Image Classification

A Core Task in Computer Vision

Today:

- The image classification task
- Two basic data-driven approaches to image classification
  - ~~K-nearest neighbor and linear classifier~~

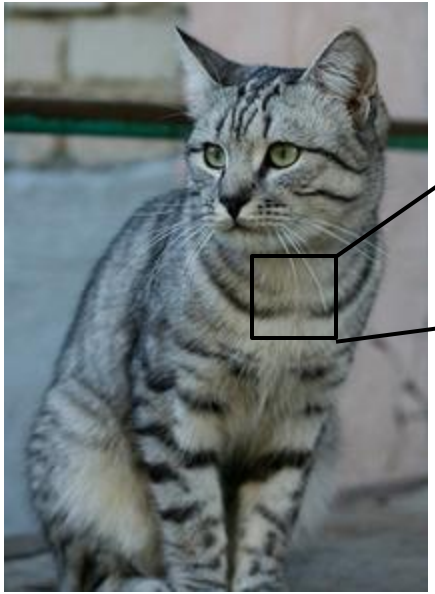
We cover only part of the original lecture.

Image Classification: A core task in Computer Vision

(assume given a set of possible labels)
{dog, cat, truck, plane, ...}

cat

The Problem: Semantic Gap




```
[[105 112 108 111 104  99 106  99  96 103 112 119 104  97  93  87]
 [ 91  98 102 106 104  79  98 103  99 105 123 136 110 105  94  85]
 [ 76  85  90 105 128 105  87  96  95  99 115 112 106 103  99  85]
 [ 99  81  81  93 120 131 127 100  95  98 102  99  96  93 101  94]
 [106  91  61  64  69  91  88  85 101 107 109  98  75  84  96  95]
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 [127 125 131 147 133 127 126 131 111  96  89  75  61  64  72  84]
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 [ 89  93  90  97 108 147 131 118 113 114 113 109 106  95  77  80]
 [ 63  77  86  81  77  79 102 123 117 115 117 125 125 130 115  87]
 [ 62  65  82  89  78  71  80 101 124 126 119 101 107 114 131 119]
 [ 63  65  75  88  89  71  62  81 120 138 135 105  81  98 110 118]
 [ 87  65  71  87 106  95  69  45  76 130 126 107  92  94 105 112]
 [118  97  82  86 117 123 116  66  41  51  95  93  89  95 102 107]
 [164 146 112  80  82 120 124 104  76  48  45  66  88 101 102 109]
 [157 170 157 120  93  86 114 132 112  97  69  55  70  82  99  94]
 [130 128 134 161 139 100 109 118 121 134 114  87  65  53  69  86]
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 [123 107  96  86  83 112 153 149 122 109 104  75  80 107 112  99]
 [122 121 102  80  82  86  94 117 145 148 153 102  58  78  92 107]
 [122 164 148 103  71  56  78  83  93 103 119 139 102  61  69  84]]
```
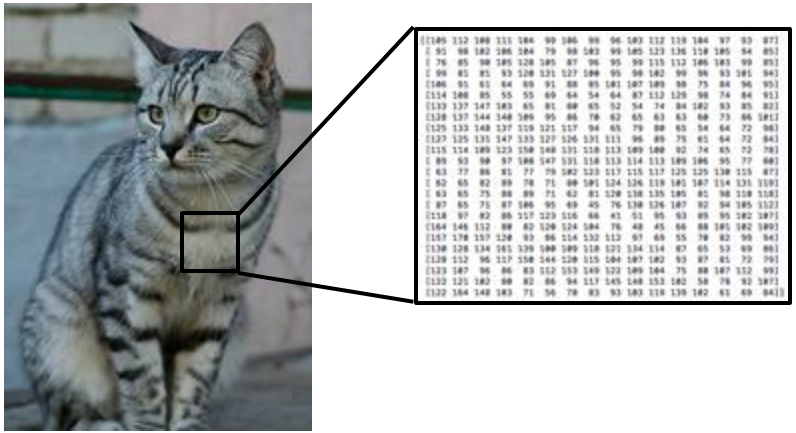

What the computer sees

An image is a tensor of integers between [0, 255]:

e.g. 800 x 600 x 3
(3 channels RGB)

Challenges: Viewpoint variation



All pixels change when the camera moves!

