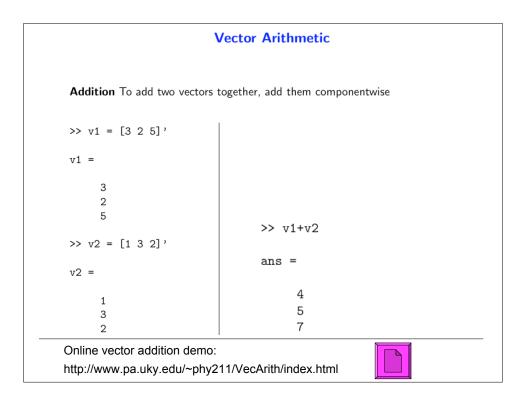
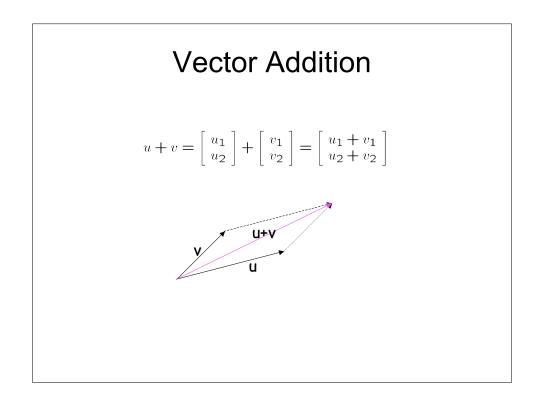


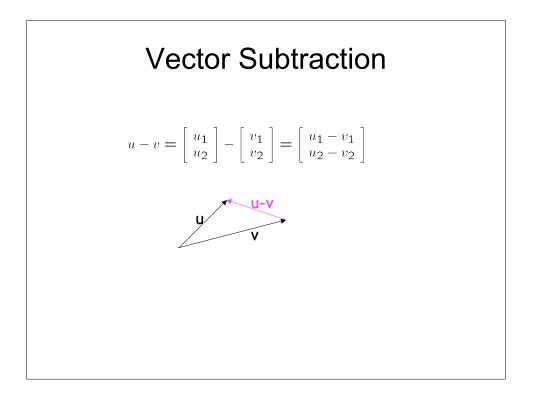
#### **Good Review Materials**

http://www.imageprocessingbook.com/DIP2E/dip2e\_downloads/review\_material\_downloads.htm (Gonzales & Woods review materials)

Chapt. 1: Linear Algebra Review Chapt. 2: Probability, Random Variables, Random Vectors









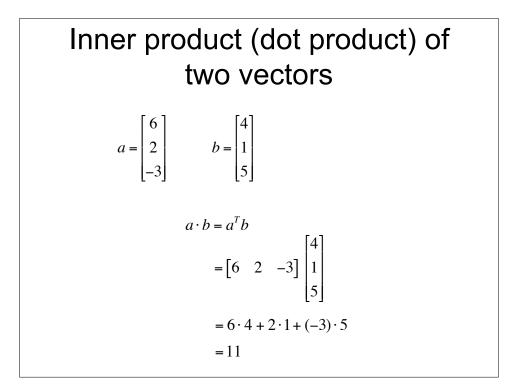
 $\boldsymbol{z} = \alpha \boldsymbol{x}$ 

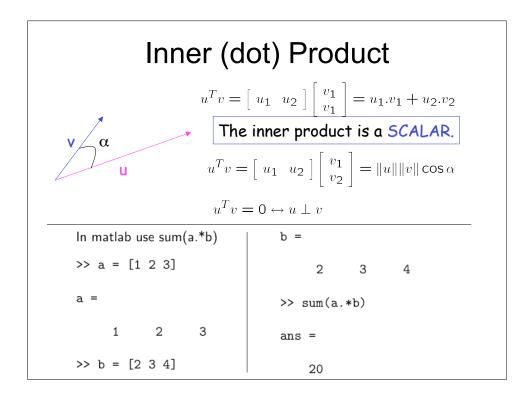
for a scalar  $\alpha$  then

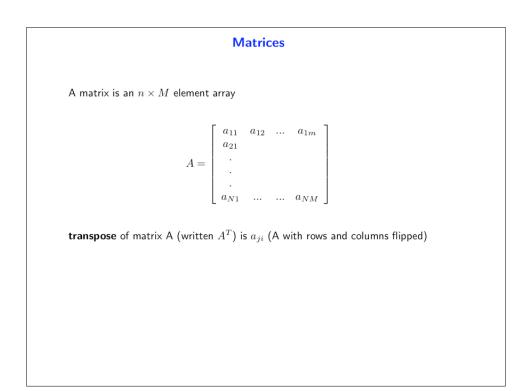
$$\boldsymbol{z} = \alpha \left( \begin{array}{c} 3\\2\\5 \end{array} \right) = \left( \begin{array}{c} 3\alpha\\5\alpha\\2\alpha \end{array} \right)$$

