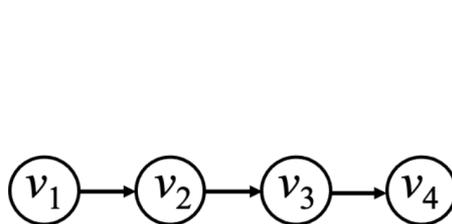
- Cloud/Edge Computing
  - Application-driven: VR/AR, video analytics using IoTs
  - Better QoE: cloud computing and mobile/edge device
  - Key indicators: latency, accuracy, energy, and privacy
  - Latency-sensitive: how to bring rich computational resources to mobile users?



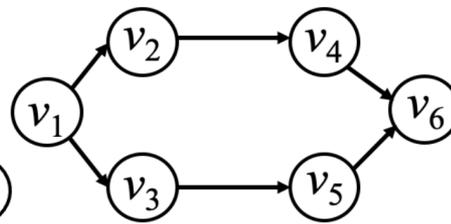
50 billion IoT devices by 2020

# DNN Inferencing

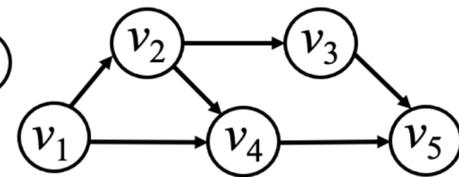
- Deep Neural Networks (**DNNs**)
  - Technologies: GPU (graphic) and TPU (tensor)
- AI applications
  - Computer vision: AlexNet, VGG-16, Inception, RandWire
  - Natural language processing: GPT-3
- Graph models of DNNs



(a) line

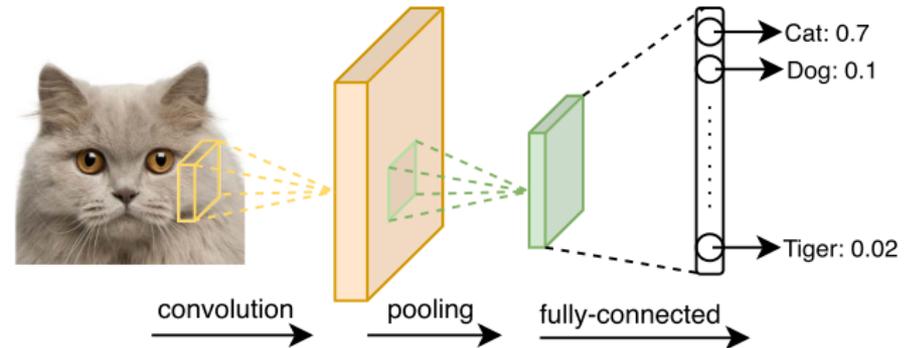


(b) multi-path

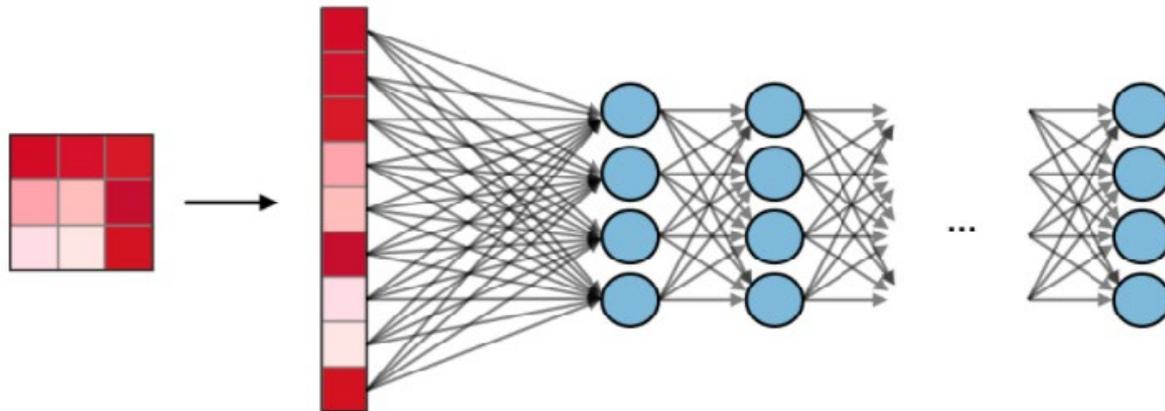
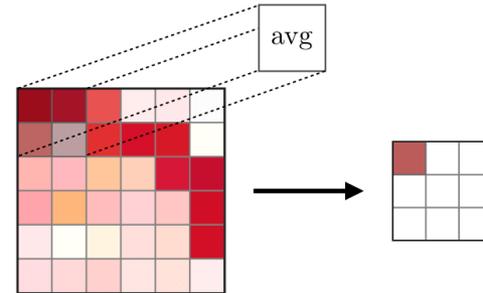
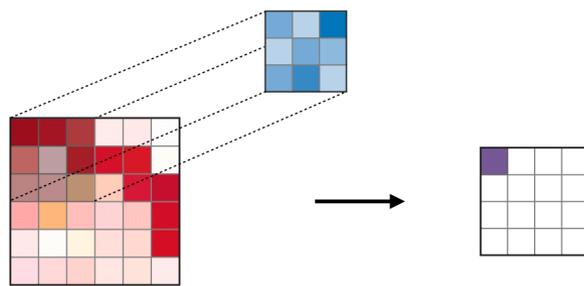


