Fault-Aware Flow Control and Multi-path Routing in Wireless Sensor Networks

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***Abstract*— Maintaining an acceptable level of network performance degradation in the presence of faults has been an active research area in wireless sensor networks (WSNs). This paper proposes a scheme to optimize the total network performance by exploiting fault-aware rate control and multi-path routing in WSNs. We use statistical information and estimation on each wireless link to characterize the effect of faults, and develop a leaky-path model. This model takes account of packet loss along fault-paths and the "shrinking" feature of effective flow at the destination node. Based on the leaky-path model, we propose a fault-correlated flow control and routing $((FC)^2R)$ approach to maximize the network utility of effective flows. A novel distributed algorithm is designed to adjust flow rates adaptively on each path, using multi-path routing. Our simulation results demonstrate that higher effective network throughput, and better fairness, can be achieved by our $((FC)^2R)$ algorithm than the standard optimal flow control in the presence of faults.**

I. Introduction

In recent years, there has been tremendous research interest in optimal flow control in wireless networks [2][3]. Most existing wireless network optimal flow control protocols assume that every node in the network delivers packets successfully, and that packet loss occurs mainly due to congestion. In a realistic environment, due to the fundamental characteristics of wireless mediums, WSNs are vulnerable to channel impairments, failure, interference and fading, etc. [4]. All of these can cause unreliable data transmission and a high packet loss rate on wireless links. As a result, the effective data rate received successfully at the destination node is lower than the transmission rate at the source. Faulty links can significantly affect the network performance in a wireless sensor network. Therefore, how to design a fault-tolerant flow control scheme to maintain an acceptable level of network performance degradation is a crucial issue for WSNs.

The majority of existing fault-tolerant communication schemes make use of network redundancy [5-7][9][10]. The rate control for multi-path routing could improve the end-to-end throughput, since it exploits the network resources by utilizing multiple source-destination paths. Since faulty links may exist on these source-destination paths, we model each path as a leaky path, over which data packets will be lost. In this paper, we approach the problem of optimizing the total network performance by jointly studying optimal flow control and multi-path routing in a wireless sensor network. We also consider the potential effect of faulty links on the resulting data throughput.

When faulty links are present in the wireless network, they have direct impacts on the ability to deliver data packets and routing protocol. Most of the existing fault-tolerant communication and resource allocation schemes classify each wireless node as either 'failed' or 'non-failed' and decide whether to choose it as a relay node or not. For example, authors of [7] proposed a traffic-aware dynamic routing algorithm to control traffic around the faulty links, and to scatter the excessive packets along multiple paths consisting of idle and under-loaded nodes. An adaptive fault-tolerant communication scheme (AFTCS) [8] adjusted the channel bandwidth allocation to fulfill the reliability requirement of sensors, according to the fault-tolerant priority and queue.

In order to characterize the effect of faulty links on effective throughput, each source must have perfect knowledge of how faulty behaviors impact various parts of the network. However, it is difficult to collect information on faulty links directly, in practice. The extent of fault at each network link relies on various unknown parameters, including malfunctioning/failure of internal components and unknown external faults in the network's environment. Hence, the impact of fault links can be characterized as probabilistic from the perspective of the network. We use statistical information and estimation on each wireless link to characterize the effect of faults as a random process, due to the uncertainty of the faults. Using statistical information about the probabilistic faulty links, each source can make resource allocations and routing decisions among multiple source-destination paths.

Most existing fault-tolerant communication schemes give preference to disjoint paths [6]. Node-disjoint multi-path routing protocols construct paths without common nodes. However, Chen et al. [9] proved that there is a theoretical limit on the security- performance tradeoff of node-disjoint

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multi-path routing with the presence of faults. Still, due to the random deployment of the sensor nodes, it is difficult to discover a large set of fully disjointed paths without any common node between multiple source nodes and multiple sink nodes. NC-RMR [10] increased the network reliability by constructing disjoint and braided multi-path and any intermediate node could forward packets received from any upstream neighbor node. Similar to [9][10], we consider that a sensor network contains multiple paths with some common nodes among them. These multiple paths turn to be failure-corrected between a source-destination pair, as any node failure in a set of paths may affect other paths that share the failed node. The correlation between non-fully-disjoint routing paths may bring out the loss of the effective throughput.