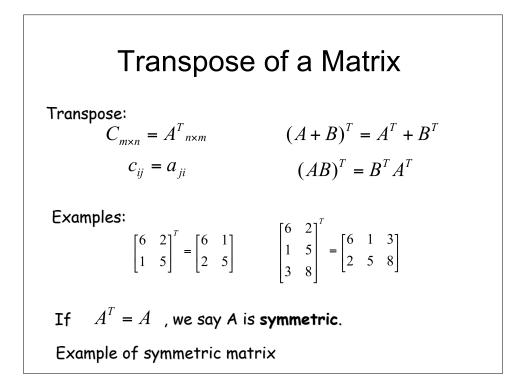
(This is just like stretching/shrinking the vector by a factor  $\alpha$ 

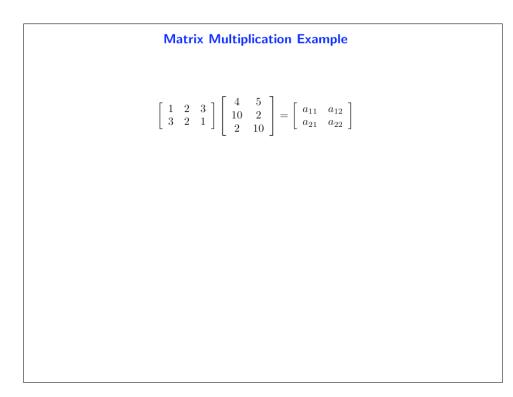
Example (on board)