(c) DAG

# Convolution NNs

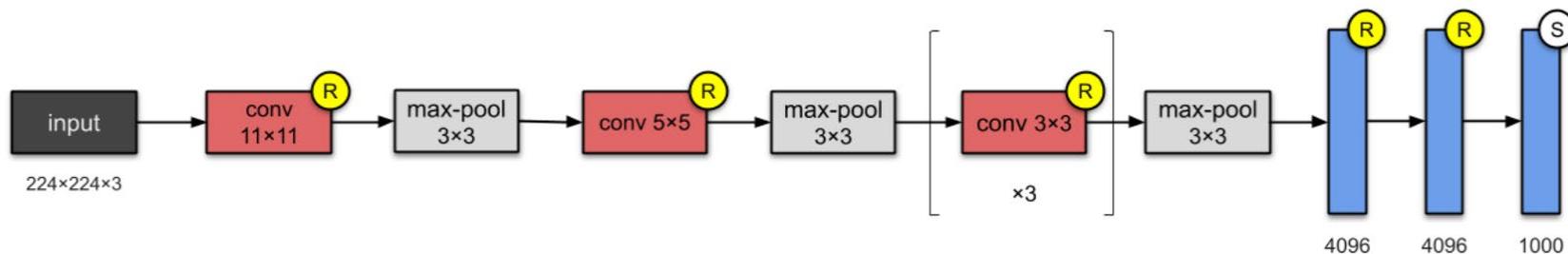


- CNNs (image classification)
- convolution (filtering), pooling (max/avg), fully-connected (neurons)

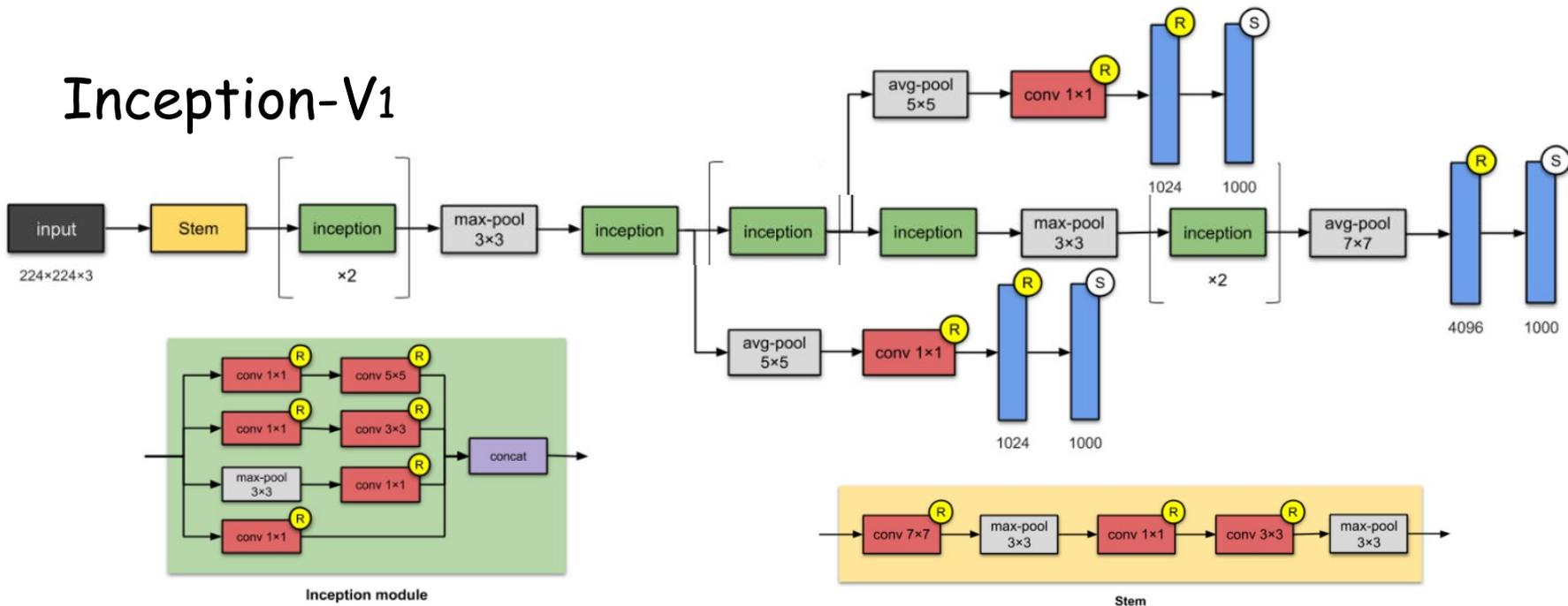


# Sample CNNs

AlexNet (Red: CONV, Gray: POOL, Blue: FC)

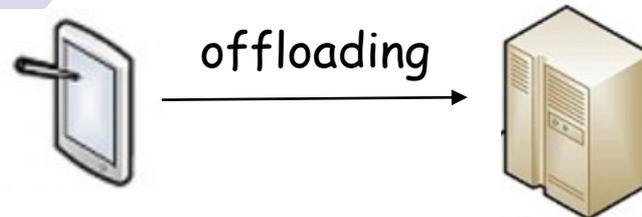


## Inception-V1



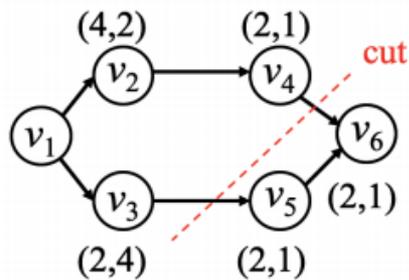
# Offloading

- Three-stage collaborative computation offloading
  - Local computation: processing on local devices
  - Communication: transmitting intermediate DNN layers' outputs
  - Remote computation: completing the remote processing in cloud
- Three models
  - On-device optimization
  - Cloud-only offloading
  - Mixed-mode offloading

