# Fault-Tolerant and Secure Data Transmission Using Random Linear Network Coding

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***Abstract*—Network coding is a technique which can be used to improve wired and wireless network's throughput and to provide reliable transmission. In network coding, the original data packets can be encoded to an infinite number of coded packets, and a subset of these coded packets is sufficient to decode the coded packets and retrieve the original data. In addition to provide reliable data transmission, network coding can be used as a lightweight security mechanism to protect data against eavesdroppers. An eavesdropper is not able to decode the coded packets and retrieve the original data unless it has access to a sufficient number of coded packets. In a data transmission application, transmitting more redundant packets increases the chance of delivering sufficient number of coded packets to the destination and as a result, increases the reliability of the data transmission. However, more redundant transmissions makes the system more vulnerable against eavesdropper attacks, as there is a higher chance that an eavesdropper receives enough coded packets. In this work, we study the application of network coding in providing reliable and secure data transmission schemes by performing a trade-off between security and reliability of the data transmission. We formulate the problem as a mixed integer and linear programming, and propose linear programming approximation to solve it. Moreover, we study the performance of our proposed methods through extensive simulations.**

***Index Terms*—Network coding, security, fault tolerance, unicast, eavesdropping, optimization, random linear network coding.**

## I. Introduction

Providing reliable data transmission, which is resilient to path failures is an important challenge in data transmission. Transmitting redundant data over different paths is an effective approach to have a fault-tolerant data transmission. Clearly, transmitting more redundant data over each of the paths provides a higher level of protection against path failures. However, more redundant transmission increases the cost of data transmission. Finding the optimal redundancy level over each of the paths to achieve a certain level of fault-tolerance has been widely studied by the research community. Also, different techniques such as erasure codes and fountain codes have been used to generate redundant data. Random linear network coding is one of these techniques that is widely used to produce redundancy.

In random linear network coding, the original packets are linearly coded with each other, and the coded coefficient for mixing the packets are selected randomly over a finite field. In this type of coding, each of the coded packets packet is in the form of $\sum_{j=1}^{m} \beta_j \times P_j$. In this equation, $\beta_j$ and $P_j$ are the random coefficients and the packets that are coded with each other, respectively. Assuming that $m$ packets are coded using random linear network coding, any $m$ linearly independent coded packets suffice for decoding the coded packets and retrieving the original packets. In this method, we can potentially produce an infinite number of coded packets. The decoding is performed using methods that solve a system of linear equations, such as Gaussian elimination.

Another challenge in transferring data in a network is the security of the data transmission. Assume that we want to securely transfer a file from a source to a destination, but the network is not trustworthy. As a result, an eavesdropper might be able to overhear some of the transmitted packets. The straightforward approach to prevent non-authorized users or eavesdroppers to get access to the original data is to use cryptographic methods. The source node can encrypt the original data before transmitting it. The encrypted data can be partitioned to multiple packets and transmitted over different paths. However, the complexity of cryptographic methods is itself a challenge.

An alternative approach to achieve data security is using network coding. In order to provide a low-complexity security mechanism, the work in [1], [2] propose a method to ensure the security of a distributed data storage that relies on network coding technique. Assuming that $m$ packets are coded with each other using random line network coding, $m$ coded packets are required to decode the coded packets and retrieve the original packets. The main idea in [1], [2] is to prevent the eavesdroppers or non-authorized users from accessing the sufficient number of coded packets that is required to decode the coded packets. As a result, the eavesdroppers are not able to use Gaussian elimination to decode the coded packets and construct the original data. Using random linear network coding as a security mechanism, confidentiality can be achieved without adding extra complexity and cost. In this work, it is assumed that only the authorized users know the location of the data storage that store a specific file. Consequently, in the proposed security scheme, the location of the data storages has the same role as a secret key in a cryptographic method. In the mentioned work, applying network coding makes the distributed data storage robust against eavesdropper attacks and storage failures at the same time.

In [3], we used network coding to design a fault-tolerant and secure distributed data storage using network coding. In a similar way, network coding can be used to make data transmission robust against eavesdropper attacks and path failures. Obviously, transmitting more redundant data over different paths can enhance fault-tolerance of the data transmission against path failures and increase the chance that the destination is able to receive enough coded packets to retrieve the original data. On the other hand, transmitting more redundant coded packets increases the vulnerability of the data transmission against eavesdropping attack. The reasons is that, transmitting more number of coded packets on each of the paths increases the chance that an eavesdropper can access to a sufficient number of coded packets. It does not mean that a secure data transmission that uses network coding is less fault-tolerant. The coded packets are transmitted over different paths with different reliability and security levels. As a result, if we consider the security and reliability of the paths in distributing data over the paths, we can use network coding to provide security and reliability concurrently.

