Flow Based XOR Network Coding for Lossy Wireless Networks

Abdallah Khreishah†, Jie Wu†, Pouya Ostovari†, and Issa M. Khalil‡
†Department of Computer & Information Sciences, Temple University, Philadelphia, PA 19122
‡Faculty of Information Technology, United Arab Emirates University, UAE

***Abstract*—The broadcast nature of wireless links makes wireless networks an attractive environment for *intersession network coding*. Most intersession network coding protocols exploit this property, but ignore the diversity among the links by turning off coding when the channels are lossy. Other protocols deal with the packets separately – not as members of flows – which makes the intersession network coding problem with lossy links untractable. In this paper, we use a different approach by looking at flows or batches instead of individual packets. We characterize the capacity region of the 2-hop relay network when the coding operations are limited to XOR. The 2-hop relay network represents all of the local intersession network coding opportunities in large multihop networks. The characterization is in terms of linear equations. We also provide a coding scheme that can achieve the capacity with almost zero feedback overhead. Simulation results show that our scheme enhances the throughput by 82% while maintaining fairness among the flows compared to the intersession network coding protocols that deal with the packets separately.**

***Index Terms*—Capacity, fairness, network coding, wireless networks.**

I. Introduction

One of the fundamental challenges in wireless network research is to characterize the capacity of such networks. The capacity refers to the set of all possible end-to-end rates that can be achieved by the users simultaneously [1]. Characterizing the capacity for wireless networks is not a straightforward extension from the wireline networks. This is due to the unique characteristics of wireless networks such as the broadcast nature, the interference among the links, the diversity, and the lossy behavior of the wireless links.

Traditionally, the broadcast nature of wireless links is considered a challenge due to the interference effect it creates and the unnecessary multiple copies of the same packet it produces. If we allow intermediate nodes to code the packets, the broadcast nature becomes an opportunity that needs to be exploited. Take Fig. 1 as an example: if the broadcast nature of wireless links is not exploited, and assuming that nodes s_1 and s_2 are out of range of each other, we need four transmissions to exchange two packets between nodes s_1 and s_2. The relay node r can exploit the broadcast nature of its output links and reduce the number of transmissions to three using network coding by XORing the two packets, as shown in the figure.

In *intersession network coding* (IRNC), intermediate relay nodes code packets from different flows at intermediate nodes. IRNC exploits the broadcast nature of wireless links and reduces the number of packets to be sent, as explained in the example in Fig. 1. In general, it is hard to perform IRNC, because the problem is NP-hard [3] and linear coding is not sufficient for the problem [4]. However, one can limit coding opportunities to be in the local neighborhood. Empirical studies have shown substantial throughput improvement in wireless networks when IRNC coding is limited to local XOR opportunities, as in COPE [2]. The example in Fig. 1 represents COPE. The local neighborhood structure is termed a *2-hop relay network* as we will discuss later. Based on the COPE approach, the problem of coding-aware routing and scheduling was studied in [5]. The formulation in [5] is linear programming that is computed centrally. The work in [6] studied the fundamental limit of how many sessions can be encoded simultaneously together when COPE is used. The fundamental limit depends on geometry, and the maximum number of sessions that can be coded together under a typical setting is limited to five. Our previous work [7] considered *pairwise* IRNC that allows coding over multihops, but limits coding to be among only two original packets. We designed its corresponding optimal scheduler and rate controller.

Based on the previous discussion, IRNC is well suited when the links are not lossy. However, IRNC does not work well when the links have a moderate loss probability of 20% as the work in [2] turns off coding in this case. In [8], IRNC with lossy links is considered. However, the authors did not optimize overhearing and limited the operations to be only XOR. The optimal solution was found to be #P-complete and several approximation algorithms were obtained. The work in [9] considered energy efficiency in lossy wireless networks with XOR-based IRNC and provided a heuristic to solve the IRNC problem.