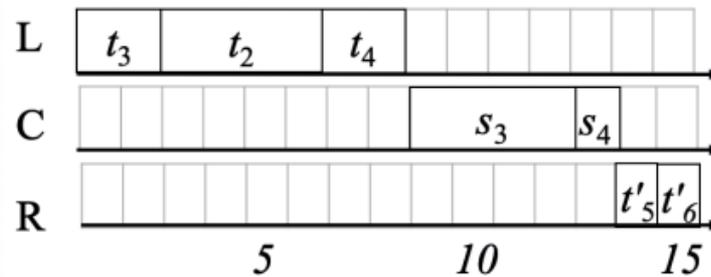


# Offloading Samples

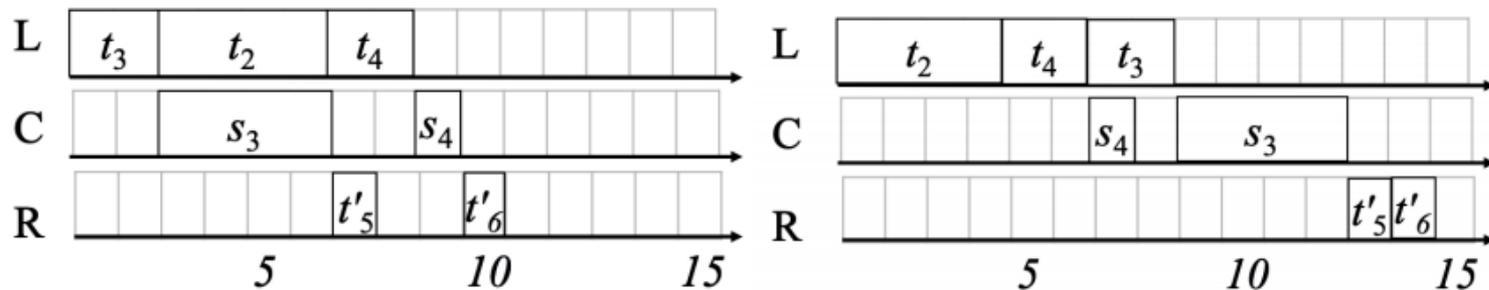
- Given a partition (i.e., cut)
  - Course-grained pipeline: local, comm, and remote
  - Fine-grained pipeline: path-based



(a) a DNN



(b) no fine-grained pipeline



(c) fine-grained pipeline



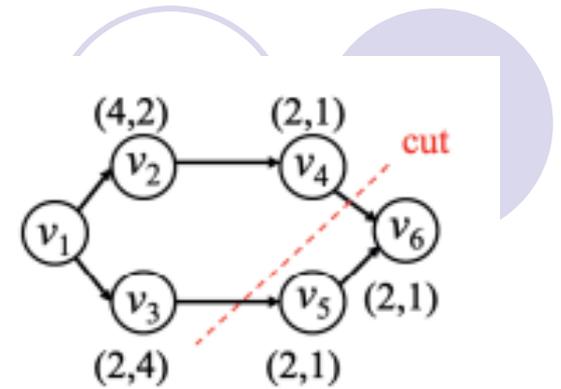
# 3. Optimal Scheduling

- DNN Computation Offloading Optimization (DCOO)
  - DCOO: optimal scheduling (in terms of minimum **makespan**) for a given partition (i.e., cut).
- Cases of DNN
  - Line-structure: trivial
  - Multi-path: hard
  - DAG: hard

Theorem 1: DCOO is NP-hard for a multi-path DNN.

Proof: Reduce 3-machine **flow-shop** to DCOO.

# Multi-Path Scheduling



- Single-path

- Straightforward solution, even without a given cut

- Multi-path

- Path: a path from input to cut or from cut to output
- Non-overlaps among paths (except input and output)
- E.g.,  $v_1-v_2-v_4$ ,  $v_1-v_3$ ,  $v_6$ ,  $v_5-v_6$

Theorem 2: In multi-path DNNs, the optimal schedule can be achieved via the **non-preemptive** path-based schedule.

# Extended Johnson Algorithm (EJA)

Path  $p(i)$  in three stages

- $P_1(i), P_2(i), P_3(i)$

Linear solution (EJA)

- Dividing paths into  $H$  and  $L$
- E.g.,  $H = \{1\}, L = \{3, 4, 2\}$

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**Algorithm 1** Extended Johnson Algorithm (EJA)

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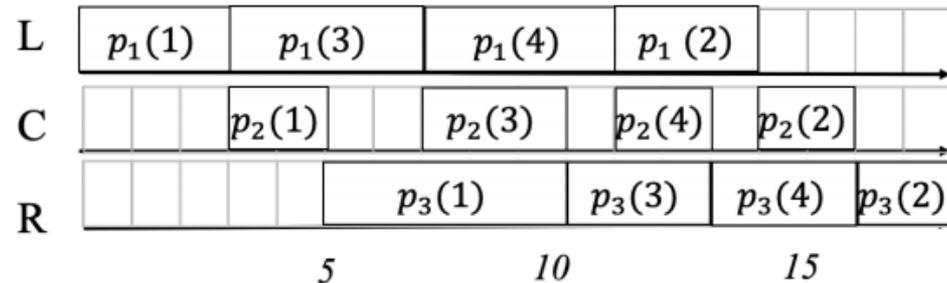
```

1:  $H \leftarrow L \leftarrow \phi$ 
2: for  $i = 1$  to  $m$  do
3:   if  $p_1(i) + p_2(i) \leq p_2(i) + p_3(i)$  then
4:      $H = H \cup p(i)$ 
5:   else
6:      $L = L \cup p(i)$ 
7: Sort  $H$  increasingly based on  $p_1(i) + p_2(i)$ 
8: Sort  $L$  decreasingly based on  $p_2(i) + p_3(i)$ 
9: Concatenate  $H$  and  $L$  to obtain  $\sigma$ 

```

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Path	$p_1(i)$	$p_2(i)$	$p_3(i)$
$i = 1$	3	2	5
$i = 2$	3	2	2
$i = 3$	4	3	3
$i = 4$	4	2	3



# Optimality

Theorem 3\*: If stage 2 is dominated by either stage 1 or 3,  $\max\{\min p_1(i), \min p_3(i)\} \geq \max p_2(i)$ , EJA is optimal.