In Figure 1, three parallel and edge disjoint paths are shown between source $S$ and destination $D$. In this example, the path failure probability of each path is 0.2. The source node transmits a file consisting of 4 packets through these 3 paths. Assume that an eavesdropper might access the transmitted packets over path 1, 2, and 3 with probabilities 0.1, 0.1, and 0.2. We consider two cases. In case one, we transmit 2 random liner network coded packets on each of paths 1 and 2. As a result, the only scenario that the destination node is able to decode the coded packets and retrieve the original packets is that when both of paths 1 and 2 successfully deliver the transmitted packets without any path failures. In this case, the probability of successful data retrieval equals to $0.8^2 = 0.64$. An eavesdropper cannot decode the coded packets without accessing all of the 4 transmitted packets. Consequently, the probability of successful eavesdropping equals to $0.1^2 = 0.01$.

In the second case, the source node transmits 2 coded packets over each of the 3 paths. As the number of original packets is 4, accessing any 4 transmitted coded packets suffice to decode them. In this scenario, the probability of successful decoding by the destination node is $0.8^3 + 3 \times 0.8^2 \times 0.2 = 0.896$, which is the case that at least two paths do not fail. In the same way, an eavesdropper can retrieve the original data from any 4 coded packets. Thus, the eavesdropping probability in this network is $0.1^3 + 2 \times 0.1^2 \times 0.9 = 0.019$. In other words, the security of the network in this case is $1 - 0.019 = 0.981$. In the second case, we could increase the reliability of the data transmission by adding more redundancy, which comes in the cost of increase in the vulnerability of the system against eavesdropping attacks. In order to make sure that increasing the redundancy does not make the system vulnerable against eavesdroppers, we need to perform a trade-off between the reliability of the network and its security. Moreover, the level of redundancy on each path needs to optimized depending on the reliability and the security of the path.

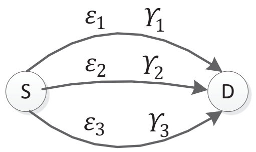


Fig. 1. Motivation example.

In this work, we use a similar approach to our secure and fault-tolerant data storage work in [3] to study the problem of secure and fault-tolerant data transmission over disjoint paths. Our work is motivated by the application of network coding in providing reliable data transfer and its application as a light-weight security mechanism. We assume that there are multiple parallel disjoint paths between a source and a destination node, and the source applies random linear network coding on the original data before the transition of its packets. In our work, we perform a trade-off between the reliability and the security of the data transmission. We formulate the problem as a mixed integer and linear programming in different cases, and find their solution using approximation methods. We also analyze the performance of our proposed methods using extensive simulation results.

The remainder of the paper is as follows. We review the related work and a background on network coding in Section II. The system model and the objective of our work is discussed in Section III. We propose our reliable and secure data transmission methods in Section IV. We analyze our proposed methods though extensive simulation results in Section V. In Section VI, we conclude our work.

## II. Related Work and Background

In the following subsections, we review the related work and discuss the preliminaries on network coding and its applications. Our proposed secure and reliable data transmission method is based on random linear network coding. For this reason, we first provide a background on network coding and its applications. Then, we review some of the applications of network coding, including providing reliable transmissions and security. We also review the previous work that use network coding as a security scheme.

### A. Network Coding Preliminaries

Network coding technique [4]–[7] generalizes the traditional store-and-forward routing in networks. Network coding is proposed and used in [8] to achieve the capacity of a multicast session in wired networks. In [8], the authors prove that the maximum multicast capacity that can be achieved using network coding equals to the min-cut from the source to the set of destinations of the multicast session, which is known as the min-cut max-flow theorem. It is proven in [9] that in order to achieve the capacity of a multicast session in a wired network, its sufficient to use liner network coding. However, selecting the coefficients of the linear coded packets is a challenge.

The authors in [10] prove that if each intermediate node select the coefficients of the coded packets randomly and in a distributed fashion, most likely the generated linear coded packets will be linearly independent. Consequently, the proposed scheme, which is called random linear network coding suffices to archive a rate close to the capacity of a multicast session. The proposed idea in random linear network coding makes the packets transmission very simple, as it removes the complexity of coefficients selection. In [11], the authors take an algebraic look at the network capacity and network coding, and derive an interesting and useful model for linear network coding.

The coded packets that are generated in linear network coding are linear combination of the original packets over a finite field (Galois field). In linear network coding, any linear mixture of coded packets is a linear coded packet as well. In random linear network coding, the coefficients of the coded packets are selected randomly over a finite field. When the coefficients of the coded packets are selected randomly, most likely the recoded packets are linearly independent. The form of the coded packets in random linear network coding is $\sum_{j=1}^{m} \beta_j \times P_j$, in which $P_j$ is a packet. This packet can be an original packet or a coded packet itself. The coefficients of the linear combinations are shown as $\beta_j$.

Selecting the coefficients randomly and in a distributed manner makes random liner network coding appropriate for distributed systems and large networks. Moreover, similar to fountain codes (rateless codes) [12]–[15], the source node can potentially generate an infinite number linearly coded packets. These coded packets can be transmitted over the network and recoded at the intermediate relay nodes. Assuming that the source has $m$ original packets to send, with a high probability the destination node is able to decode the coded packets using any $m$ coded packets, which is done using Gaussian elimination. This characteristic makes random linter network coding a great tool in achieving reliable transmissions without need to feedback messages.