In this paper, we characterize the effect of faulty links as probabilistic, and incorporate the statistical information and estimation into the computation of the throughput. Then a leaky path model is proposed. In this model, the effective flow at the destination node is smaller than that from the source node since the data flow traverses over the probabilistic fault-paths. Based on this model, we generalize the OFC approach to obtain new problem formulations, namely fault-correlated flow control and routing ($(FC)^2R$), which maximize the total effective utility for multi-path WSNs. We further develop the distributed $(FC)^2R$ algorithm to obtain the effective rate allocation on multiple wireless paths. The numerical results indicate that higher effective throughput and better fairness among effective flow rates can be achieved by the $(FC)^2R$ algorithm than the standard OFC with multi-path routing.

The rest of the paper is organized as follows. We introduce our system model in Section II, and present the standard OFC approach with multi-path routing in WSNs in Section III. In Section IV, we describe the leaky-path model and present the $((FC)^2R)$ approach. In Section V, the $((FC)^2R)$ algorithm is given. The performance of our algorithm is evaluated in Section VI. Finally, Section VII concludes our work.

II. System Model

A. Network Model

We model our wireless network by a graph $G(\mathcal{V}, \mathcal{L}, \mathcal{C})$, where $\mathcal{V} = \{1, 2, ..., V\}$ is the set of nodes, and $\mathcal{L} = \{1, 2, ..., L\}$ is the set of links. We denote a link as a pair of nodes (i, j), where $i \in \mathcal{V}$ is the transmitter of the link and $j \in \mathcal{V}$ is the receiver. Let $\mathcal{S} = \{1, 2, ..., S\}$ be the set of sources, and $S \subseteq V$. We denote $\mathcal{C}$ as the set of fixed data rate c_l over $l \in \mathcal{L}$. Each source $s \in \mathcal{S}$ has k_s available paths or routes from the source to destination. We let a set $S(l)$ be the set of sources whose flows traverse through link l. The set of all the available paths of source s is defined by

$$R_s = [R_{s,1}, R_{s,2}, ..., R_{s,k_s}]$$

and the total number of paths in the network is defined by a $L \times K$ routing matrix R,

$$R = [R_1, R_2, ..., R_s]$$

where $K = k_1 + k_2 + + k_s$ is the total number of paths.

Denote the $k_s \times 1$ vector $A_{s,l}$ as the set flow s' paths pass through link l, whose nth element is equal to 1 if the path $R_{s,n}$ of flow s contains link l, and 0 otherwise. Each path $R_{s,n}$ is given by a subset $p_l \subseteq \mathcal{L}$ in the network.

For each source s, let $x_{(s,n)}$ be the rate of source s on the path $R_{s,n}$, and $x_s = \sum_{n=1}^{k_s} x_{s,n}$ be the total source rate. Let

$$X_s = [x_{s,1}, ..., x_{s,n_s}]$$

and

$$X = [X_1, X_2, ..., X_S]$$

be the path rate vectors of source s, and all sources.

A wireless sensor network consists of a group of wireless nodes connected by wireless channels, which are shared mediums, and are interference-limited. Under the MAC strategies such as time-division multiple access and random access, the users compete for exclusive access to the physical channel. We first assume that the transmission range equals the interference range. In the contention graph [3], only one link in the same maximal clique can be active at a time. Let $\Omega(l)$ be the set of cliques that link l belongs to, and $\mathcal{L}(\omega_l)$ be the set of links in the ωth maximal clique consisting of link l. Link l transmits data with a persistence probability p_l, and the sum of all transmission probabilities in a clique must be less than $1 : \sum_{l \in \mathcal{L}(\omega_l)} p_l \leq 1$.

B. Faulty Link Model

We consider the network which has no knowledge of the faulty behaviors: malfunctioning/failure of internal components, or some external faults in the network's environment. We assume that packet loss occurs only due to faulty links, rather than congestion, which can be managed by the provision of the underlying network protocols. In the presence of faulty links, the increased probability of collision usually leads to the increased packet loss ratio of each wireless link for the persistence transmission mechanism. As the faulty links' actions are unknown and uncertain to the network, the impact of fault is probabilistic from the perspective of the network. The behavior of node v is formulated as a random variable $H(v)$. The outcomes of $H(v)$ are defined as follows: $H(v) = 1$, if v receives the packet form the transmitter successfully; $H(v) = 0$, otherwise. The data packet ratio over each wireless link (v, j) can be formulated as a random variable using statistics from node v.