If Theorem 3 fails, EJA still achieves an approximation ratio of  $5/3^+$ .

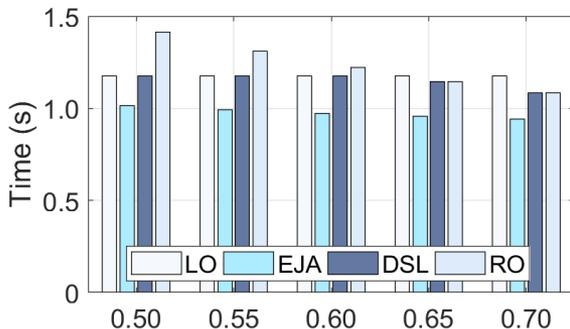
Path	$p_1(i)$	$p_2(i)$	$p_3(i)$
$i = 1$	3	2	5
$i = 2$	3	2	2
$i = 3$	4	3	3
$i = 4$	4	2	3

\*Chen et al, A new heuristic for three-machine flow shop scheduling, *OR*, 1996.

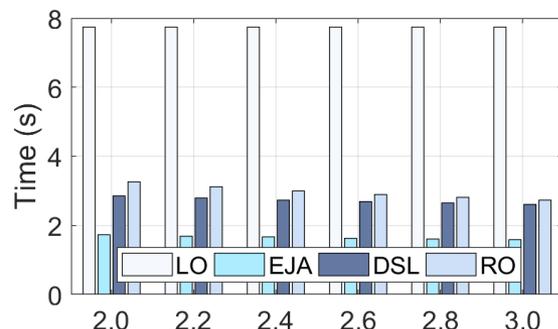
+Framinan et al, A review and classification of heuristics for permutation flow shop scheduling with makespan objectives, *JORS*, 2004.

# Simulation

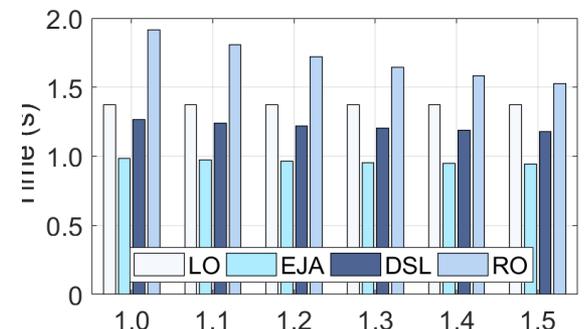
- Local and Cloud
  - Local: Raspberry Pi, Cloud: Amazon EC2
- Algorithms
  - LO: local only, **EJA**: Extended Johnson's Algorithm, DSL: no fine-grained pipeline, RO: remote only



(a) AlexNet: Bandwidth (MB/s)



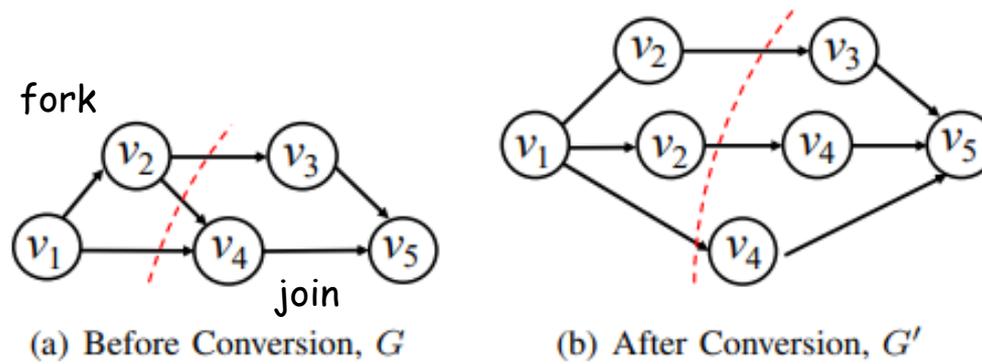
(b) Inception: Bandwidth (MB/s)



(c) RandWire: Bandwidth (MB/s)

# Extensions

- General structure: DAG
  - Conversion to multi-path
  - Replicated nodes at **join** and **fork**
- Heuristic solution
  - Scheduling: EJA on multi-path
  - Execution: Replicated node **executed once** (the first time)



# Multiple DNNs Offloading to Edges

## Internet of Vehicles: smart city

- Autonomous driving systems: perception is a key
- Multiple cameras/sensors: multiple (identical) DNNs
- V2X: V (vehicle), I (infrastructure), N (network), P (pedestrian)



# 4. Optimal Partition and Scheduling

- Multiple line-structure DNNs
  - AlexNet and VGG-16
  - Video analytics and VR/AR
- Optimal partition and scheduling
  - Brute force:  $O(k^n)$   
n: # of copies, k: # of layers
- Existence of a better solution?
  - Exploring special [application properties](#)

# Johnson Algorithm (JA)

- Closer look at the optimality for EJA
  - $\max\{\min p_1(i), \min p_3(i)\} \geq \max p_2(i)$
- However,  $p_3(i) \approx 0$ , reduced to 2-stage pipeline