The reason that the optimal solution for lossy 2-hop relay netwoks is #P-complete is that the packets were considered separately, not as members of flows. In this paper, we tackle the problem from a different angle as we consider flows instead of individual packets, and we use light feedback. This allows us to optimize the overhearing and characterize the capacity region when only XOR operations are used. Our characterization is in terms of linear equations, which makes the capacity region computable using a linear program with different objective functions. These objective functions can represent the sum of the throughput, strict fairness, or proportional fairness. Our simulation results show that the optimal solution for the capacity region can increase the throughput by 82% while enhancing the fairness compared

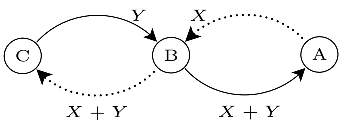

Fig. 1. A network with two flows.

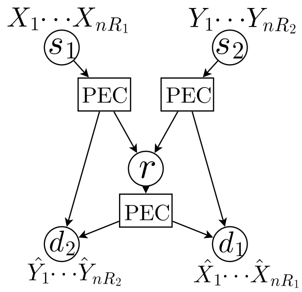

Fig. 2. A 2-hop relay network with two sessions.

to the state-of-the-art approaches.

Note that in our previous work [10] and in [11], the single-hop intersession network coding problem in lossy networks is considered. The authors optimized overhearing, did not limit to XOR, considered flows instead of packets and assumed limited feedback. The capacity region for the problem is characterized using linear equations when the number of sessions is less than 3. For more than three sessions, a near-optimal coding scheme is provided and its performance is characterized using linear equations. In this work, we limit the coding operations to be XOR, as the nodes in many wireless networks have limited computational power and cannot perform operations over large finite fields. The works in [12], [13] consider a similar objective for the reverse carpooling scenario which is a special case of our problem.

The rest of the paper is orginized as follows: In Section II, we describe the network settings and then present the capacity characterization in Section III. We present the simulation results in Section IV and conclude the paper in Section V.

II. The Settings

The two-hop relay network consists of N sessions as defined in [14], where each session i is represented by the source node s_i, the destination node d_i, and the rate R_i that should be supported between s_i and d_i. The destination node d_i can not overhear the source node packets, but can overhear other sources' packets. Therefore, we use the relay node r to code different session packets and send the coded packets through its outgoing broadcast link so that the overall capacity region can be enhanced. Node r receives a limited number of feedback messages from $d_i, \forall i$ about the overheard packets to help in deciding the coded combination. Fig. 2 represents the 2-hop relay network for two sessions, i.e. $N = 2$. In the figure, PEC stands for *packet erasure channel*. PEC is a broadcast channel where every sent packet can be received by any subset of the receivers. The reception at the receivers depends on the probability of reception between the source and any individual receiver. We use p_{uv} to denote the reception probability at node v of the packet sent by node u. We assume that the reception processes across the individual links of the PEC are independent.

For example, when $N = 2$, each of s_1, s_2, and r can use the corresponding PEC n times, respectively. Source s_1 would like to send $n \times R_1$ packets $X_1, \cdots, X_{nR_1}$ to destination d_1, and s_2 would like to send $n \times R_2$ packets $Y_1, \cdots, Y_{nR_2}$ to d_2. We are interested in the largest achievable rate pair (R_1, R_2) that guarantees recoverability of $X_1, \cdots, X_{nR_1}$ from the coded packets $\hat{X}_1, \cdots, \hat{X}_{nR_1}$ at d_1 and the recoverability of $Y_1, \cdots, Y_{nR_2}$ from the coded packets $\widehat{Y}_1, \cdots, \widehat{Y}_{nR_1}$ at d_2 with close-to-1 probability for sufficiently large n when node r is limited to perform only XOR operations.

To model the "reception report" suggested by practical implementations, we enforce the following sequential, round-based feedback schedule: s_1 and s_2 transmits n symbols, respectively. After the transmission of $2n$ symbols, two reception reports are sent from d_1 and d_2, respectively, back to relay r so that r knows which packets have successfully arrived at which destinations. After the reception reports, no further feedback is allowed and relay r has to make its own decision on how to use the available n PEC usages to guarantee decodability at d_1 and d_2. In our setting, we also assume that all nodes know the success probability parameters of all PECs and all of the coding operations. The only unknown parts are the values of the X and Y symbols.