Challenges: Illumination

This image is CC0 1.0 public domain

This image is CC0 1.0 public domain

This image is CC0 1.0 public domain

This image is CC0 1.0 public domain

Challenges: Background Clutter

This image is CC0 1.0 public domain

This image is CC0 1.0 public domain

Challenges: Occlusion

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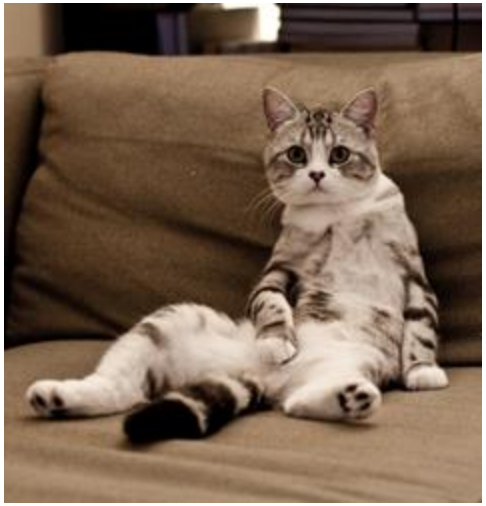
Challenges: Deformation



Challenges: Intraclass variation

Challenges: Context

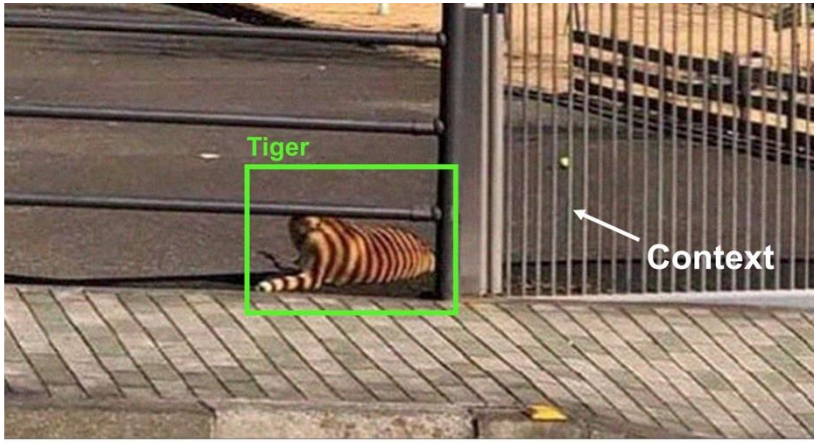


Image source: https://www.linkedin.com/posts/ralph-aboujaoude-diaz-40838313_technology-artificialintelligence-computervision-activity-6912446088364875776-h-Iq?utm_source=linkedin_share&utm_medium=member_desktop_web

Linear Classifier

Parametric Approach

Image

Array of 32x32x3 numbers
(3072 numbers total)

f(x,W)

W
parameters
or weights

10 numbers giving
class scores

Parametric Approach: Linear Classifier

$$f(x,W) = Wx$$

Image

Array of 32x32x3 numbers
(3072 numbers total)

f(x,W)

10 numbers giving
class scores

W

parameters
or weights

Parametric Approach: Linear Classifier

$$f(x, W) = Wx$$

3072x1

10x1

10x3072

Image

Array of 32x32x3 numbers
(3072 numbers total)

f(x,W)

10 numbers giving
class scores

W

parameters
or weights

Parametric Approach: Linear Classifier

$$f(x,W) = Wx + b$$

3072x1 (x)

10x1 (b)

10x1 (f(x,W))

10x3072 (W)