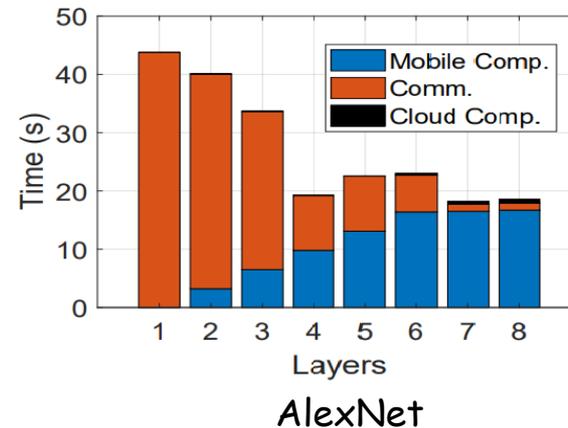
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## Algorithm 2 Johnson Algorithm (JA)

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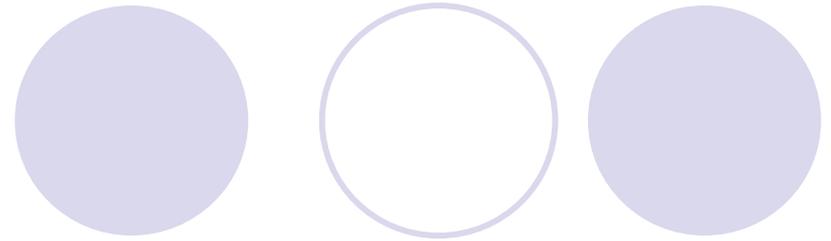
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7: Sort  $H$  increasingly based on  $p_1(i)$ 
8: Sort  $L$  decreasingly based on  $p_2(i)$ 
9: Concatenate  $H$  and  $L$  to obtain  $\sigma$ 
```

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Johnson, Optimal Two- and Three-Stage Production Schedules With Set-up Time Included, *Naval Research Logistics Quarter*, 1954.

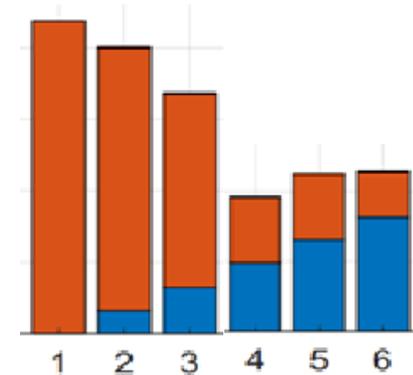
# JA in Illustration



- Optimality is guaranteed: JA on 2-stage pipeline

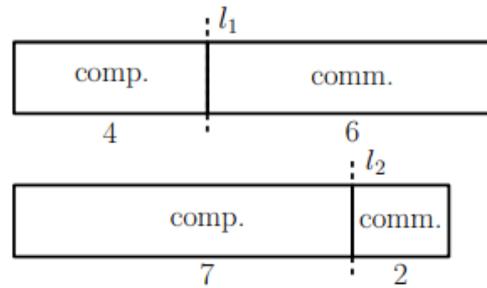
- First six layers of AlexNet

- One copy for each partition 6 copies
- $H = \{1, 2, 3\}$ , increasing order of blue
- $L = \{4, 5, 6\}$ , decreasing order of red
- Comm.-domination (or comp.-domination)

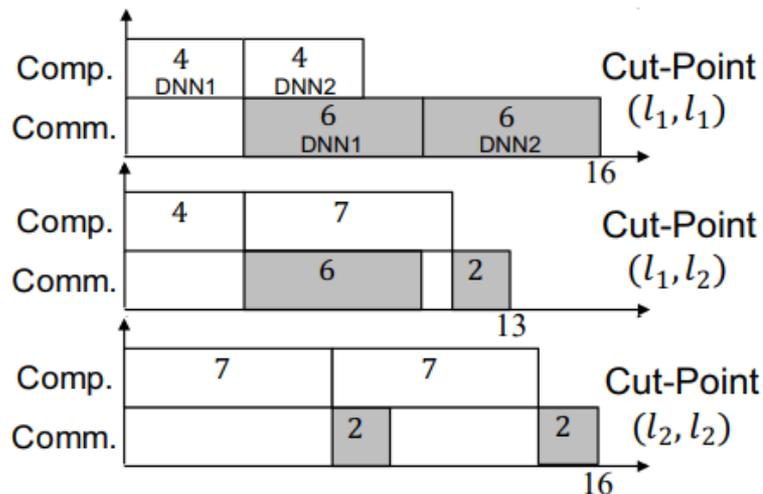


# Multiple Line-Structure Example

- Two copies of line-structure DNN

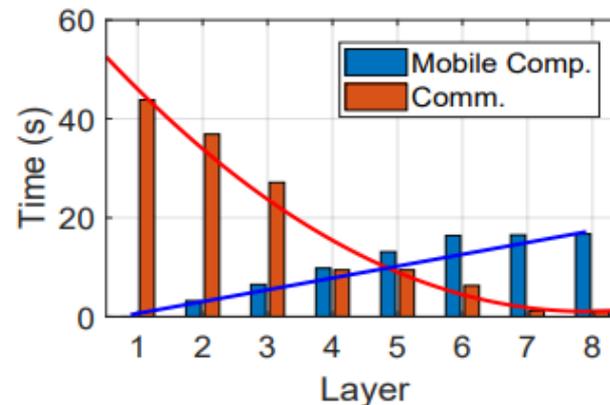
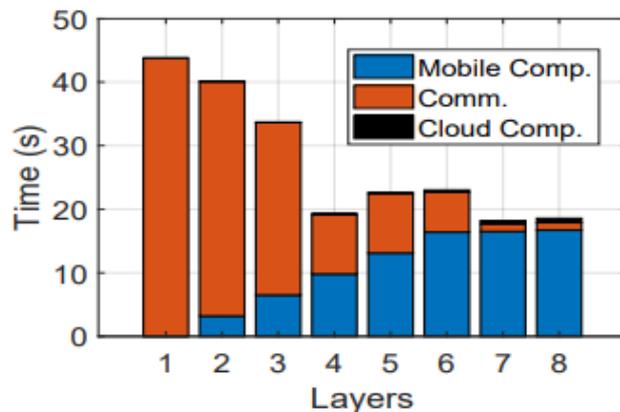


- Three possible partitions and scheduling