Due to the packet loss in wireless links, the data rate of a flow becomes lower and lower along its routing path, and the effective throughput received at the destination node is lower than the throughput at the source node. A path $p = [v_1, v_2, ..., v_p]$ can also be formulated as a random variable $T(p)$. The outcomes of $T(p)$ are defined as follows: $T(p) = 1$, p delivers the observed data packets successfully; $T(p) = 0$, otherwise. We jointly optimize flow control and routing over multiple paths based on statistical characterization of the impact of faulty links.

III. The Optimal Flow Control Approach for Multi-Path Routing

We first study the routing and optimal flow control problem in the wireless multi-path network without considering faulty links. We use Protocol Model [11] to describe when a transmission is successful. In a time slot, link l transmits data packets successfully if no other link in the contention range transmits packets simultaneously. As discussed in Section II, the set of links in one maximal clique Ω that contend with each other is denoted by $\mathcal{L}(\omega_l)$. We denote that link l transmits data with a persistence probability p_l to contend for the channel resource in its clique. Hence, the probability of the transmission over link l is successful $p_l \prod_{d \in \mathcal{L}(\omega_l)}(1-p_d)$. The average capacity on link l can be obtained as $c_l p_l \prod_{d \in \mathcal{L}(\omega_l)}(1-p_d)$, where c_l is the fixed data rate of link l. We impose the MAC resource constraint that the total flow rate over link l should be no more than the average capacity.

The cross-layer rate optimization across the transport layer and the MAC layer can be formulated as the maximization of the network utility under the constraints coming from MAC protocol and multi-path routing. The objective of optimal flow control is to choose the rates X, so as to maximize the total utility $\sum_{s \in S} U_s(x_s)$ subject to the constraints:

$$\text{Problem: } \max \textstyle\sum_{s \in S}(U_s(\sum_{n=1}^{k_s} x_{s,n}))$$

$$\begin{aligned} s.t.: \sum_{s \in S(l)} A_{s,l} X_s &\leq c_l p_l \prod_{d \in \mathcal{L}(\omega_l)} (1-p_d) \\ x_s^{min} &\leq \sum_{n=1}^{k_s} x_{s,n} \leq x_s^{max} \\ 0 &\leq \sum_{l \in \mathcal{L}(\omega_l)} p_l \leq 1 \end{aligned} \tag{1}$$

where x_s^{max} and x_s^{min} are the maximum and minimum flow data rates of s. The utility function $U_s(\cdot)$ is assumed to be [12]:

$$U_s^{\alpha}(x_s) = \begin{cases} log x_s & if\, \alpha = 1 \\ (1-\alpha)^{-1} x_s^{1-\alpha}, & if\, \alpha \geq 0, \alpha \neq 1 \end{cases} \tag{2}$$

Appealing to the Lagrangian dual method, a dual algorithm for the updates of source rates and link prices is given by:

$$x_{s,n}(t+1) = [x_{s,n}(t) + \gamma L_{x_{s,n}}(x(t), \lambda(t))]_{x_s^{min}}^{x_s^{max}} \tag{3}$$

$$\lambda_l(t+1) = [\lambda_l(t) - \gamma L_{\lambda_l}(x(t), \lambda(t))]^+ \tag{4}$$

where $L_{x_{s,n}}(\cdot)$ is the gradient of L with respect to $x_{s,n}$ and $L_{\lambda_l}(\cdot)$ is the gradient of L with respect to λ_l, and $[z]_a^b = min\{b, max\{z, a\}\}$, $[z]^+ = max\{z, 0\}$.

IV. Fault-Correlated Flow Control and Routing Approach

A. Leaky-path Model

In our wireless network, the ratios of packet loss on wireless links decline when faulty links exist in WSNs. We define the ratio of packets successfully delivered over link $l(i,j)$ as $h_l = Pr\{H(i) = 1\}$, a random variable, due to the uncertainty in the impact of faults.