Image

Array of 32x32x3 numbers
(3072 numbers total)

f(x,W)

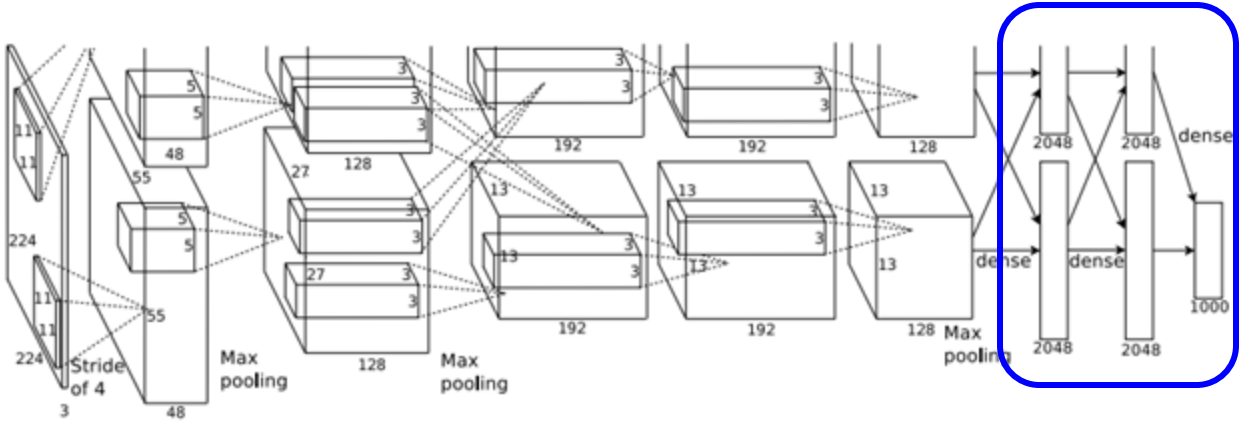
10 numbers giving class scores

W

parameters or weights

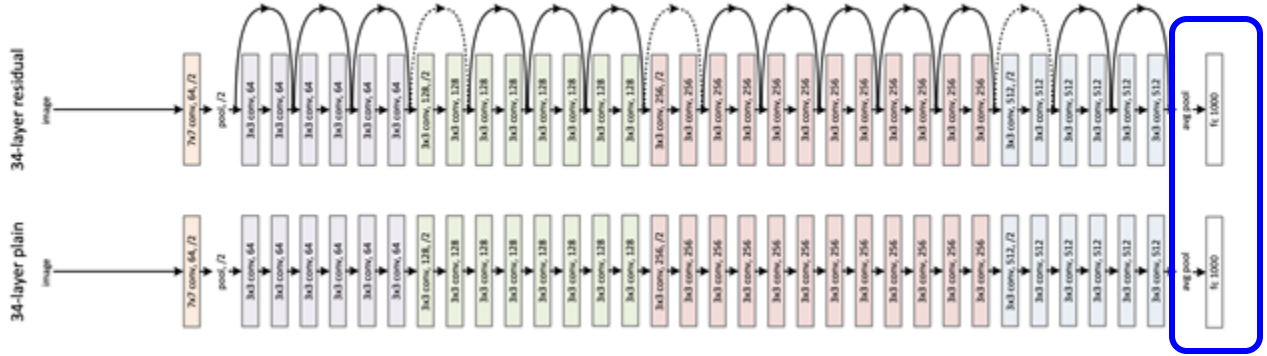
Neural Network

Linear classifiers



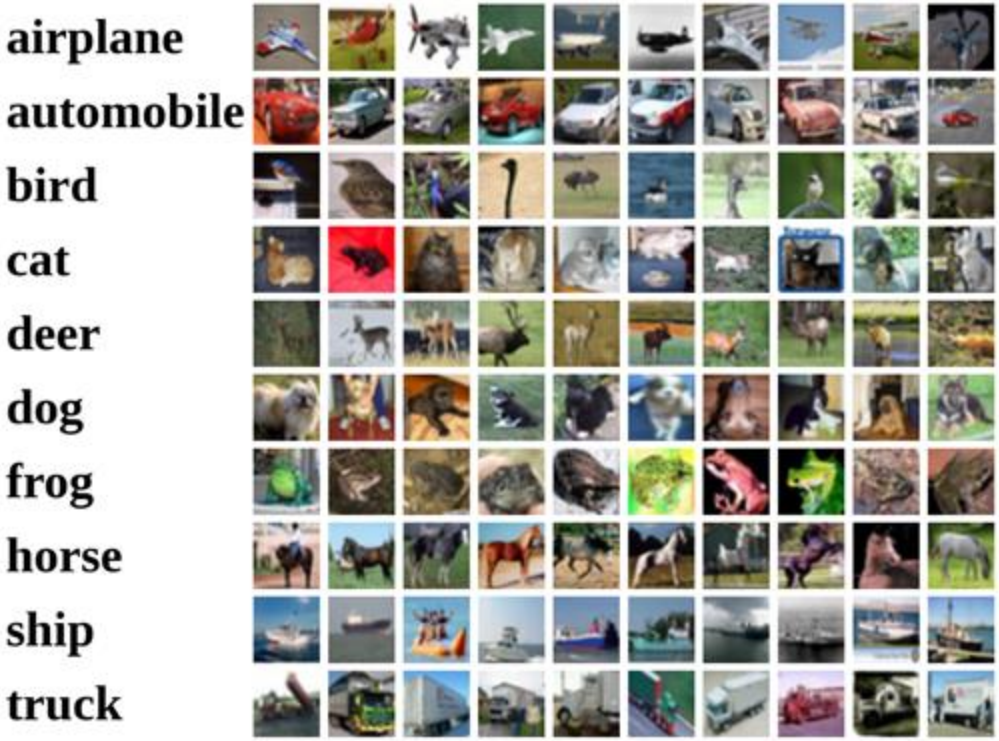
[Krizhevsky et al. 2012]

Linear layers



[He et al. 2015]

