Quantum Bayesian Inference on Rejection Sampling: Grover-Accelerated Rare Event Amplification

Temple University — CIS 5603 Project Presentation

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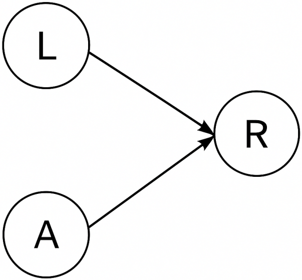
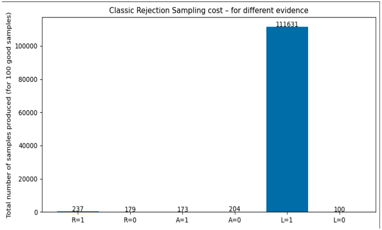
Bayesian Network Problem

Why Classical Inference Fails for Rare Events

- Marginalization becomes expensive: $P(L) = \sum_{A,R} P(L, A, R)$
- Rejection sampling is extremely inefficient for rare evidence: $P(L = 1) = 0.001 \Rightarrow$ requires $\sim 100{,}000$ samples
- Reason:
  - Almost all samples are discarded
  - Classical inference becomes too slow





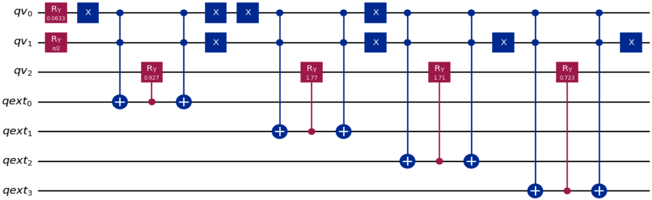
Quantum Solution: Grover Amplitude Amplification

Turning a Rare Event Into a High-Probability Outcome

- The Bayesian Network state is encoded into a quantum register
- An oracle is defined that marks the state where $L = 1$
- Grover behavior after k iterations:

$$\mathcal{Q}^k \left|\Psi_{\text{init}}\right\rangle = \sin\left(\frac{(2k+1)\theta}{2}\right)\left|\text{Good}\right\rangle + \cos\left(\frac{(2k+1)\theta}{2}\right)\left|\text{Bad}\right\rangle$$

- Required number of Grover iterations: $k \approx \dfrac{\pi}{4\sqrt{p}} \approx 24$



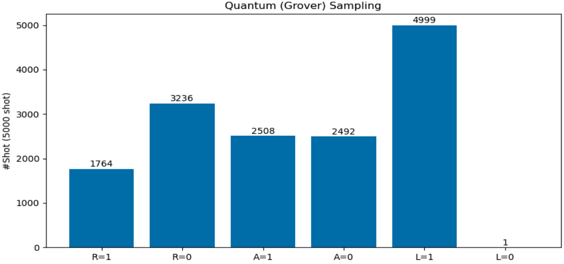
Simulator Results

Noise-free Qiskit Simulator (5000 shots)

- 5000 shots were executed
- The target rare event $L = 1$ is amplified to **over 99%**
- Clear quantum speedup:

$$\text{Classical requires } \sim 100{,}000 \text{ samples}$$
$$\text{Quantum requires just 1 shot per sample}$$

- Number of Grover iterations used: **24**



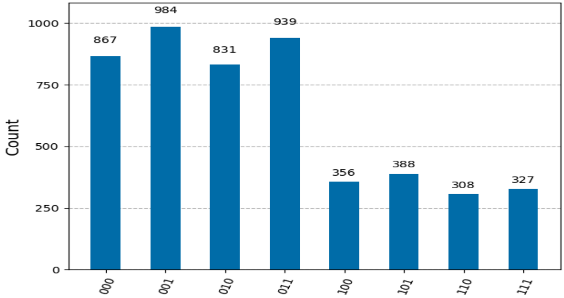
Real Quantum Hardware Results

IBM Torino (133 qubits — Free Tier)

- Circuit transpilation expanded the gates to over 1000 operations
- Hardware noise significantly degrades amplitude amplification
- Measured result:

$$P(L = 1) \approx 0.27$$

- Despite noise, still **better** than classical rejection sampling
- Grover iteration count: **24**



Conclusion: Quantum Speedup Demonstrated

For our test case (rare evidence with $P(L = 1) = 0.001$), producing just 100 good samples requires either:

- Classical rejection sampling $\rightarrow$ **100,000 total samples**
- Quantum amplitude amplification $\rightarrow$ **24 Grover iterations**

- Simulator result: **99% success for L=1**
- Real hardware: **27% success** (noise effects), still far above classical
- Project success: Bayesian inference demonstrated on both simulator and real quantum hardware

References

M. de Oliveira and L. S. Barbosa.
Quantum Bayesian decision-making.
Preprint, 2023.

G. H. Low, T. J. Yoder, and I. L. Chuang.
Quantum inference on Bayesian networks.
Physical Review A, 89(6):062315, 2014.

P. Sakkaris.
QuDot Nets: Quantum Computers and Bayesian Networks.
arXiv preprint arXiv:1601.07035, 2016.

M. Steffen, D. P. DiVincenzo, J. M. Chow, T. N. Theis, and M. B. Ketchen.
Quantum computing: An IBM perspective.
IBM Journal of Research and Development, 55(5):13:1–13:11, 2011.

Qiskit Documentation.
`https://qiskit.org`