### *B. Applications of Network coding*

The first application of network coding was in solving the bottleneck problem and throughput maximization in wired networks. However, these days network coding has a wide range of applications including but not limited to providing reliable transmissions, throughput enhancement, protocol simplification, and security. In fact, network coding is more attractive in wireless networks that wired networks. This is due the the unreliability of the wireless links and the broadcast nature of the medium, which makes network coding more beneficial in wireless networks. In the following subsections, we briefly review some of the applications of network coding and the previous work on providing security using network coding.

*1) Throughput/Capacity Enhancement:* COPE method [4] is one of the first practical methods proposed for data transmission in wireless networks using network coding. This method benefits from the broadcast nature of wireless medium and the overhearing among the nodes to augment the throughput of the system. The main idea in COPE is that, in the case that two crossing flows meet at a relay node and each of the destinations overhears the flow destined to the other destination, the relay node can combine these two flows to reduce the number of transmission. The authors extend this idea for more number of crossing flows in their proposed method.

The work in [16]–[18] propose a one-hop reliable broadcasting using network coding. In order to make sure that all of the packets are received by the destination nodes, feedback messages are used. The proposed method has transmission and retransmission phases. In the first phase, the original packets are transmitted. Then, based on the feedback messages that are received from the destination nodes, the source decide how to use XOR network coding to reduce the number of required retransmissions to deliver the missing packets to the destinations. When network coding is used, each coded packet can deliver multiple lost packets to the different destination nodes, which reduces the number of retransmissions.

*2) Reliable Transmission:* One of the most important applications of network coding is its application in reliable transmission methods, specially in wireless networks which are more prone to packet erasures. Feedback messages are widely used in reliable transmission methods, such as ARQ (automatic repeat request) method [19]. However, feedback messages have overhead, which is a major problem in the case of multicasting. In order to reduce this overhead, hybrid-ARQ methods [20], [21] can be used, in which ARQ and a forward error correcting code (FEC) [22]–[24] are combined. Hybrid-ARQ methods reduce the number of feedback messages, but feedback messages are still needed. Linear network coding can be used in reliable transmission methods to provide reliability without feedback messages or with the minimum possible number of them. The important feature of linear network coding that makes it suitable for this application is that each of the coded packets contributes the same amount of information to the destination nodes. Consequently, the destination node only needs to receive a sufficient number of coded packets instead of receiving particular packets. Using linear network coding, the source node keeps transmitting coded packets until the destination (or destinations) receives a sufficient number of coded packets.

*3) Protocol Simplification:* One of the important applications of network coding is in simplifying network protocols. For example, one of the challenges in peer-ro-peer (P2P) networks [25]–[27] is retrieving a file from different peers that store different parts of the file. Since the original file is stored on different peers, a tracking mechanism is needed to know the location of different parts of data on different peers. Network coding simplifies the distribution of the file and its retrieval [28]. Network coded packets are distributed over the peers, and instead of knowing the peers that store a particular part of the file, we just need to know the number of coded packets that are stored on each peer.

Another example of the application of network coding in protocol simplification is in content distribution. Many content distribution problems are NP-complete, so they cannot be solved in polynomial time [29], [30]. Network coding can modify these problems to new problems that can be solved in polynomial time using techniques such as linear programming optimizations [31], [32].

*4) Security:* In [33], the authors propose a low-complexity cryptographic mechanism using random linear network coding. The authors propose to encrypt the coefficients of the network coded packets instead of encrypting the original data. In this way, the complexity and overhead of the encryption is reduced. The reasons is that, the size of the coefficients is much less than the size of the original data, which reduces the amount of the data that needs to be encrypted.

In [34], the idea of coding the coefficients is extend and used for broadcasting multi-resolution videos [35]–[39]. In multi-resolution (multi-layer) videos, a video is divided to multiple videos, including a base layer an several enhancement layers. The base layer is required to watch the video, but enhancement layers increase the quality of the video [40]. Multi-resolution videos are useful in multicasting or broadcasting a video to a set of users with different channel conditions. Another application of multi-resolution is to provide different video qualities to a set of users that are subscribed to different services with different quality of services. In the mentioned work, the authors encrypt the coefficients of the network coding packets of each layer with a different key, to prevent unauthorized users to receive the video layers.

## III. System Model

We consider a network consisting of a source, a destination and multiple relay nodes. We assume that it is possible to find $n$ parallel paths between the source and destination node, that are node and link disjoint. Each of these paths might fail with a given probability. The failure can be due to different reasons, such as node failure, link failure, interference, or noise. We represent the failure probability of the $i$th path between the source and destination as $\epsilon_i$. Moreover, each path is subject to eavesdropping attack. The probability that an eavesdropper has access to the packets transmitted over the $i$th data path is represented as $\gamma_i$.

The source node has a file to transmit to the destination node. In order to provide fault tolerance, the source node applies random linear network coding on the original file, and network coded packets are transmitted through the $n$ disjoint data paths. In more details, the original file is first partitioned into $m$ packets, and then random linear network coding is performed among the $m$ packets to generate coded packets. Using random linear network coding, the source node can transmit redundant linear coded packets through the $n$ different paths. The destination node needs to receive at least $m$ coded packets to be able to decode the coded packets and retrieve the original packets. Once the decoding is successful, the destination node can merge the original packets to generate the file.