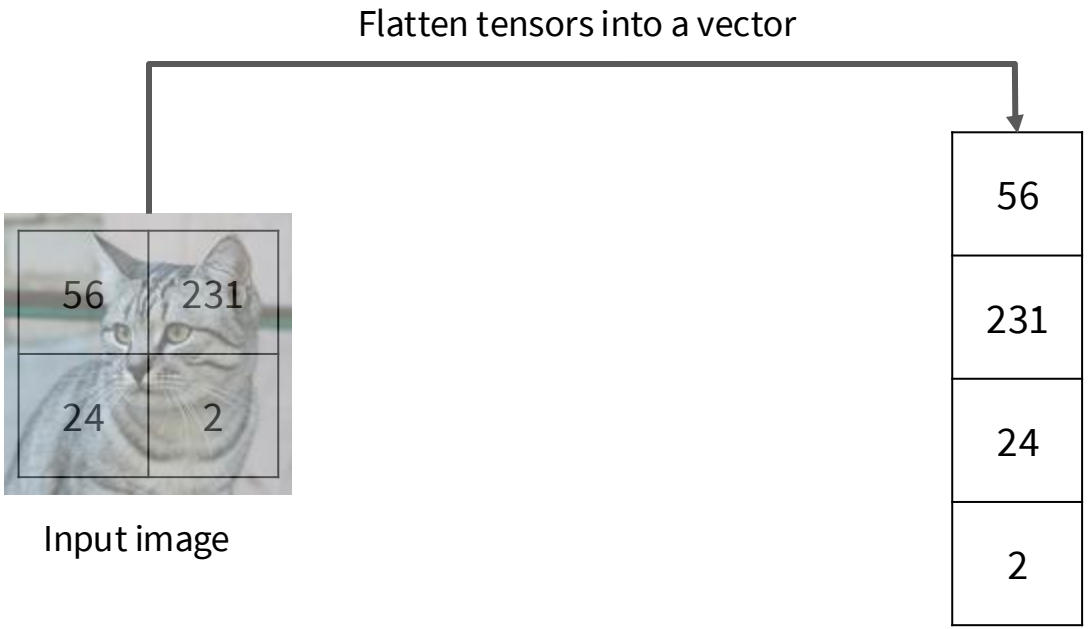
Recall CIFAR10



50,000 training images
each image is 32x32x3

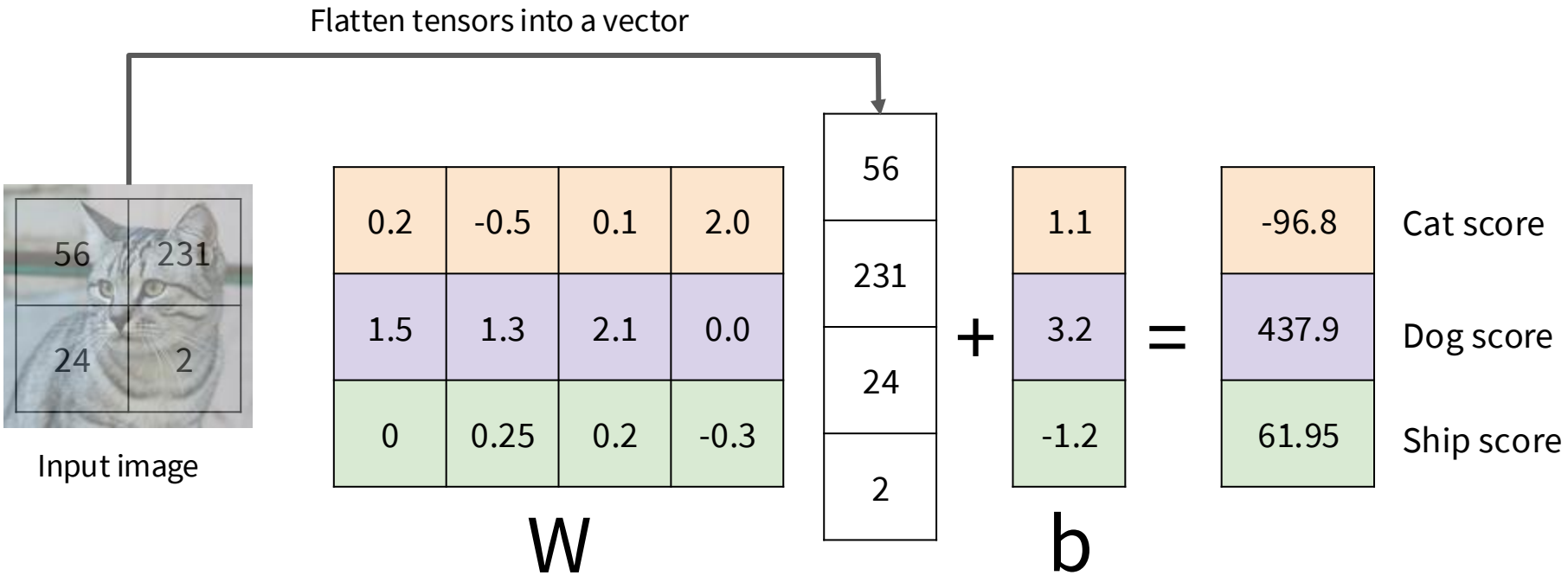
10,000 test images.