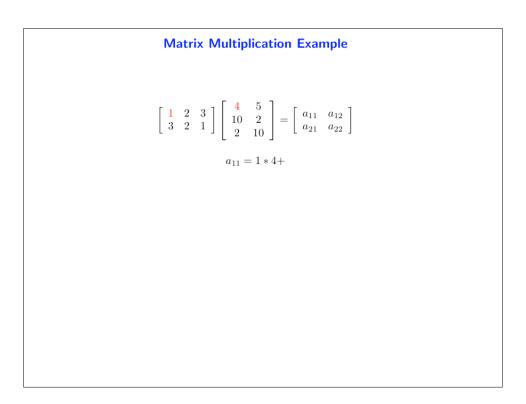


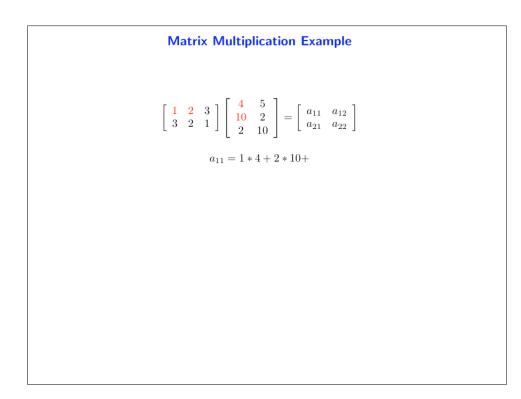


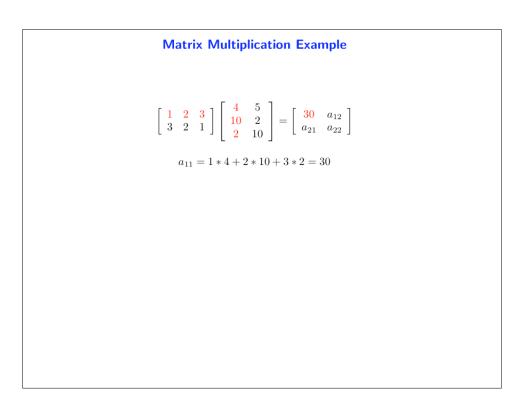


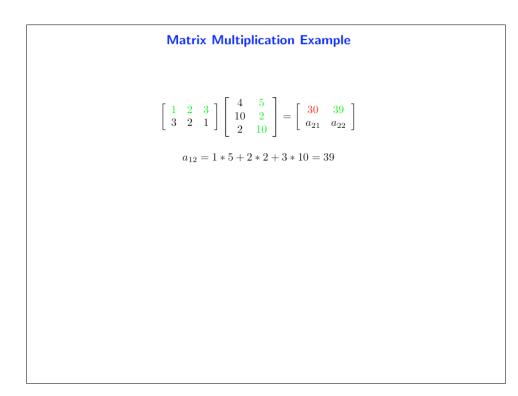


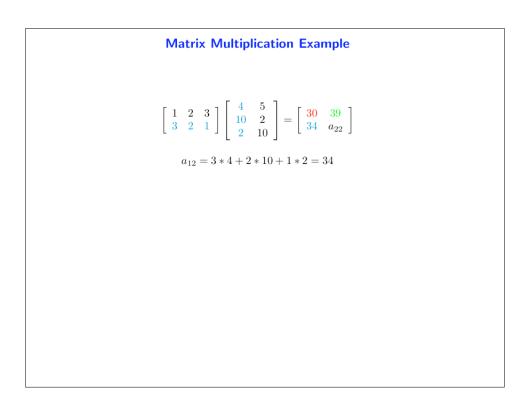


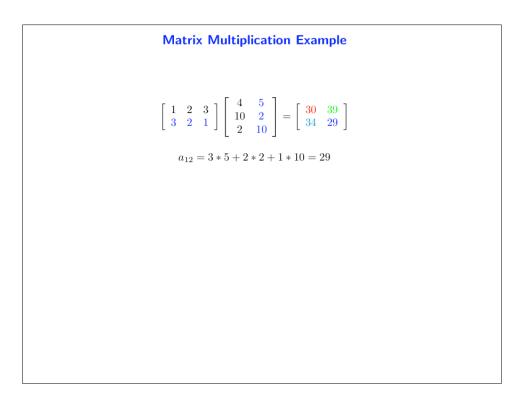


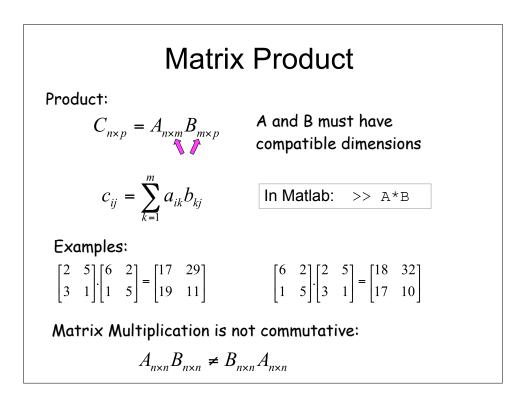


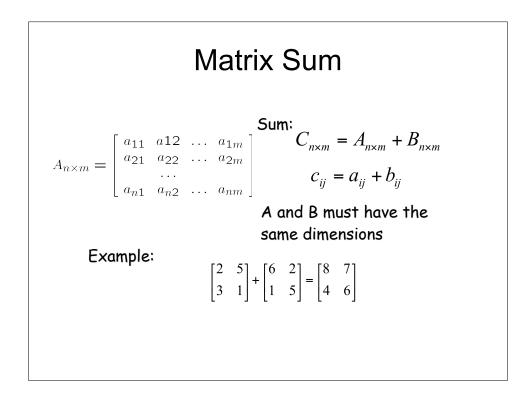


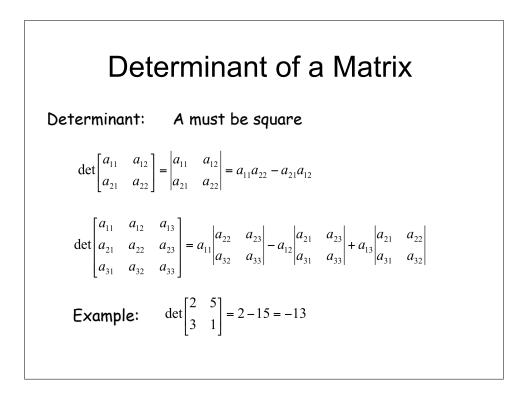


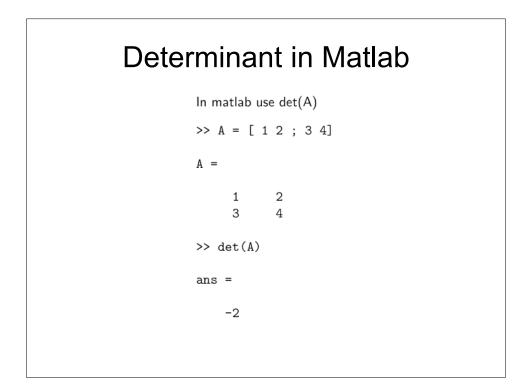