In traversing the path $R_{s,n}$, the source s estimates the effective end-to-end packet delivery probability. The effective flow rate of source s is reduced to $x_{s,n} \prod_{l \in R_{s,n}} h_l$ at the destination node. The end-to-end packet success ratio for the path can be formulated as

$$g_{s,n} = \prod_{l \in R_{s,n}} h_l \tag{5}$$

which is also a random variable because of the random variable h_l. We denote $\theta_{s,n}$ as the mean of random variable $g_{s,n}$ and Θ_s as the $L_s \times \mathbf{1}$ vector of the mean $\theta_{s,n}$ of random variable $g_{s,n}$.

Necessary conditions of optimal flow control were introduced previously; the first constraint in (1) shows that the average data rate should be no more than average capacity, without considering faulty links. For a wireless sensor network under probabilistic faults, we must consider the average capacity and the reduction of flow rate, due to faults at intermediate links. At the intermediate node i of flow on path $R_{s,n}$, the correctly received data rate can be represented as $g_{s,n}^{(i)} x_{s,n}$. The capacity constraint on the average data rate, imposed by faults, can be given as follows:

$$\sum_{Q_s} g_{s,n}^{(i)} x_{s,n} \leq c_{(i,j)} p_{(i,j)} \prod_{d \in \mathcal{L}(\omega_{(i,j)})} (1-p_d) \tag{6}$$

where $Q_s = s \in S((i,j)) \bigwedge R_{s,n} \in R_s \bigwedge (i,j) \in R_{s,n}$. We can see that the effective data rate traveling over the path $R_{s,n}$ becomes lower and lower along its route, and we consider a path with faulty links to be a leaky path.

Since multiple paths may have common nodes while there is no shared link between paths, the random variable $g_{s,n}$ may not be independent. From the above subsection, $\theta_{s,n}$ is the mean of random variable $g_{s,n}$. We then compute the covariance of two paths $R_{s,n}$ and $R_{s,m}$ as follows:

$$\phi_{s,n,m} = E[g_{s,n} g_{s,m}] - E[g_{s,n}]E[g_{s,m}] \tag{7}$$

Then we define a $n_s \times n_s$ matrix Φ_s with (n,m) entry $\phi_{s,n,m}$ to measure the correlation of multiple paths between source s and its sink. This correlation means that any packet loss in a set of correlated paths sharing the faulty nodes may decrease the effective throughput. The variance of source s' throughput can be expressed as $X_s \Phi_s X_s^T$ which is based on the variance between correlated paths.

B. $(FC)^2R$ Approach with Multiple paths

At the destination node of flow s, the correctly received data rate can be obtained as $\sum_{n=1}^{k_s} \theta_{s,n} x_{s,n}$. Each flow s has a utility function associated with the effective rate $\sum_{n=1}^{k_s} \theta_{s,n} x_{s,n}$ and the variance between correlated paths. Our principle objective is to maximize the overall effective network utility of all flows:

$$\text{Problem: } \max \textstyle\sum_{s \in S}(U_s(\sum_{n=1}^{k_s} \theta_{s,n} x_{s,n})) - k_s X_s \Phi_s X_s^T$$

$$s.t.: \sum_{Q_s} g_{s,n}^{(i)} x_{s,n} \le c_{(i,j)} p_{(i,j)} \prod_{d \in \mathcal{L}(\omega_{(i,j)})} (1-p_d)$$
$$x_s^{min} \le \sum_{n=1}^{k_s} x_{s,n} \le x_s^{max} \quad (8)$$
$$0 \le \sum_{l \in \mathcal{L}(\omega_l)} p_l \le 1$$

where k_s is the weight on the overall utility and variance between correlated paths. We use a change of variables $\tilde{x}_{s,n} = log(x_{s,n})$, $\tilde{X}_s = [e^{\tilde{x}_{s,1}}, ..., e^{\tilde{x}_{s,n_s}}]$ and $\tilde{U}_s(\tilde{x}_{s,n}) = \sum_{s \in S}(U_s(\sum_{n=1}^{k_s} \theta_{s,n} e^{\tilde{x}_{s,n}})) - k_s \tilde{X}_s \Phi_s \tilde{X}_s^T$. This reformulation turns the problem into:

$$\text{Problem: } \tilde{U}_s(\tilde{x}_{s,n})$$

$$s.t.: log \sum_{Q_s} g_{s,n}^{(i)} e^{\tilde{x}_{s,n}} - \log c_{(i,j)} - \log p_{(i,j)}$$
$$- \sum_{d \in \mathcal{L}(\omega_{(i,j)})} \log(1-p_d) \le 0$$
$$x_s^{min} \le \sum_{n=1}^{k_s} e^{\tilde{x}_{s,n}} \le x_s^{max} \quad (9)$$
$$0 \le \sum_{l \in \mathcal{L}(\omega_l)} p_l \le 1$$