More redundancy in the data transmission enhances the fault-tolerance of the system against path failures. However, that might increase the vulnerability of the system against eavesdropping attack, as more number of transmitted packets increase the chance that an eavesdropper is able to get enough coded packets, which is $m$ in our model. In this work, we want to perform a trade-off between the reliability and the robustness against the eavesdropper. A data path might be robust against failure, but that might not be a secure data paths. In this case, we need to make a decision about the number of transmitted packets on that path.

TABLE I
The set of symbols used in this paper.

| Notation | Definition |
|---|---|
| $n$ | Number of parallel node and edge disjoint data paths |
| $m$ | Number of packets in the original file |
| $d_i$ | The $i$th data path |
| $\epsilon_i$ | Failure probability of data path $d_i$ |
| $\gamma_i$ | Access probability of the eavesdropper to the transmitted data over data path $d_i$ |
| $R_j$ | The set of paths that did not fail |
| $S_j$ | The set of paths overheard by the eavesdropper |
| $p_j$ | Failure probability of the data paths not in set $R_j$ |
| $q_j$ | Access probability of the eavesdropper to the data paths in set $S_j$ |
| $x_i$ | Portion of transmitted file on the $i$th data path |
| $y_j$ | Boolean variable which shows whether paths in set $R_j$ transmits $m$ coded packets |
| $z_j$ | Boolean variable which shows whether paths in set $S_j$ transmits $m$ coded packets |
| $U$ | Utility function |
| $\alpha_1$ | The assigned weights to security |
| $\alpha_2$ | The assigned weights to fault tolerance |
| $t_1/t_2$ | Threshold for fault tolerance/security |

## IV. Secure Data Transmission

In this work, our objective is to design a fault-tolerant and secure data transmission scheme. Since more redundancy can reduce the security and increase fault-tolerance, we need to perform a trade-off between security and reliability. In the following subsections, we first formulate our problem as mixed integer and linear programming optimizations. We then propose our secure and fault-tolerant data transmission methods, which find the number of packets that need to be transmitted on each of the parallel paths from the source to the destination in such a way that a certain level of security and fault-tolerance is met.

### *A. Formulation*

Similar to our previous work on fault-tolerant and secure distributed data storage [3], we can formulate the discussed fault-tolerant and secure data transmission problem in the following three cases.

- Case 1: In this case, we assume that a certain level of fault tolerance needs to be met. For this purpose, we fix the fault tolerance into a specific threshold, denoted as $t_1$, and set it as a constraint of the optimization. This threshold is the minimum required fault tolerance of the system. We then set the objective as minimization of the eavesdropping probability.
- Case 2: This case is the reverse of the optimization in Case 1. We assume that a certain level of security needs

to be met. Therefore, we set a threshold for the security of the system, denoted as $t_2$, and use it as a constraint of the optimization. The objective of this optimization is to maximize the fault-tolerance of the data transmission.

- Case 3: In the third case, there is no limit on the security and fault-tolerance of the data transmission. Instead, we define the objective function as a maximizing of a function of fault tolerance and security.

The eavesdropper needs to receive at least $m$ linearly independent coded packets to be able to decode the coded packets and retrieve the original file. In the other words, if the eavesdropper receives less that $m$ coded packets, it will not be able to retrieve the original file. It can be proved that if we use a sufficiently large finite field to linearly code the packets, with a high probability any $m$ out of the random linear coded packets will be linearly independent and sufficient to retrieve the original file [10]. For any paths failure case, we represent the set of paths that do not fail as $R_j$, and the set of all of these sets as $R$. Moreover, for any eavesdropping scenario, we represent the set of paths that are overheard by the eavesdropper as $S_j$, and the set of all of these sets as $S$. Additionally, we use boolean variables $y_j$ and $z_j$ to show whether the destination and the eavesdropper are able to retrieve the original file from the set of packets are transmitted over paths in $S_j$ and $R_j$, respectively. The set of notations used in this work are shown in Table I.

Clearly, if at least $m$ coded packets are transmitted on the set of paths in $S_j$, the eavesdropper can retrieve the file. In this case, $z_j$ has a value equal to 1; otherwise, its value is 0. In the same way, if at least $m$ packets are transmitted over the paths in $R_j$, the value of variable $y_j$ is 1, which means the destination node is able to decode the coded packets and retrieve the original file. Consequently, if we represent the probability that $R_j$ and $S_j$ happen as $q_j$, the probability that an eavesdropper and the destination can retrieve the original file equal $q_j z_j$ and $p_j y_j$, respectively. The probability that an eavesdropper can receive the transmitted packets on path $d_i$ equals $\gamma_i$. Thus, the probability that an eavesdropper has only access to data transmitted on the set of paths in $S_j$ can be calculated as follows:

$$q_j = \prod_{d_i \in S_j} \gamma_i \prod_{d_i \notin S_j} (1 - \gamma_i) \tag{1}$$

Moreover, the failure probability of path $d_i$ is denoted as $\epsilon_i$. Thus, the probability that none of the paths in set $R_j$ fails and the rest of the data paths fail can be calculated as:

$$p_j = \prod_{d_i \in R_j} (1 - \epsilon_i) \prod_{d_i \notin R_j} \epsilon_i \tag{2}$$

In the following subsections, we formulate our problem in the discussed three cases.