# Special Application Property

- Line-structure (as the layer increases)
  - Computation time: linear increasing (convex) function
  - Communication time: monotonic decreasing convex function
- Computation vs. communication
  - Data size: 2 - 12 MB
  - Speed (uplink): 2-5 Mbps (4G) and 6-54 Mbps (WiFi)



# Optimization Approximation

- Two functions on the continuous space
  - Both comp. and comm. are convex
  - One increasing and one decreasing

Theorem 4: A uniform partition of  $n$  line DNNs at the intersection will guarantee an approximation of  $1 + \frac{1}{n}$ .

## Formal Proof: convex optimization

- Intersection point has the  $\min \{ \max \{ \text{comp.}, \text{comm.} \} \}$  for  $n$  copies
- Strong duality, then KKT, the uniform partition has the least  $\max \{ \sum \text{comp.}, \sum \text{comm.} \}$

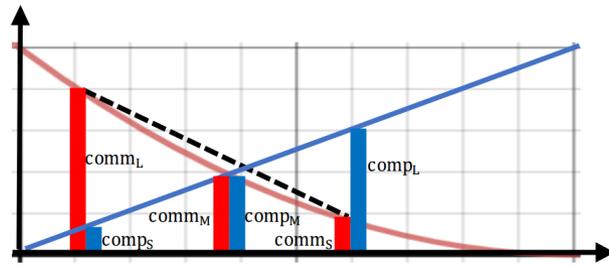
# Optimization Approximation (cont'd)

- Informal proof

- Pair-wise "merge" and "replaced" by the middle-point

$$\frac{f(x) + f(x')}{2} \geq f\left(\frac{x + x'}{2}\right)$$

- Height of the intersection  $x^* \leq \text{any } \max\{\text{comp.}, \text{comm.}\}$



- Two gaps: first pair in comm. and last pair in comp.