Note that problem (9) is separable, but may not be a convex optimization problem, since the objective function $\tilde{U}_s(\tilde{x}_{s,n})$ may not be a (strictly) concave function.

Lemma 1: The function $\tilde{U}_s(\tilde{x}_{s,n})$ is strictly concave with $k \ge 1$.

Proof: We omit it for the sake of the space. ■

The Lagrangian function is given by Equation (10) at the top of the next page. Here, $\overline{\lambda} = [\overline{\lambda}_1, ..., \overline{\lambda}_s]^T$, $\underline{\lambda} = [\underline{\lambda}_1, ..., \underline{\lambda}_s]^T$, $\lambda = [\lambda_1, ..., \lambda_L]^T$ and $u = (\overline{\lambda}, \underline{\lambda}, \lambda)$ are all nonnegative. The objective function of the dual problem is given by

$$D(\lambda, \overline{\lambda}, \underline{\lambda}) = max_{\{\tilde{x}\}} L(\tilde{x}, \lambda, \overline{\lambda}, \underline{\lambda}) \quad (11)$$

Based on the Arrow-hurwizz gradient method [14, pp. 154-165], we can obtain:

$$\tilde{x}_{s,n}(t+1) = [\tilde{x}_{s,n}(t) + \gamma L_{\tilde{x}_{s,n}}(\tilde{x}(t), \lambda(t))]^+ \quad (12)$$

$$L_{\tilde{x}_{s,n}}(\tilde{x}(t), \lambda(t)) = U^{'}(\sum_{n=1}^{k_s} \theta_{s,n} e^{\tilde{x}_{s,n}(t)}) - \bar{\lambda}_s(t) e^{\tilde{x}_{s,n}(t)}$$
$$+ \underline{\lambda}_s(t) e^{\tilde{x}_{s,n}(t)} - g_{s,n}^{(i)} e^{\tilde{x}_{s,n}(t)} \sum_{(i,j) \in \mathcal{L}(s)} (\frac{\lambda_{(i,j)}(t)}{\sum_{Q_k} g_{k,n}^{(i)} e^{\tilde{x}_{k,n}(t)}})$$
$$- (2 \sum_{i=1}^{n_s} \phi_{s,n,i} e^{2\tilde{x}_{s,n}} + \sum_{i=1}^{n_s} (\phi_{s,i,n} + \phi_{s,n,i}) e^{\tilde{x}_{s,i} + \tilde{x}_{s,n}})$$
$$(13)$$

where γ is a small step size. The master dual problem is

$$min_{\{\lambda, \overline{\lambda}, \underline{\lambda}\}} D(\lambda, \overline{\lambda}, \underline{\lambda}) \quad (14)$$

We have the Lagrangian multipliers for the dual by the gradient method, as follows:

$$\lambda_{(i,j)}(t+1) = [\lambda_{(i,j)}(t) - \gamma(\sum_{d \in \mathcal{L}(\omega_{(i,j)})} \log(1-p_d(t))$$
$$+ \log p_{(i,j)}(t) + \log c_{(i,j)} - \log(\sum_{Q_s} g_{s,n}^{(i)} e^{\tilde{x}_{s,n}(t)})]^+ \quad (15)$$

$$\bar{\lambda}_s(t+1) = [\bar{\lambda}_s(t) + \gamma(x_s^{max} - \sum_{n=1}^{k_s} e^{\tilde{x}_{s,n}(t)})]^+ \quad (16)$$

$$\underline{\lambda}_s(t+1) = [\underline{\lambda}_s(t) - \gamma(x_s^{min} - \sum_{n=1}^{k_s} e^{\tilde{x}_{s,n}(t)})]^+ \quad (17)$$

The persistent probability can be yielded [13]:

$$p_{(i,j)}(t) = \frac{\lambda_{(i,j)}(t)}{\sum_{k \in \mathcal{L}(\omega_{(i,j)})} \lambda_k(t)} \quad (18)$$

V. Fault-Correlated Flow Control and Routing Algorithm

In this section, we propose **Algorithm 1** based on the problem formulation of fault-correlated flow control and routing. It is designed in a fully distributed manner, in which each update computation is only based on the local information of a source or a link.