We use t_r^A to represent the fraction of time that the relay node sends XORed packets formed by the packets of the sessions in set A. We also use x_i^A to represent the achievable rate for session i from the auxiliary session formed by XORing packets from the sessions in set A. Symbol x_i^{AB} represents the achievable rate for session i from the auxiliary session formed by XORing packets from the sessions in set A with the constraint that session i packets used in XORing are received by exactly all of the nodes in $r \bigcup (\bigcup_{j \in B} d_j)$ before being XORed. We use $R_{i,B}$ to represent the rate at which packets sent by s_i are overheard by r and exactly all of the nodes d_j, $j \in B, i \neq j$. Throughout the paper, we use the term "auxiliary session" to refer to the session formed by XORing different packets from different sessions.

III. The Capacity Region

A. The Characterization

The following theorem characterizes the capacity region of the 2-hop relay networks when the relay node r is limited to performing XOR operations.

Theorem 1: The capacity region of the 2-hop relay network when only XOR operations are allowed, can be represented by the following set of equations:

$$R_i \leq \sum_{A: i \in A} x_i^A, \forall i \tag{1}$$

$$x_i^A \leq t_r^A p_{rd_i}, \forall A, i \in A \tag{2}$$

$$x_i^A = \sum_{B:(A \setminus i) \subseteq B} x_i^{AB} \forall A, i \in A \tag{3}$$

$$\sum_{A:(A \setminus i) \subseteq B} x_i^{AB} = R_{i,B}, \forall B, i \notin B \tag{4}$$

Note that the summation in both (1) and (4) is over A, and in (3) is over B.

Proof: We prove our theorem by showing that the constrains are necessary and sufficient.

Necessity: Using XOR coding, any coded packet is formed by XORing packets of sessions i, $\forall i \in A$, where A is a set of sessions belonging to the power set of all sessions. Constraint (1) states that the total rate of session i is the sum of the achievable rate for session i from all of the auxiliary sessions A, where $i \in A$.

Since t_r^A is the frequency of sending XORed packets by the relay node formed by XORing packets of the sessions in set A, node d_i will receive XORed packets for the auxiliary session A from the relay node at rate $t_r^A p_{rd_i}$. Therefore, constraint (2) should be satisfied for any achievable XOR-based code.

Note also that (2) does not require the coded packet for the auxiliary session A to be received by all of d_i, $i \in A$, every time it is sent, any one of the d_i that receive this packet can decode it and it will count as a decodable packet.

For any auxiliary session A and $i \in A$, the set of the packets for session i that are XORed in this auxiliary session should be received from s_i by all of the nodes in the set $r \bigcup (\bigcup_{j \in A, j \neq i} d_j)$. The reason for that is because r should be able to relay the XORed packets formed in part by these packets, and also because all d_j should have enough remedy packets to remove the components corresponding to these packets from the XORed packets, and recover their respective packets. Also, the set of packets for session i that are received from s_i by any super set of $r \bigcup (\bigcup_{j \in A, j \neq i} d_j)$ can be used in the XORed auxiliary session A, because this will guarantee that all of the nodes in the set $r \bigcup (\bigcup_{j \in A, j \neq i} d_j)$ have received these packets. This explains the constraint (3).

The right hand side of (4) $R_{i,B}$ represents the rate of session i packets received by exactly all of the nodes in the set $r \bigcup (\bigcup_{j \in B, j \neq i} d_j)$ after being sent by s_i. These packets can be used by any auxiliary session A such that $(A \backslash i) \subseteq B$. This is because this guarantees that all of the nodes $d_j, j \in A, i \neq j$ will have enough remedy packets to remove session i components in the XORed packets. Therefore, we have constraint (4). We postpone calculating a closed form expression for $R_{i,B}$ to the end of this section.

Note that the packets sent by s_i can be divided among all of the auxiliary sessions A, $i \in A$. This is due to the following:

- Because the right hand side of (4) represents the rate at which an exact specific set of nodes are receiving the packets from s_i. Therefore, every triple (i, A, B) can be assigned an exclusive share of these packets.
- Because each x_i^{AB} appears only once in (3), the packets of session i that are used in the auxiliary session A will be $\bigcup_{B:(A \backslash i) \subseteq B} Y_i^{AB}$, where Y_i^{AB} are the set of packets assigned for the triple (i, A, B).