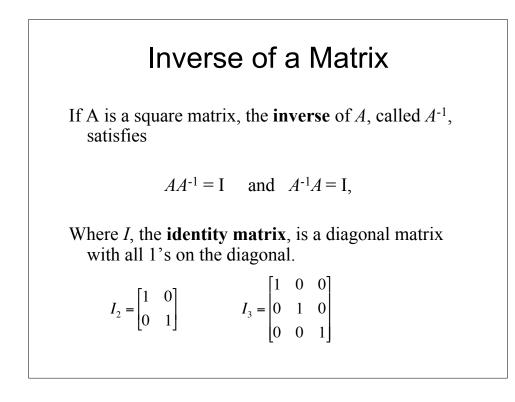


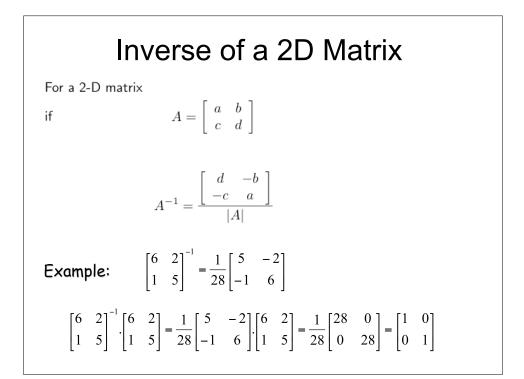


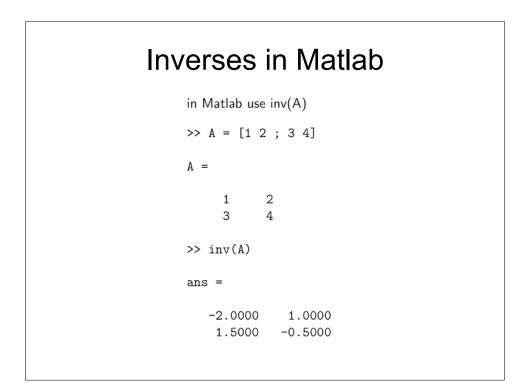


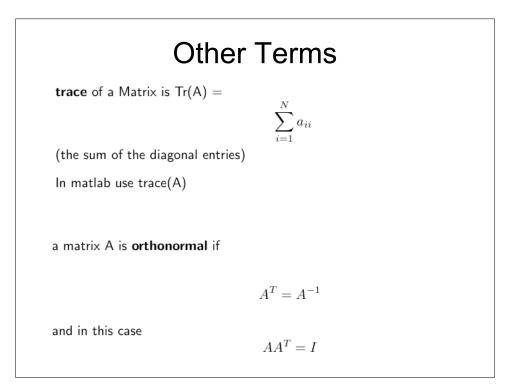


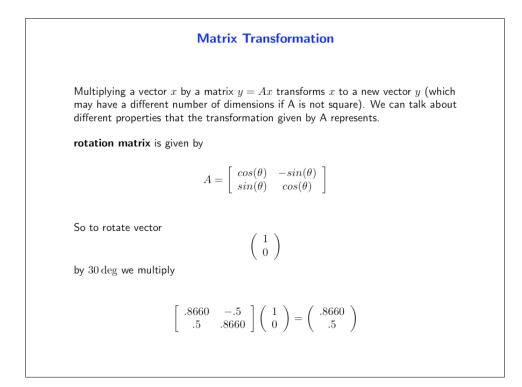


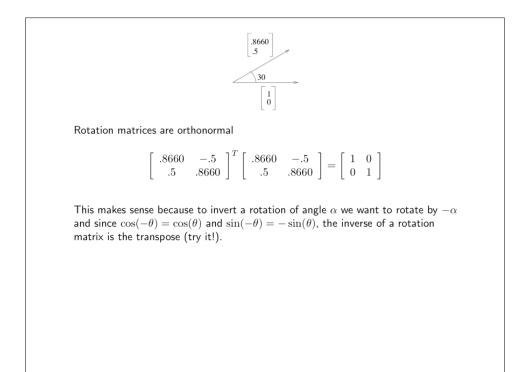


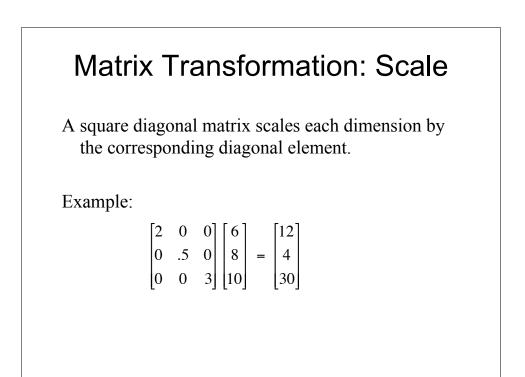


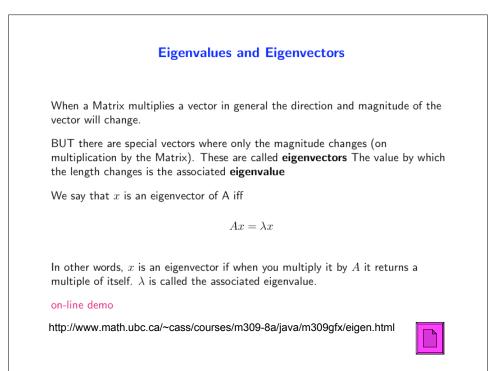










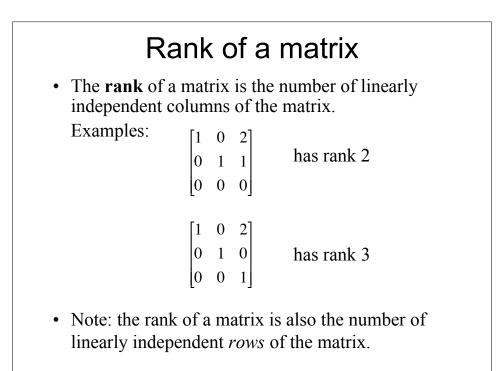


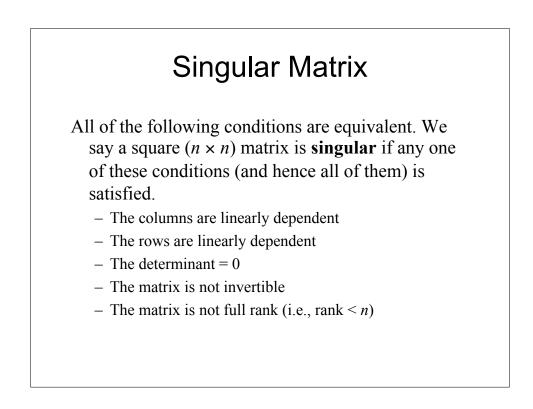
Eigenvalues and Eigenvectors in Matlab In Matlab use $[V,D] = eig(A)$ to get a matrix V whose columns are the eigenvectors of A and a <b>diagonal</b> matrix D whose entries on the diagonal are the corresponding eigenvalues.		
Α =		
1 2 3 4		
>> [V,D] = eig(A)		
V =	D =	
-0.8246 -0.4160 0.5658 -0.9094	-0.3723 0 0 5.3723	

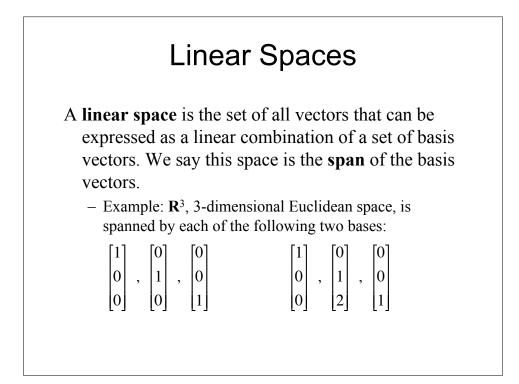
### Some Properties of Eigenvalues and Eigenvectors