Algorithm 1 $(FC)^2R$ Algorithm

- Link l's algorithm
  At each time t = 1, 2, ..., each link l:
  1) Aggregates flow rates $x_{s,n}(t)$ and $\tilde{x}_{s,n}(t)$ for all paths $R_{s,n}$ that contain link l;
  2) Updates the persistence probabilities by using (18);
  3) Computes a new link error price by formula (15);
  4) Generates the parameters $\frac{\lambda_{(i,j)}(t)}{\sum_{Q_k} g_{s,k}^{(i)} e^{\tilde{x}_{s,k}(t)}}$ to be fed back to the source node along the routing path;
  5) Communicates the new price λ_l to all sources whose paths $R_{s,n}$ contain link l.
- Source s's algorithm
  At each time t = 1, 2, ..., each source s:
  1) Receives $\sum_{(i,j) \in \mathcal{L}(s)} (\frac{\lambda_{(i,j)}(t)}{\sum_{Q_k} g_{s,k}^{(i)} e^{\tilde{x}_{s,k}(t)}})$ from the paths for all its paths $R_{s,n}, n = 1, ..., k_s$;
  2) Computes $\tilde{x}_s(t+1)$ using Eqs.(12) and (13);
  3) Updates the path rate $x_{s,n}(t+1) = e^{\tilde{x}_{s,n}(t+1)}$ and source rate $x_s(t+1) = \sum_{n=1}^{k_s} x_{s,n}(t+1)$;
  4) Communicates the upper and lower bound price $\overline{\lambda}$ and $\underline{\lambda}$ for the next step, according to (16) and (17);
  5) Communicates the new parameter $\tilde{x}_{s,n}(t+1)$ to all the links which are contained in path $R_{s,n}$.

There are usually oscillations in the Lagrangian algorithm for the multi-path network. In order to improve the convergence speed and eliminate the effect of oscillation, we introduce an augmented variable $f_{s,n}$ to the following modified

$$L(\tilde{X}, f, \lambda, \underline{\lambda}, \bar{\lambda}) = \sum_{s \in S} (U(\sum_{n=1}^{k_s} \theta_{s,n} e^{\tilde{x}_{s,n}}) + \bar{\lambda}(x_s^{max} - \sum_{n=1}^{k_s} e^{\tilde{x}_{s,n}}) - \underline{\lambda}(x_s^{min} - \sum_{n=1}^{k_s} e^{\tilde{x}^{s,n}})) - \sum_{(i,j)} \lambda_{(i,j)} (\log(\sum_{Q_s} g_{s,n}^{(i)} e^{\tilde{x}_{s,n}}) - \log c_{(i,j)} - \log p_{(i,j)} - \sum_{d \in \mathcal{L}(\omega_{(i,j)})} \log(1 - p_d)) \quad (10)$$

objective function, which replaces the objective function of formulation (9):

$$\max \sum_{s \in S} (U_s(\tilde{\theta}_s \sum_{n=1}^{k_s} e^{\tilde{x}_{s,n}})) - \sum_{s \in S} \sum_{n=1}^{k_s} \frac{1}{2} (\tilde{x}_{s,n} - f_{s,n})^2 \quad (19)$$

Let $\tilde{x}_{s,n}^*$ denote the optimal value of (9), then $\tilde{x}_{s,n} = \tilde{x}_{s,n}^*$, $f_{s,n} = f_{s,n}^*$ is the optimal value of the utility maximization problem with objective function (19). Hence, the optimal value of (19) coincides with that of (9). Thus, Eq. (12) is slightly modified by applying the new objective function:

$$\tilde{x}_{s,n}(t+1) = [(1-\gamma)\tilde{x}_{s,n}(t) + \gamma f_{s,n}(t) + \gamma(L_{\tilde{x}_{s,n}}(\tilde{x}(t), \lambda(t)))]^+$$
$$f_{s,n}(t+1) = (1-\gamma) f_{s,n}(t) + \gamma \tilde{x}_{s,n}(t) \quad (20)$$