Sufficiency (an achievable coding scheme):

- Node s_i, $\forall i$ keeps trying to send its nR_i packets one-by-one until all of them are received by the relay node.
- Feedback messages from all d_i about the overheard packets are sent to the relay node r.
- For every set A, the relay node chooses the corresponding feasible $x_i^A, \forall i$ from the linear program depending on the objective function. It also assigns nx_i^A packets for every A and i, such that these packets are received by r and all $j \in A, j \neq i$. As was explained before, we can assign unique packets for every A.
- For every A, the relay node XORs one packet from each nx_i^A packet for all $i \in A$, and then sends it. If this packet is received by d_j for $j \in A$, this means that the packets belonging to session j in the XORed packets can be recovered by d_j. Therefore, we remove this packet from the set of packets assigned to j and A at the relay node. The relay node keeps performing the XORing and sends until all of the packets assigned for the set A at the relay node are sent.

This proves our theorem. ∎

Note that the last step in the achievable coding scheme assumes instant feedback. To avoid such an assumption, the relay node can use *fountain codes* [15] and achieves the same rates asymptotically, using only XOR operations. The fountain codes can be used as follows: (1) The relay node applies a fountain code on every set of packets Y_i^{AB} separately. (2) The relay node performs XOR on these packets, as was explained before. (3) Upon receiving these coded packets, the destination nodes can recover the fountain coded packets, because they overheard the remedy packets. (4) The destination nodes apply the inverse of the fountain code to retrieve the original packets.

Note that the approach for finding the feasible coding sets As would be to run the linear program by the relay node.

B. Computing $\mathbf{R_{i,B}}$

In this section, we provide a closed form expression for $R_{i,B}$. The closed form solution is not straightforward, because every packet has to be received by the relay node. Therefore, every s_i has to keep sending a packet until it is received by the relay node. We have:

$$
\begin{aligned}
R_{i,B} &= (\text{delivery rate from } s_i \text{ to } r) \\
&\quad \times (\text{probability that } r \text{ receives a} \\
&\text{symbol and by the time the symbol is received by } r \\
&\quad \text{it is received by only the nodes in } d_j, \\
&\quad j \in B,\ j \neq i) \\
&= p_{s_i r} \sum_{n=1}^{\infty} \text{Probability}\{r \text{ receives the packet on time} \\
&\text{slot } n\} \times \text{Probability}\{\text{only the nodes in } d_j,\ j \in B \\
&\text{receive the packet in time slots } 1, \ldots, n\} \\
&= p_{s_i r} \Big[\sum_{n=1}^{\infty} p_{s_i r} (1 - p_{s_i r})^{n-1} (\Pi_{j \notin B} (1 - p_{s_i d_j})^n) \times \\
&\Pi_{j \in B} (1 - (1 - p_{s_i d_j})^n) \Big]
\end{aligned}
$$

$$= p_{s_i r}^2 \times \sum_{n=1}^{\infty} \Pi_{j \notin B}(1 - p_{s_i d_j}) \Big[(1 - p_{s_i r})$$
$$\Pi_{j \notin B}(1 - p_{s_i d_j}) \Big]^{n-1} \left[\Pi_{j \in B}(1 - (1 - p_{s_i d_j})^n) \right]$$

Therefore, we have:

$$R_{i,B} = p_{s_i r}^2 \Pi_{j \notin B}(1 - p_{s_i d_j}) \times \sum_{n=0}^{\infty} \Big[(1 - p_{s_i r})$$
$$\Pi_{j \notin B}(1 - p_{s_i d_j}) \Big]^n \left[\Pi_{j \in B}(1 - (1 - p_{s_i d_j})^{n+1}) \right]$$
$$= p_{s_i r}^2 \Pi_{j \notin B}(1 - p_{s_i d_j}) \times \sum_{n=0}^{\infty} \Big[[(1 - p_{s_i r})$$
$$\Pi_{j \notin B}(1 - p_{s_i d_j})]^n [\sum_{H: H \subseteq B} (-1)^{|H|} \Pi_{k \in H}(1 - p_{s_i d_k})^{n+1}] \Big]$$