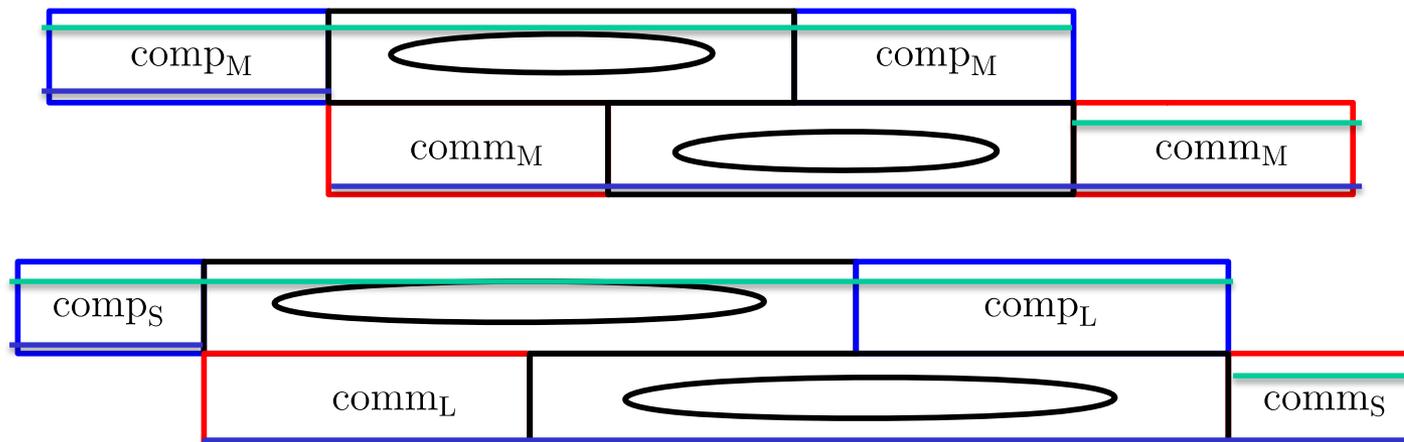
Only one is counted in comm.-domination or comp.-domination

- When  $n \rightarrow \infty$ ,  $1 + \frac{1}{n} = 1$

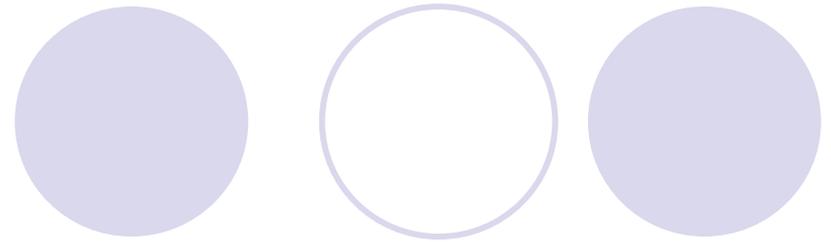
# Sufficient Condition for Optimality

- For a given set of partitions
  - Left/right most partition:  $(\text{comp}_s, \text{comm}_l) / (\text{comp}_l, \text{comm}_s)$
- Intersection partition:  $(\text{comp}_m, \text{comm}_m)$

Theorem 5: The uniform partition beats the given set if  $3\text{comp}_m < \text{comp}_s + \text{comp}_l + \text{comm}_s$  and  $3\text{comm}_m < \text{comp}_s + \text{comm}_l + \text{comm}_s$



# Simulation (cont'd)

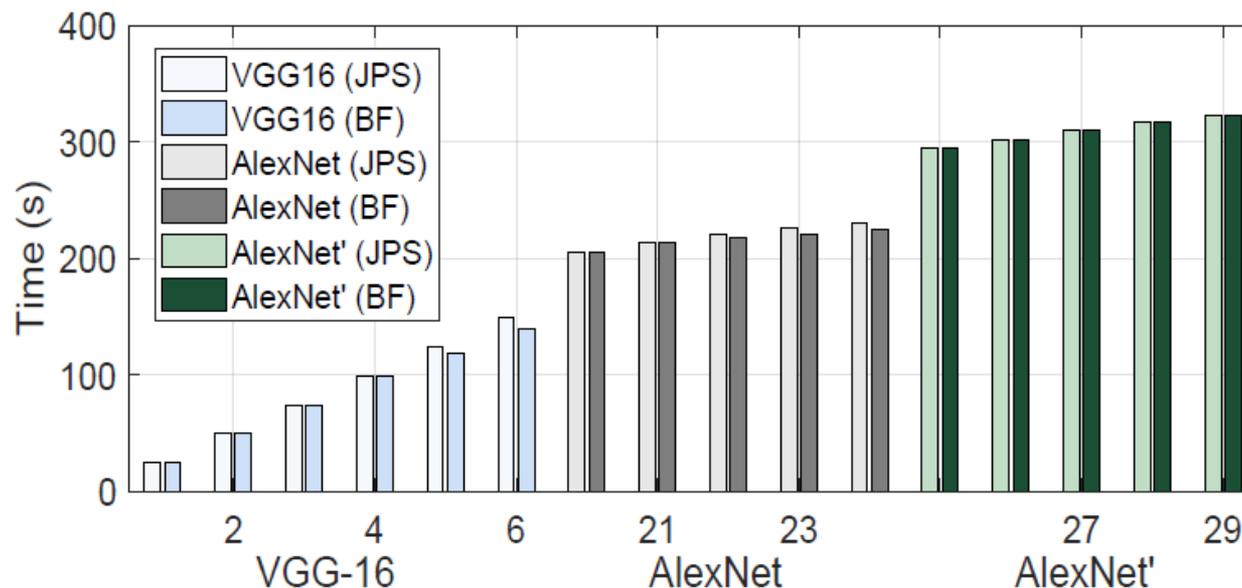


- Partition methods

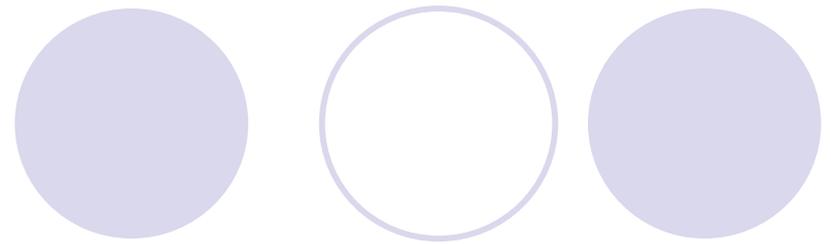
- Joint Partition and Scheduling: JPS, Brute Force: BF

- Application

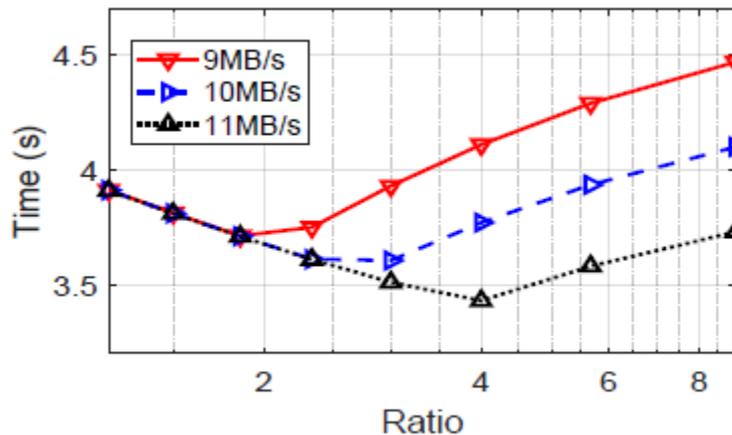
- VGG-16, AlexNet, and AlexNet' (curve fitting) with  $n = 1, \dots, 29$



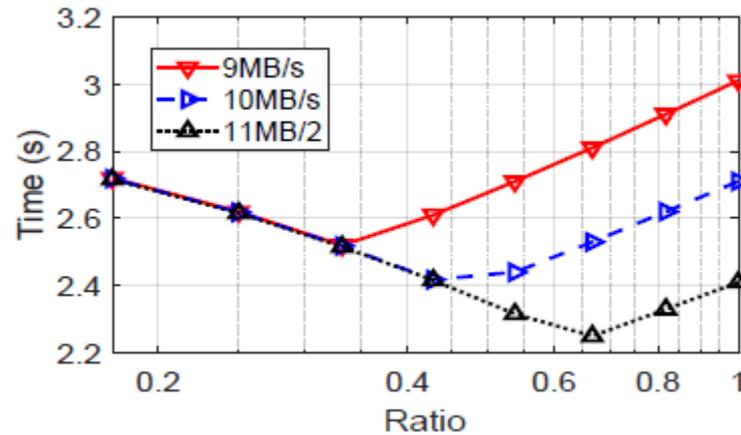
# Simulation (cont'd)



- Discrete version of intersection
  - Intersection:  $x^*$ , right:  $\lceil x^* \rceil$  and left:  $\lfloor x^* \rfloor$
  - Ratio:  $\frac{|x^* - \lfloor x^* \rfloor|}{|x^* - \lceil x^* \rceil|}$



AlexNet



VGG-16