VI. Performance Evaluations

In this section, we present numerical results to demonstrate the efficiency of our solutions. Consider a wireless network given in Fig. 1 with 10 links, 9 nodes, and two sources $s1$ and $s2$. $s1$ routes its flow along two paths (A→B→D→E) with path rate $x_{1,1}$, and (A→C→D→E) with path rate $x_{1,2}$. $s2$ routes its flow along two paths, (F→C→G→I) with path rate $x_{2,1}$, and (F→H→G→I) with path rate $x_{2,2}$. The average data rate of each link is set as 1 in Mbps. Each fault parameter $h_{i,j}$ is modeled as an independent beta random variable with parameters ($\delta_{i,j}$,$\beta_{i,j}$) as $h_{i,j} = \frac{\delta_{i,j}}{\delta_{i,j}+\beta_{i,j}}$. For example, link (A, C) with $h_{A,C} = 0.9$ has corresponding parameters $\delta_{A,C} = 39.6$ and $\beta_{A,C} = 4.4$. The utility function is the algorithm utility function where $\alpha = 1$, $x_s^{min} = 0$ and $x_s^{max} = 1$, $\forall s$. The step size γ is set to 0.01.

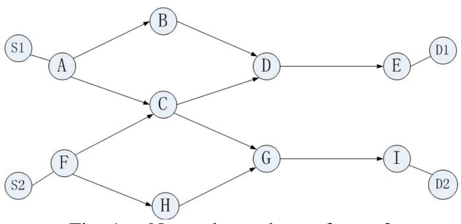

Fig. 1. Network topology of case 2

We denote that x' is the data rate of one flow at the source node, and x as the effective data rate at the destination node. In the OFC approach, we use the algorithm in Equation (3) and (4), where $L_{x,n}(x(t), \lambda(t))$ and $L_{\lambda_l}(x(t), \lambda(t))$ are calculated based on the flow rate on each link and link price on each path. Fig. 2 shows the flow rates at the source node among four paths using the the OFC approach. Two path rates $x'_{1,1}$ and $x'_{1,2}$ of source s_1 converge to (0.09, 0.054), while $x'_{2,1}$ and $x'_{2,2}$ of source s_2 converge to (0.054, 0.09). Without considering the effect of faulty links, the source algorithm computes $x'_{1,2}$ and $x'_{2,1}$ to share the bottleneck node C with an equal flow rate 0.054. The OFC approach provides a fair rate allocation in which $x'_{1,1}$ equals $x'_{2,2}$, and $x'_{1,2}$ equals $x'_{2,1}$ at the source node.