By Fubini's theorem [16], we have:

$$R_{i,B} = p_{s_i r}^2 \Pi_{j \notin B}(1 - p_{s_i d_j}) \sum_{H: H \subseteq A} (-1)^{|H|} \Pi_{k \in H}(1 - p_{s_i d_k})$$
$$\left[\sum_{n=0}^{\infty} [(1 - p_{s_i r}) \Pi_{j \notin B}(1 - p_{s_i d_j}) \Pi_{k \in H}(1 - p_{s_i d_k})]^n \right]$$
$$= p_{s_i r}^2 \Pi_{j \notin B}(1 - p_{s_i d_j}) \sum_{H: H \subseteq B} (-1)^{|H|} \Pi_{k \in H}(1 - p_{s_i d_k})$$
$$\left[\frac{1}{1 - [(1 - p_{s_i r}) \Pi_{j \notin B}(1 - p_{s_i d_j}) \Pi_{k \in H}(1 - p_{s_i d_k})]} \right]$$

Note that our results can be extended to the case of flexible scheduling, such that every source node s_i is scheduled for t_i fraction of the time. This can be done by multiplying the closed form for $R_{i,B}$ by t_i. Note also that by using our approach, we can maximize or minimize any objective function. This makes our approach more flexible as we will see in our simulation results.

IV. Simulations

In this section, we present simulation results to show the effectiveness of our flow-based scheme over the schemes that deal with packets separately.

We construct a unit circle with the relay r placed at the center. We then place N source nodes s_i, and N destination nodes d_i, in the circle at random (see Fig. 3). The only condition we impose is that for each (s_i, d_i) pair, d_i must be in the 90-degree pie area opposite to s_i (see Fig. 3). For each randomly constructed network, we use the Euclidean distance between each node to determine the overhearing probability. More explicitly, for any two nodes separated by distance D, we use the Rayleigh fading model to decide the overhearing probability $p = \int_{T^*}^{\infty} \frac{2x}{\sigma^2} e^{-\frac{x^2}{\sigma^2}} dx$, where we choose $\sigma^2 \triangleq \frac{1}{(4\pi)^2 D^{\alpha}}$, the path loss order $\alpha = 2.5$, and the decodable SNR threshold $T^* = 0.06$. Fig. 4 respresents the relationship between the overhearing probability p and the distance D. We assume that the overhearing event among different receivers is independent.

For each randomly generated network, we compute the overhearing probabilities and use the corresponding linear constraints on the time-sharing variables' ts and the rate variables' Rs to compute the achievable rate of each scheme.

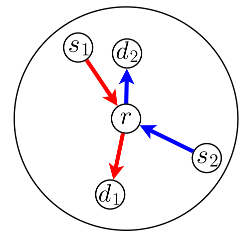

Fig. 3. A figure representing the settings of the simulations.

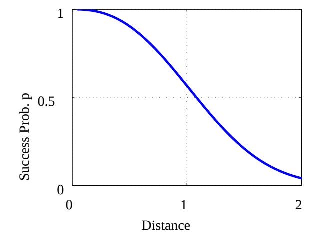

Fig. 4. The relationship between the distance and the signal strength for the Rayleigh fading channel.

Given a randomly generated network, the achievable sum rates are computed for all of the schemes. We then repeat this computation for 1,000 randomly generated networks. Let $R^*_{\text{scheme},k}$ denote the achievable sum rate of the given scheme for the k-th randomly chosen topology. We are interested in the following two performance metrics: The average sum rate over 1000 topologies, $\frac{1}{1000}\sum_{k=1}^{1000} R^*_{\text{scheme},k}$ and per topology improvement $\triangleq \frac{R^*_{\text{scheme},k} - R^*_{\text{baseline},k}}{{}^{*}_{\text{baseline},k}}$.

Fig. 5 represents the average sum rate over the 1000 topologies for different values of N and different schemes. The simulated schemes are: (1) COPE, from [2], which is the basic XOR-based coding scheme; (2) CLONE [8], which is the state-of-the-art loss-aware coding scheme that deals with the packets separately, not as members of flows. Two versions of CLONE are simulated. These are CLONE-binary and CLONE-multi. The details of the two CLONE schemes are described in [8]. It's worth noting that CLONE-multi has a very large complexity, which makes it difficult to report the results for $N = 6$; (3) Our optimal scheme. Since our optimal scheme can be casted with different objective functions, we simulate three objective functions. These are maximizing the total throughput "Cap-Sum", achieving strict fairness "Cap-Strictf", and achieving proportional fairness "Cap-PrFair". Cap-Strictf means that the rates of all sessions should be the same. Cap-PrFair means that each session i is assigned a weight w_i such that $\frac{R_i}{R_j} = \frac{w_i}{w_j}$, $\forall i, j$.