Example with an image with 4 pixels, and 3 classes (cat/dog/ship)

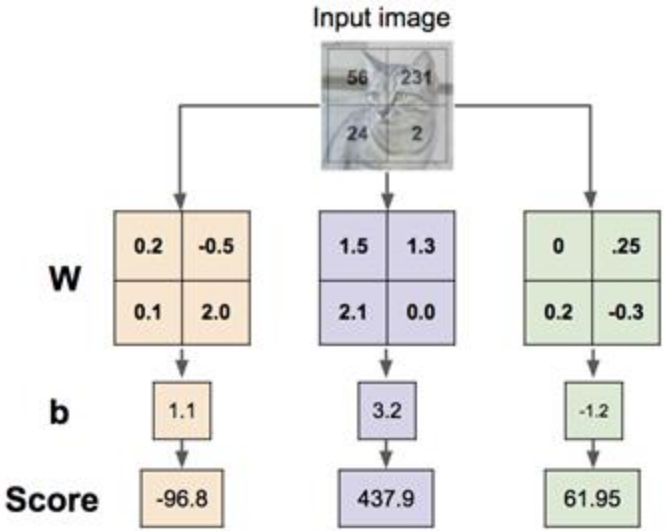


Example with an image with 4 pixels, and 3 classes (cat/dog/ship)