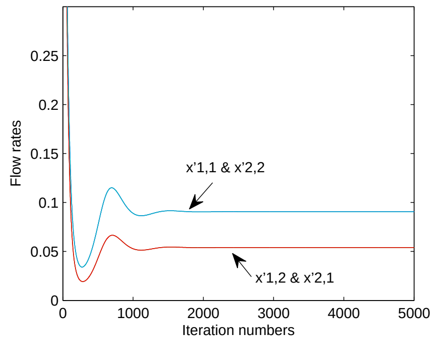

Fig. 2. The flow rates at the source nodes of OFC

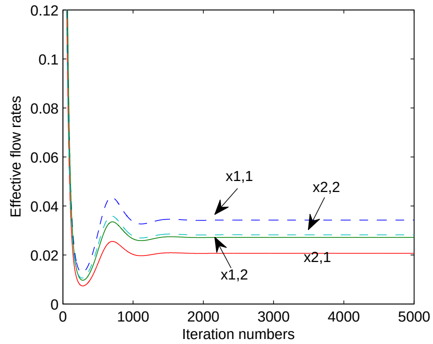

Fig. 3. The effective flow rates at the destination nodes of OFC

In fact, the effective rates of four flows cannot maintain the fairness at their destination node after traveling along the leaky-paths. The effective rates of four paths in the OFC approach are depicted in Fig. 3. The effective flow rates decrease to (0.034, 0.027, 0.02, 0.028) among four paths. For $(FC)^2R$, the effective rates in Algorithm 1 are shown in Fig. 4. It can be seen that $(FC)^2R$ yields higher effective rates (0.087, 0.04, 0.037, 0.086) for four flows than OFC. Fig. 5 clearly shows that the effective throughput of $(FC)^2R$ can be higher than that of OFC. We take a closer look at rate allocation among flows and effective flows in Fig. 2 and Fig. 3. In Fig. 2, four flows share a fair rate allocation that $x'_{1,1}$ is equal to $x'_{2,2}$, and $x'_{1,2}$ is equal to $x'_{2,1}$. However, the fairness is broken due to different faulty effects on four paths. $x_{2,2}$

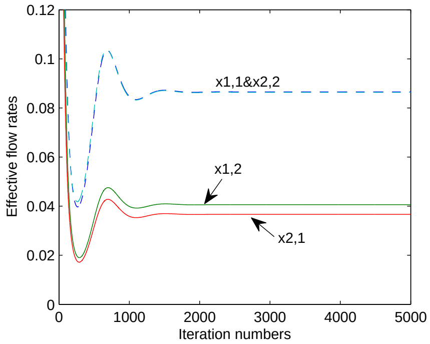

Fig. 4. The effective flow rates at the destination nodes of $(FC)^2R$

and $x_{2,1}$ are lower than $x_{1,1}$ and $x_{1,2}$, respectively, in Fig. 3. In Fig. 4, the effective rates $x_{2,2}$ and $x_{1,1}$ in $(FC)^2R$ are closer to each other than those in the OFC approach. The same situation happens in $x_{2,1}$ and $x_{1,2}$. It demonstrates that better fairness is attained among effective flow rates by $(FC)^2R$. The source adjusts its flow rate on each path adaptively to compensate for the data loss by multi-path routing in our algorithm, which takes into account the effect of faulty links in utility functions and constraints. It is clear that the network performance under faulty links is improved through both higher effective throughput and better fairness among effective flows by our proposed algorithm.

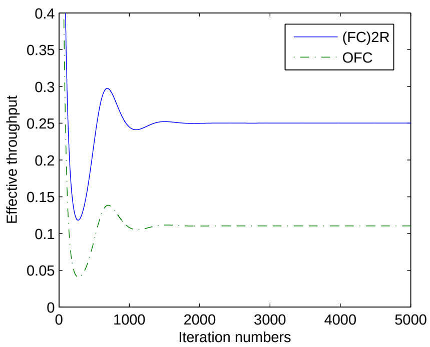

Fig. 5. The comparison of effective throughput of $(FC)^2R$ and OFC

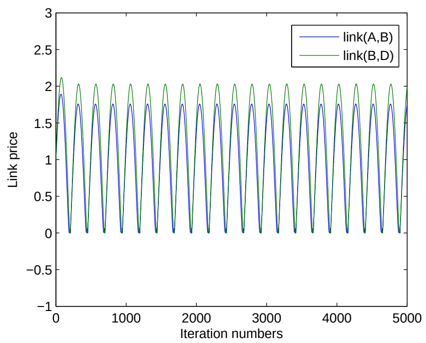

Fig. 6. Oscillations of link price

In the simulation, we run the algorithm with the original objective function in (9). The oscillation is observed in Fig. 6, which motivates the modification of the objective function with the augmented variable $f_{s,n}$ for each $\tilde{x}_{s,n}$. To prevent potential oscillations, we replace Eq.(12) with Eq.(20) in Algorithm 1. The effective rates for two sources over four paths, convergence to (0.087,0.04,0.037,0.086) is in Fig. 4.

VII. Conclusion

In this paper, we investigate the problem of rate control and multi-path routing in the wireless network in the presence of faults whose effects can be characterized statically. We incorporate the impact of faulty links into the problem of fault-aware flow control for multi-path routing in WSNs. Due to faulty links, the data flow becomes "smaller and smaller" along its leaky-path. We formulate the problem as a holistic optimization problem and derive the objective function with effective flows over leaky-paths. We present the $((FC)^2R)$ approach that maximizes the overall effective network utility. A distributed algorithm is developed using the decomposition technique. Simulations show that $((FC)^2R)$ performs better than OFC.

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