# 5. Conclusions and Future Work

- **Offloading as a Service**

- Edge/cloud networks

- **Different DNNs**

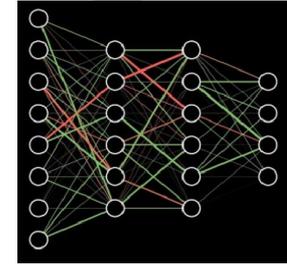
- Single path, multi-path, and DAG

- **Joint partition and scheduling**

- Johnson's rule and its extensions using pipelines
- Unique properties of comp. and comm. of DNNs

- **Future work**

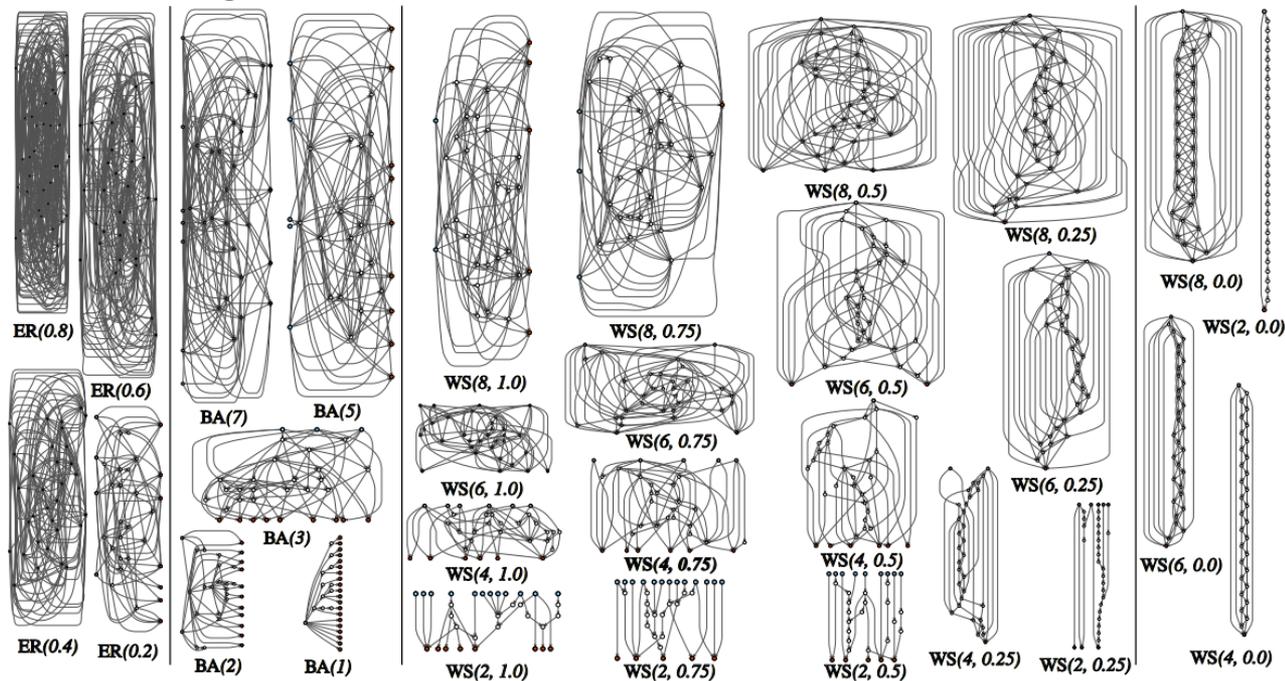
- Multi-tier offloading: edge and then cloud pipeline
- Optimal partition and scheduling of special DAGs



# 6. Some Reflections

Back to the past: interconnection networks

- Randomly wired NNs (random graphs): neuroscience
- Erdos-Renyi (ER): random, Barabasi-Albert (BA): preferential
- Watts-Strogatz (WS): small-world



# Avoiding: Reinventing the Wheel

## Reference searching practice

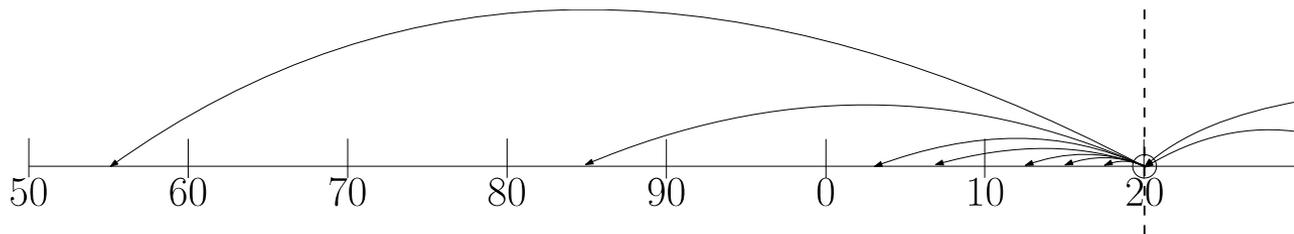
- 1 to 3 iterative process: references, references' references

## Knowledge span

- Career span: 5 to 7 years in MS + PhD period

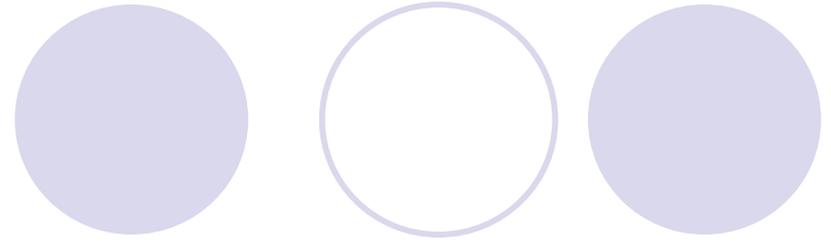
## Art of citations

- Good practice for citation: yearly distributions



Zheng and Wu, Snowballing Effects in Preferential Attachment, ICCCN'15

# Questions



Collaborators: Ning Wang (Rowan U.) and Yubin Duan (Temple U.)