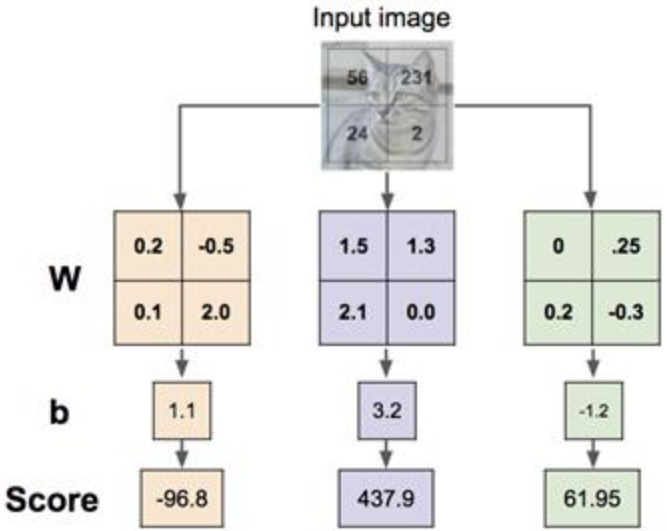
Algebraic Viewpoint



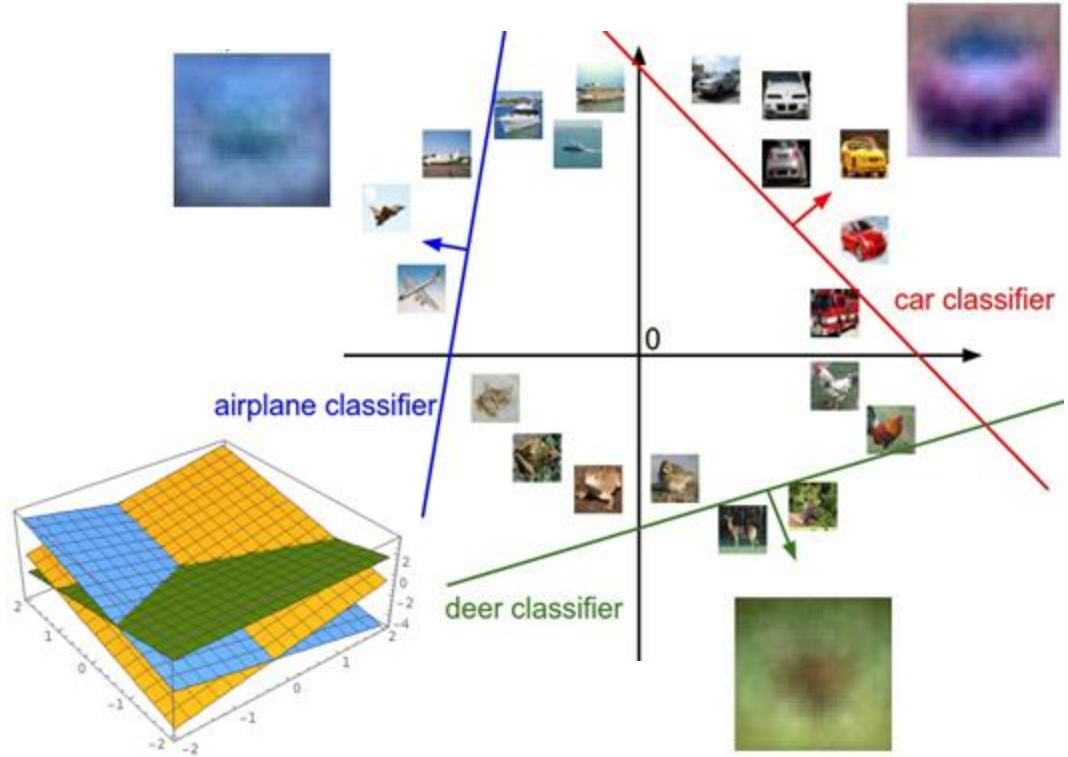
Interpreting a Linear Classifier



Interpreting a Linear Classifier: Visual Viewpoint



Interpreting a Linear Classifier: Geometric Viewpoint



$$f(x,W) = Wx + b$$

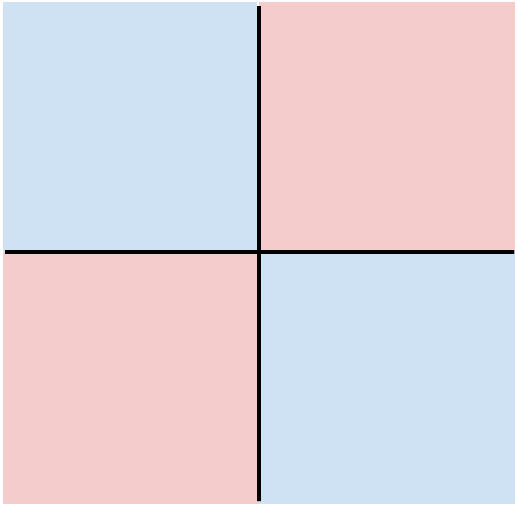
Array of 32x32x3 numbers
(3072 numbers total)

Plot created using Wolfram Cloud

Hard cases for a linear classifier

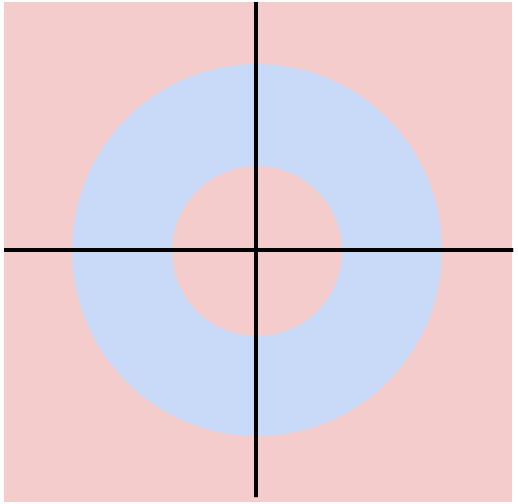
Class 1:
First and third quadrants

Class 2:
Second and fourth quadrants



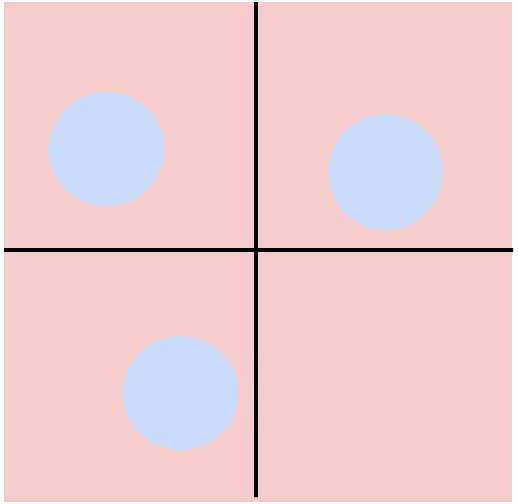
Class 1:
1 <= L2 norm <= 2

Class 2:
Everything else



Class 1:
Three modes

Class 2:
Everything else



Linear Classifier – Choose a good W

airplane	-3.45	-0.51	3.42
automobile	-8.87	**6.04**	4.64
bird	0.09	5.31	2.65
cat	**2.9**	-4.22	5.1
deer	4.48	-4.19	2.64
dog	8.02	3.58	5.55
frog	3.78	4.49	**-4.34**
horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

