**II. Game Playing** (21 pts.)

There’s a simple game you can play with silver dollars. (A silver dollar is a coin worth $1.) There are five (5) silver dollars on the table and you and your opponent take turns picking up 1 or 2 coins until none is left. You get to keep each silver dollar you pick up. *But*, if you pick up the last coin, you have to pay $2 to your opponent. The object of the game is to finish with the most money.

1. (4 pts.) Consider building a game tree to solve the above problem. What would each state in the game tree represent? What is/are the operator(s)?

   **Solution:** Each state should indicate the following:
   
   (a) The number of silver dollars on the table
   
   (b) The number of silver dollars that the first and second players have [Note: only one of these is necessary]
   
   (c) Whether it is the first or second player’s turn

   The operators are:
   
   (a) Pick up one silver coin
   
   (b) Pick up two silver coins if at least two are left

2. (3 pts.) What cost function, i.e., static evaluation function, do you use to evaluate terminal nodes?

   **Solution:** First pay the fine of $2. The static evaluation function can be either one of the following:
   
   - The number of coins that the first player has
   
   - The number of coins that the first player has minus the number of coins that the second player has

   The first one is used in the rest of this problem, but the second one will produce similar results.

3. (6 pts.) Draw the (complete) resulting minimax tree for this problem.

   **Solution:** Note that the nodes for which it is the first player’s turn are maximizing nodes, and the other nodes are minimizing nodes.

   Start On table=5, First=0, Second=0, Turn=First, Value=4

   - Start:1 On table=4, First=1, Second=0, Turn=Second, Value=4
   
     - Start:1:1 On table=3, First=1, Second=1, Turn=First, Value=5
     
     * Start:1:1:1 On table=2, First=2, Second=1, Turn=Second, Value=1
     
     - Start:1:1:1:1 On table=1, First=2, Second=2, Turn=First, Value=1
4. (4 pts.) Your opponent courteously offers to let you go first. If you accept, what is your first move? How much will you earn?

Solution: Take one coin. I will earn $4 unless the opponent takes just one coin in his first move, in which case I will earn $5.

5. (4 pts.) Clearly indicate one example of an alpha-beta pruning in this tree. If there isn’t one, say how you would rearrange the tree to facilitate pruning.

Solution: Let us say that when evaluating Start : 1 : 1, we first evaluate Start : 1 : 1 : 1 : 2 : 1. Note that Start : 1 : 1 is a max node, and we know that its value is at least 5. If we next evaluate Start : 1 : 1 : 1 : 2, whose value is 4, we can prune the other child of Start : 1 : 1 : 1 because its value will never be greater than 4.