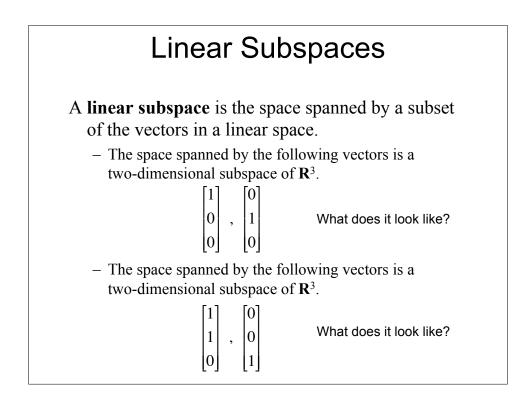
- If  $\lambda_1, ..., \lambda_n$  are *distinct* eigenvalues of a matrix, then the corresponding eigenvectors  $e_1, ..., e_n$  are linearly independent.
- A real, symmetric square matrix has real eigenvalues, with eigenvectors that can be chosen to be orthonormal.

# Linear Independence• A set of vectors is linearly dependent if one of<br/>the vectors can be expressed as a linear<br/>combination of the other vectors.Example: $\begin{bmatrix} 1\\0\\0 \end{bmatrix}, \begin{bmatrix} 0\\1\\0 \end{bmatrix}, \begin{bmatrix} 2\\1\\0 \end{bmatrix}$ • A set of vectors is linearly independent if none<br/>of the vectors can be expressed as a linear<br/>combination of the other vectors.Example: $\begin{bmatrix} 1\\0\\0\\0\end{bmatrix}, \begin{bmatrix} 0\\1\\0\\0\end{bmatrix}, \begin{bmatrix} 2\\1\\0\\1\\3\end{bmatrix}$









#### Orthogonal and Orthonormal Bases

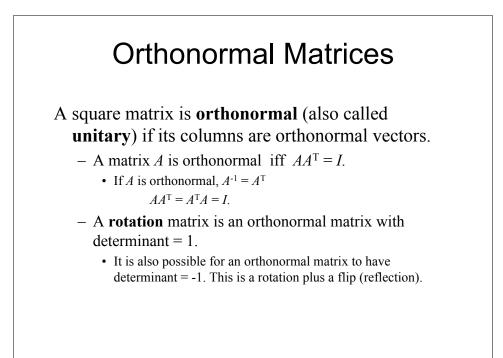
*n* linearly independent real vectors span  $\mathbf{R}^n$ , *n*-dimensional Euclidean space

- They form a **basis** for the space.
- An **orthogonal** basis,  $a_1, ..., a_n$  satisfies
  - $a_i \cdot a_j = 0$  if  $i \neq j$
- An **orthonormal** basis,  $a_1, \ldots, a_n$  satisfies

 $a_i \cdot a_j = 0$  if  $i \neq j$ 

$$a_i \cdot a_i = 1$$
 if  $i = j$ 

- Examples.



#### 

SVD in Matlab		
>> x = $[1 2 3; 2 7 4; -3 0 6; 2 4 9; 5 -8 0]$ x = 1 2 3 2 7 4 -3 0 6 2 4 9 5 -8 0 >> $[u,s,v] = svd(x)$ u = -0.24538 0.11781 -0.11291 -0.47421 -0.82963 -0.53253 -0.11684 -0.52806 -0.45036 0.4702 -0.30668 0.24939 0.79767 -0.38766 0.23915 -0.64223 0.44212 -0.057905 0.61667 -0.091874 0.38691 0.84546 -0.26226 -0.20428 0.15809	$s = 14.412  0  0 \\ 0  8.8258  0 \\ 0  0  5.6928 \\ 0  0  0 \\ 0  0  0 \\ v = 0.01802  0.48126  -0.87639 \\ -0.68573  -0.63195  -0.36112 \\ -0.72763  0.60748  0.31863 \\ \end{array}$	

#### Some Properties of SVD

- The rank of matrix A is equal to the number of nonzero singular values  $\sigma_i$
- A square  $(n \times n)$  matrix A is singular iff at least one of its singular values  $\sigma_1, \ldots, \sigma_n$  is zero.

## **Geometric Interpretation of SVD** If *A* is a square $(n \times n)$ matrix, $\begin{array}{c} A & U & D & V^{T} \\ ( \cdot \cdot \cdot ) \\ ( \cdot \cdot ) \\ \end{array}$ $\begin{array}{c} D & D & V^{T} \\ ( \cdot ) & U & D \\ ( \cdot ) & U$