Supplemental Material for “On Data Center Network Architectures for Interconnecting Dual-Port Servers”

Dawei Li and Jie Wu, *Fellow, IEEE*

1 Proof of Theorem 1

For $c = 0$, the lengths of a server-to-server-direct hop and a server-to-server-via-a-switch hop are equal. We consider the maximum number of other servers that a server S can reach within distance d. Within distance 1, S has two choices to reach other servers: the first one is to connect two other servers directly, and the second one is to connect two switches, each of which connects $n-1$ other servers, resulting in a total of $2(n-1)$ servers. Obviously, the second choice is better because S reaches more other servers, and more servers has one port remaining for further expansion. Within distance 2 of S, based on the second choice, the $2(n-1)$ servers connect to $2(n-1)$ switches, each of which connects $n-1$ other servers, resulting in another $2(n-1)^2$. Extending to distance d, S can reach at most $2(n-1)+2(n-1)^2+\cdots+2(n-1)^d$ other servers. Plus the original server S itself, the maximum number of dual-port servers that any network can accommodate is: $N_v \leq 1+2(n-1)+2(n-1)^2+\cdots 2(n-1)^d$ $= (2(n-1)^{d+1}-n)/(n-2) = N_v^{ub}$.

2 Proof of Theorem 2

Consider the maximal number of servers that a server S in a DCN can reach within distance d. For $1 \leq d < 1+c$, S can reach at most 2 other servers through server-to-server-direct hops; $\lceil d/(1+c) \rceil = 1$; the theorem holds.

For $d \geq 1+c$, we consider three choices of S to reach as many other servers as possible within two hops (server-to-server-direct hop(s) and/or server-to-server-via-a-switch hop(s)).

The first one is to reach other servers only by server-to-server-direct hops; in this case, it can reach at most 4 other servers (if possible), 2 of which have one port remaining for further outreaching, and S's remaining outreaching distance is $d-2$.

The second choice is connecting S's two ports to two switches; by doing this, it can reach $2(n-1) > 4$ other servers, all of which have one port remaining for further outreaching, and S's remaining outreaching distance is $d-(1+c) \geq d-2$. Thus, compared with the first choice, the second one is always better.

The third choice is to connect S's two ports to two other servers first; next, the two new servers connect to two switches, each of which connecting $n-1$ other servers, if $d \geq 1+(1+c)$. By the third choice, S can reach at most $2+2(n-1) = 2n$ other servers, of which $2(n-1)$ have one port remaining. However, if the next step of the third choice is possible, i.e. $d \geq (1+c)+1$, in the second choice, the $2(n-1)$ servers can also connect to $2(n-1)$ other servers within distance $(1+c)+1$. The second choice results in $4(n-1) > 2n$ new servers; there are also $2(n-1)$ servers with one port remaining. Thus, the second choice is also better than the third one.

Based on the analysis of these three choices, we can see that S should always try to reach other servers via server-to-server-via-a-switch hops, if the remaining outreaching distance allows it to do so. Within $\lfloor d/(1+c) \rfloor$ server-to-server-via-a-switch outreaching hops, S can reach at most $2(n-1)+2(n-1)^2+\cdots+2(n-1)^{\lfloor d/(1+c) \rfloor}$ other servers. Exploiting the remaining outreaching distance $d-(1+c)\lfloor d/(1+c) \rfloor$, S can reach at most another $2(n-1)^{\lfloor d/(1+c) \rfloor}$ servers, if possible. Thus, the maximal number of servers in any network with diameter less than or equal to d is $N_v \leq (2(n-1)^{\lfloor d/(1+c) \rfloor+1}-n)/(n-2)+2(n-1)^{\lfloor d/(1+c) \rfloor} \leq (2(n-1)^{\lceil d/(1+c) \rceil+1}-n)/(n-2) = N_v^{ub}$.

• *Dawei Li and Jie Wu are with the Department of Computer and Information Sciences, Temple University, Philadelphia, PA 19122, E-mail: {dawei.li, jiewu}@temple.edu*