*1) Case 1:* In the first case, we want to achieve a minimum fault-tolerance of $t_1$. Also, our objective is to minimize the probability that an eavesdropper can receive $m$ coded packets and retrieve the original packets. As a result, the distribution of packets over different paths needs to be done in such a way that does not violate the minimum fault-tolerance threshold, and it also needs to minimize the eavesdropper probability. This problem can be formulated as the following mixed integer and linear programming:

$$\min \ U = \sum_{S_j \in S} q_j z_j \tag{3}$$

$$s.t \quad \sum_{R_j \in R} p_j y_j \geq t_1 \tag{4}$$

$$y_j \leq \sum_{d_i \in R_j} x_i \quad \forall \ R_j \in R \tag{5}$$

$$z_j \leq \sum_{d_i \in S_j} x_i \quad \forall \ S_j \in S \tag{6}$$

$$y_j, z_j \in \{0, 1\} \quad \forall R_j \in R, \ S_j \in S \tag{7}$$

The objective function of this optimization is $\min \sum_{R_j \in R} q_j z_j$, which is minimizing the probability of successful eavesdropping. Also, Constraint (4) represents the minimum reliability of the system that should not be less than threshold $t_1$. Here, $\sum_{R_j \in R} p_j y_j$ is the probability that the original file can be retrieved by the destination node. Variables $y_j$ and $z_j$ are integer variables with value 0 or 1, which denote whether a given failure and eavesdropping scenario will result in a successful eavesdropping and file delivery to the destination node. We represent the fraction of the original file that is transmitted on data path $d_i$ as $x_i$. Constraint (5) sets $y_j$ to 1 in the case that the destination node can retrieve the original packets by successfully receiving the packets transmitted over the paths in $R_j$. Also, Constraint (6) sets $z_j$ to 1 if an eavesdropper can retrieve the original file by overhearing the packets transmitted on the set of paths in $S_j$.

*2) Case 2:* In the second case, which is the reverse of case 1, our objective is to maximize the system fault-tolerance against path failures. Moreover, the probability of a successful eavesdropping should not be greater that threshold $t_2$. In this case, the problem can be formulated as the following mixed integer and linear programming:

$$\max \ U = \sum_{R_j \in R} p_j y_j \tag{8}$$

$$s.t \quad \sum_{R_j \in R} q_j z_j \leq t_2 \tag{9}$$

$$y_j \leq \sum_{d_i \in R_j} x_i \quad \forall \ R_j \in R \tag{10}$$

$$z_j \leq \sum_{d_i \in S_j} x_i \quad \forall \ S_j \in S \tag{11}$$

$$y_j, z_j \in \{0, 1\} \quad \forall \ R_j \in R, S_j \in S \tag{12}$$

In this optimization, the objective function (8) is maximizing the probability of successful delivery of the file to the destination node. Constraint (9) is the constraint on the maximum vulnerability of the system. Similar to case 1, Constraints (11) and (10) set $z_j$ and $y_j$ to 1 or 0 depending on whether the transmitted packets over paths in $R_j$ result in a successful eavesdropping and successful delivery of the packets to destination node or not.

*3) Case 3:* In the last case, instead of setting a threshold for the fault-tolerance or the security of the system, we perform a trade-off between security and fault tolerance. Transmitting more redundant data on each of the paths enhances the robustness against path failures. However, it makes the data transmission more vulnerable against eavesdropping attacks.

In order to perform a trade-off, we define a weighted sum of the security and fault-tolerance as the objective function. The objective function of the optimization is $U = \alpha_1 u_1 - \alpha_2 u_2$, in which $u_1$ is the probability that the eavesdropper can retrieve the original file from the overheard transmissions. Also, $u_2$ represents the probability that the destination node can retrieve the original file from the received coded packets. Furthermore, constants $\alpha_1$ and $\alpha_2$ are the assigned weights the security and reliability of the system. We can perform the trade-off using the following mixed integer and linear programming:

$$\min\ U = \sum_{S_j \in S} \alpha_1 q_j z_j - \sum_{R_j \in R} \alpha_2 p_j y_j \tag{13}$$

$$y_j \leq \sum_{d_i \in R_j} x_i \quad \forall\ R_j \in R \tag{14}$$

$$z_j \leq \sum_{d_i \in S_j} x_i \quad \forall\ S_j \in S \tag{15}$$

$$y_j, z_j \in \{0, 1\} \quad \forall\ R_j \in R, S_j \in S \tag{16}$$

The objective function of this optimization is a weighted sum of security and fault tolerance. Similar to Cases 1 and 2, variables $y$ and $z$ are integer variable. The set of Constraints (14), (15), and (16) sets the value of these two variables to 0 or 1.