As can be seen from the figure, COPE performs poorly under the lossy links environment. Also, the average throughput using COPE decreases as the number of sessions increases. CLONE-binary and CLONE-multi perform better than COPE, but the average throughput does not increase as the number of sessions increases. Our optimal scheme outperforms all of the other schemes. When the objective is to maximize the total throughput, our scheme enhances the average throughput by $1.8 - 3.7$ fold compared to COPE, depending on the number of sessions. Our scheme also enhances the average throughput over CLONE-multi by $1.5 - 1.8$ fold and about $1.2 - 1.45$ fold over CLONE-binary, depending on the number of sessions. Even when the objective function is strict fairness or proportional fairness, our scheme enhances the throughput over the best state-of-the-art scheme by around 20%. This

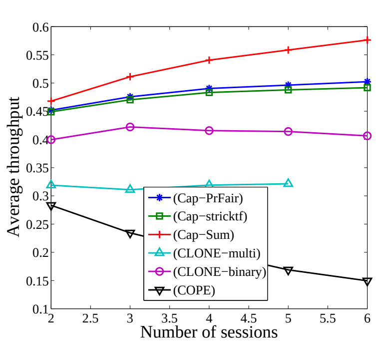

Fig. 5. The average throughput for 1,000 topologies with different values of N.

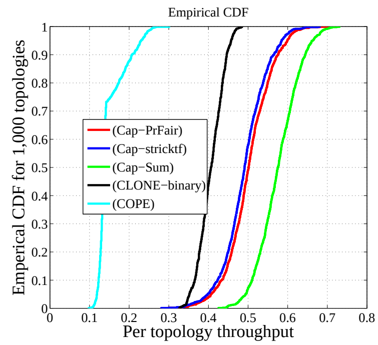

Fig. 6. The CDF of the total achievable rate for the 1,000 topologies when $N = 6$.

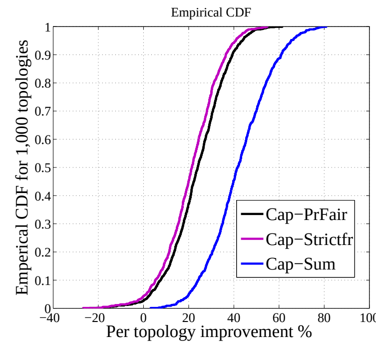

Fig. 7. The CDF of the per topology rate improvement compared to CLONE-binary for the 1,000 topologies when $N = 6$.

shows that our scheme outperforms all of the previous ones in both fairness and throughput. Fig. 6 represents the CDF function of the per topology throughput for different schemes when $N = 6$. The results in the figure confirm our results.

Fig. 7 represents the CDF for the per topology percentage gain that can be obtained by our schemes compared to CLONE-binary. As can be seen from the figure, for some topologies, the gain of our scheme, when the objective is to maximize the throughput, is about 82%. Also, for 20% of the topologies, the gain is above 58%. This means that we can find toplogies where our scheme can almost double the capacity of the network over the state-of-the-art with lower complexity. Fig. 7 also shows that when the objective is to achieve strict or proportional fairness, there are topologies such that our scheme can do, while increasing the throughput by 60%. These results show that our schemes can achieve fairness and maximize the throughput by a moderate amount simultaneously. This joint objective has been targeted by many works [17], but none has been able to get moderate improvement in both directions. It is worth mentioning that for only less than 2% of the simulated topologies, our schemes reduced the throughput compared to CLONE-binary in order to achieve the fairness objective.

V. Conclusion

In this work, we took a different look at the local inter-session network coding problem in lossy wireless networks. We considered the case where the coding operations at the relay node are limited to XOR operations. We also considered flows instead of individual packets and characterized the corresponding capacity region. Our characterization turned out to be in terms of linear constraints, which is tractable compared to the characterization without flows. We also provided a coding scheme that achieves the capacity. Our simulation results showed the superiority of our scheme in terms of throughput and fairness.

VI. Acknowledgments

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