## B. Data Transmission Scheme

All of the three optimizations that are discussed for the three cases are mixed integer and linear programming. In general, the solution of mixed integer and linear programming optimization cannot be found in polynomial time. As a result, we need to modify the proposed three optimization problems to optimizations that can be solved faster. One of the possible approaches to find an approximation solution for a mixed integer and linear programming is to relax it to a linear programming optimization. Linear programming optimizations can be efficiently solved using different available techniques, such as Gradient method.

Using relaxation technique, the problem in case 1 can be formulated as the following linear programming, denoted as LP1 optimization:

$$\min\ U = \sum_{S_j \in S} q_j z_j \tag{17}$$

$$s.t \quad \sum_{R_j \in R} p_j y_j \geq t_1 \tag{18}$$

$$y_j \leq \sum_{d_i \in R_j} x_i \quad \forall\ R_j \in R \tag{19}$$

$$z_j \leq \sum_{d_i \in S_j} x_i \quad \forall\ S_j \in S \tag{20}$$

$$y_j, z_j \in (0, 1) \quad \forall\ R_j \in R, S_j \in S \tag{21}$$

In this optimization, we relaxed the integer variables $z_j$ and $y_j$ to variables with real values, which changes the optimization from mixed integer and linear programming to linear programming. Linear programming optimizations can be solved using using gradient method and the other standard optimization techniques. However, the complexity of these methods is a polynomial function of the number of the variables and constraints in the optimization. In LP1, the number of sets in set $R$ is exponential. As a result, in the case that the number of parallel paths between the source and the destination node is high, the complexity of the solution to LP1 will be exponential. Consequently, we propose an approximation of the LP1 optimization, denoted as LP2:

$$\min\ U = \sum_{d_i \in D} \gamma_i x_i \tag{22}$$

$$s.t \quad \sum_{r_i \in S} (1 - \epsilon_i) x_i \geq t_1 \tag{23}$$

$$x_i \in (0, 1) \quad \forall\ d_i \in D \tag{24}$$

Here, $D$ represents the set of all paths between the source and the destination node. If we apply the discussed two relaxations on the mixed integer and linear programming for case two, the optimization in case 2 becomes as follows:

$$\max\ U = \sum_{d_i \in D} (1 - \epsilon_i) x_i \tag{25}$$

$$s.t \quad \sum_{r_i \in S} \gamma_i x_i \geq t_2 \tag{26}$$

$$x_i \in (0, 1) \quad \forall\ d_i \in D \tag{27}$$

Finally, the optimization in case 3 can be relaxed to the following optimization:

$$\max\ U = \sum_{d_i \in D} \alpha_1 \gamma_i x_i - \alpha_2 (1 - \epsilon_i) x_i \tag{28}$$

$$s.t \quad x_i \in (0, 1) \quad \forall\ d_i \in D \tag{29}$$

## C. Extension

In the previous sections, we considered a general model, in which the eavesdropping probability of the links are random numbers. Also, we assumed that there are $m$ parallel paths between the source and the destination node. This model can be easily extended to the more general model in Figure 2, in which the eavesdropping probability follows a special pattern without affecting our proposed method. For example, it is logical to assume that the eavesdropping probability of the links decreases based on the distance of the links from eavesdropper. The links that are closer to the eavesdropper are more probable to be overheard by the eavesdropper. The links that are not close to the eavesdropper, can be still overheard,

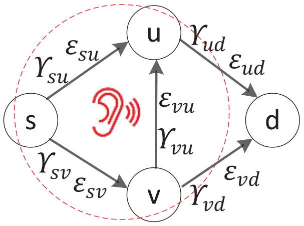


Fig. 2. Extended system model.

but with a lower probability. For example, we can assume the Rayleigh fading model [41] to calculate the overhearing probability:

$$P = \int_{T^*}^{\infty} \frac{2x}{\sigma^2} e^{-\frac{x^2}{\sigma^2}} dx \tag{30}$$

where

$$\sigma^2 \triangleq \frac{1}{(4\pi)^2 L^\theta} \tag{31}$$

Here, $T^*$ and $L$, are the decodable SNR threshold and the distance between two nodes. Also, $\theta$ is the path loss order. This model is typically used to model the loss rate of the wireless links between two nodes. In the case that the distance between two nodes is more, the loss probability of the link between them will be more, which is due to a higher noise level. Noise cannot affect the communication of two nodes that are close to each other. The same idea can be used to model the behavior of eavesdroppers. If an eavesdropper is too close to a link, there is a higher chance to overhear the transmitted packets over that link. The links that are far from the eavesdropper are less vulnerable against eavesdropping. In order to use our proposed methods in the previous section, we first need to find parallel paths between the source and the destination node. For this purpose, the methods that can find disjoint paths such as [42] can be used.

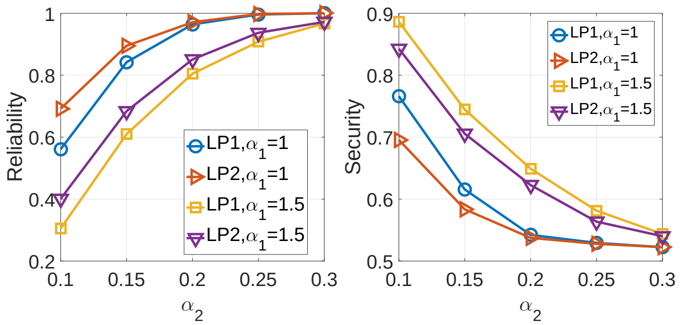
## V. Evaluations

In this section, we report the simulation results of our proposed secure and fault-tolerant data transmission schemes. We first discuss the simulation environment and the setting. Then, we present the simulation results and the summary of our findings.

### A. Simulation Setting

We implemented our simulations in the Matlab environment. In order to find the solution of the proposed optimization in the previous section, we used a built-in optimization tool of Matlab, called Linprog. This tool solves linear programming optimization. Linprog does not accept equality constraint. Because of that, we had to convert the equality constraint to inequality equations. For this purpose, each equality constraint needs to be replaced with one greater than or equal, and one less than or equal constraint.

In order to have a reasonable confidence interval, we run our simulations on 1000 networks, with random paths reliability and eavesdropping probability. In our simulations, we compare the security and reliability of the data transmissions in case 3, by measuring the effect of the following metrics:



(a)

(b)

Fig. 3. The effect of $\alpha_2$ on the reliability and the security of data transmission. $\epsilon_i \in [0, 0.1]$, $\gamma_i \in [0, 0.3]$. (a): Data transmission reliability. (b): Data transmission security.

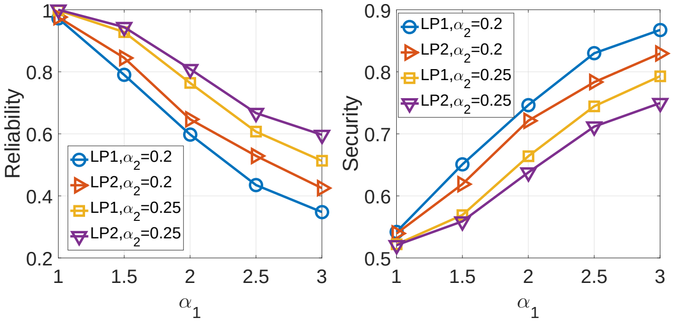
- $\alpha_1$: is the assigned weight to the security. A greater $\alpha_1$ gives more importance to the security, which results in a more secure data transmission.
- $\alpha_2$: is the weight of the fault tolerance in the optimizations. In contrast with $\alpha_1$, a greater $\alpha_2$ results in a more reliable data transmission scheme.
- $\epsilon$: is the probability of a data path failure. In the simulations results, we study the effect of the changes in the path failure probability on the fault-tolerance and the security of our proposed methods.

In the simulations, LP1 and LP2 represent the two relaxed optimizations in case 3. In all of the simulations, there are 4 parallel (disjoint) paths between the source and the destination. Also, the number of original packets to send is 10 packets.

### B. Simulation Result

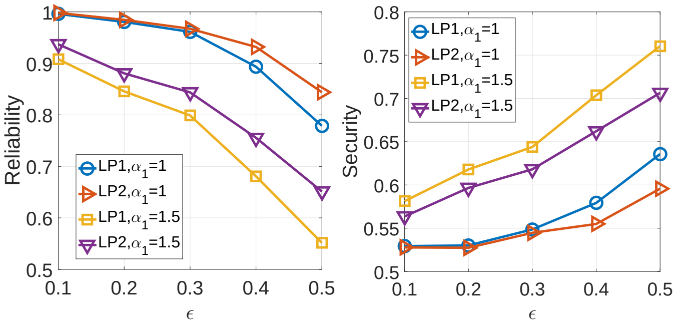
In Figure 3(a), we show the reliability of using LP1 and LP2 for case 3. The path failure probability of the links is chosen randomly in the range of $[0, 0.1]$. Moreover, the eavesdropping probability of each path ($\gamma$ ) is a random number in the range of $[0, 0.3]$. In Figure 3(a), the reliability of LP1 and LP2 methods are compared in the cases of $\alpha_1 = 1$ and $\alpha_1 = 1.5$. Increasing $\alpha_2$ gives more importance to the reliability of the system than its security. For this reason, as we increase $\alpha_2$, our proposed methods find a more reliable transmission schemes. That is why all of the curves in Figure 3(a) have a positive slope. Also, the figure shows that the reliability of LP1 and LP2 methods with $\alpha_1 = 1$ are more that that with $\alpha_1 = 1.5$. The reason is that, a greater $\alpha_1$ increase the weight assigned to the security of the transmission in the optimizations. The simulation results depict that the performance of the LP1 and LP2 methods are close. As discussed in the previous section, the complexity of LP2 method is less than that of LP1.

In our second experiment, we analyze the security of our data transmission methods. Security is defined as the probability of a successful eavesdropping, which means receiving $m$



(a) (b)

Fig. 4. The effect of $\alpha_2$ on the reliability and the security of the system. $\epsilon_i \in [0, 0.1]$, $\gamma_i \in [0, 0.5]$. (a): Measuring the reliability of the system. (b): Measuring the security of the system.



(a) (b)

Fig. 5. The effect of $\epsilon$ on the reliability and the security of the system. $\gamma_i \in [0, 0.5]$. (a): Measuring the reliability of the system. (b): Measuring the security of the system.

coded packets by and eavesdropper. In Figure 3(b), we change $\alpha_1$ from 0.1 to 0.3, and measure its effect on the security of the data transmission. The failure of the parallel paths and their eavesdropping probability are selected randomly in the range of $[0, 0.1]$ and $[0, 0.3]$. The security of the LP1 and LP2 methods decreases as we increases $\alpha_2$, which is due to giving more importance to the reliability of the data transmission. A greater $\alpha_2$ results in the transmission more redundant coded packets. Thus, there is a higher chance that an eavesdropper can receive $m$ coded packets. The security of the LP1 and LP2 methods in Figure 3(b) are close. From Figures 3(a) and (b) we can infer that as the reliability of our data transmission methods increase, their security decrease.

In Figures 4(a) and (b), we repeat the two previous experiments, but this time we fix $\alpha_2$ and change $\alpha_1$. In Figures 4(a), the failure probabilities are selected in the range of $[0, 0.1]$. In addition, the eavesdropping probability of each link is in the range of $[0, 0.3]$. As the figure shows, increasing $\alpha_1$ reduces the reliability of data transmission. Also, the reliability of the data transmission methods with $\alpha_2 = 0.25$ is more than that of with $\alpha_2 = 0.2$. Similar to the previous experiments, there is a little gap between the LP1 and LP2 methods.

In the next experiment, we fix $\alpha_2$, and change $\alpha_1$ from 1 to 3. Similar to the previous experiments, we select the failure rates of each path randomly in the range of $[0, 0.1]$, and the eavesdropping probabilities in the range of $[0, 0.3]$. Figure 4(b) shows that increasing $\alpha_1$ enhances the security of the data transmission. However, as shown in Figure 4(a), this security enhancement comes with the cost of decrease in the reliability of the data transmission.

In the next two experiments, we analyze the effect of path failure probability on the reliability and the security of our data transmission methods. For this purpose, we increase the range of failure probability from $[0, 0.1]$ to $[0.4, 0.5]$ in steps of 0.1, and measure the performance of the LP1 and LP2 methods. The eavesdropping probability of the paths are selected in the range of $[0, 0.5]$. We set $\alpha_2$ to 0.25, and run the simulations with $\alpha_1 = 1$ and $\alpha_1 = 1.5$. In Figure 5(a), the reliability of the data transmission decreases as the path failure rates increases. Also, the reliability of the LP2 method is slightly more than that of the $LP1$ method. Also, a greater $\alpha_1$ reduces the reliability of the methods.

In our last experiment, we study the effect that the path reliability has on the security of the data transmission. In Figure 5(b), we increase the path failure probability and measure the probability of a successful eavesdropping. The figure shows that as we increase the path failure rate, the security of the data transmission enhances. The reason is that, our objective function is a linear function of the security and reliability. In order to maximize the objective function, depending on the weights that are assigned to the reliability and security, security or reliability needs to be increased. When the failure rate of a path increases dramatically, its hard to combat with the failure rate. As a result, in the case that $\alpha_1$ is high, transmitting less number of packets and increasing the security of the system maximizes the objective function.

### C. *Simulation Summary*

Our finding from the simulation results can be summarized as follows:

- When random linear network coding is used to provide security and reliability, the reliability and security have negative correlation.
- The performance of the LP1 and LP2 methods in terms of reliability and security are very close to each other.
- In the case of a high $\alpha_1$, increasing the path failure rate increases the security of the system.

## VI. Conclusion

Random linear network coding is a technique in which packets are mixed to each other using an algebraic approach. This technique has many applications in wired and wireless networks, including but not limited to enhancing the network throughput, reliable transmissions, and fault-tolerant data storages. Using this technique, a set of packets can be potentially encoded to an infinite number of coded packets, and only a subset of these linearly coded packets are needed to recover the original packets. In addition to the mentioned applications, linear network coding can be used to protect the original packets from unauthorized access. An unauthorized user, e.g.

an eavesdropper, is not able to retrieve the original packets without receiving a certain number of coded packets.

In this paper, we study the application of network coding in designing a secure and fault-tolerant data transmission mechanism. In general, more redundancy increases the fault-tolerance of a system. On the other hand, more redundancy can make the system vulnerable against eavesdropper attacks, since it increase the chance that an unauthorized user receives a sufficient number of coded packets. In our work, we use random liner network coding to achieve security and fault-tolerance at the same time. We assume that the packets can be transmitted from a source node to a destination through different paths. Based on this model, we propose 3 different optimization methods to find the number of packets that should be transmitted through different paths. We analyze our methods through extensive simulation results.

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