1. Define a loss function that quantifies our unhappiness with the scores across the training data.
2. Come up with a way of efficiently finding the parameters that minimize the loss function. (optimization)

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$



cat	3.2	1.3	2.2
car	5.1	4.9	2.5
frog	-1.7	2.0	-3.1

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$

A loss function tells how good our current classifier is



cat	3.2	1.3	2.2
car	5.1	4.9	2.5
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With some W the scores $f(x, W) = Wx$



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A loss function tells how good our current classifier is

Given a dataset of examples

$$\{(x_i, y_i)\}_{i=1}^{N}$$

Where x_i is image and y_i is (integer) label

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$



cat	3.2	1.3	2.2
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frog	-1.7	2.0	-3.1

A loss function tells how good our current classifier is

Given a dataset of examples

$$\{(x_i, y_i)\}_{i=1}^{N}$$

Where x_i is image and
y_i is (integer) label

Loss over the dataset is a average of loss over examples:

$$L = \frac{1}{N} \sum_i L_i(f(x_i, W), y_i)$$

Softmax classifier

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as probabilities

cat	3.2
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as probabilities

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

cat	3.2
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as probabilities

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Probabilities must be >= 0

cat	3.2	exp →	24.5
car	5.1		164.0
frog	-1.7		0.18

unnormalized probabilities

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as probabilities

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Probabilities must be >= 0

Probabilities must sum to 1

		exp		normalize	
cat	3.2	→	24.5	→	0.13
car	5.1	→	164.0	→	0.87
frog	-1.7	→	0.18	→	0.00

unnormalized probabilities

probabilities

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as probabilities

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Probabilities must be >= 0

Probabilities must sum to 1

	Unnormalized log-probabilities / logits	exp	unnormalized probabilities	normalize	probabilities
cat	3.2		24.5		0.13
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as probabilities

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Probabilities must be >= 0

Probabilities must sum to 1

$$L_i = -\log P(Y = y_i|X = x_i)$$

	Unnormalized log-probabilities / logits	exp	unnormalized probabilities	normalize	probabilities
cat	3.2		24.5		0.13
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

→ L_i = -log(0.13) = 2.04

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as probabilities

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$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Probabilities must be >= 0

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$$L_i = -\log P(Y = y_i|X = x_i)$$

	Unnormalized log-probabilities / logits	exp	unnormalized probabilities	normalize	probabilities
cat	3.2		24.5		0.13
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

→ L_i = -log(0.13) = 2.04

Maximum Likelihood Estimation
Choose weights to maximize the likelihood of the observed data
(See CS 229 for details)

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as probabilities

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

$$L_i = -\log P(Y = y_i|X = x_i)$$

Probabilities must be >= 0

Probabilities must sum to 1

	Unnormalized log-probabilities / logits	exp	unnormalized probabilities	normalize	probabilities	compare	Correct probs
cat	3.2		24.5		0.13		1.00
car	5.1		164.0		0.87		0.00
frog	-1.7		0.18		0.00		0.00

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as probabilities

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

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Probabilities must be >= 0

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	Unnormalized log-probabilities / logits	exp	unnormalized probabilities	normalize	probabilities	compare	Correct probs
cat	3.2		24.5		0.13		1.00
car	5.1		164.0		0.87		0.00
frog	-1.7		0.18		0.00		0.00

Kullback–Leibler divergence

$$D_{KL}(P\|Q) = \sum_y P(y) \log \frac{P(y)}{Q(y)}$$

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as probabilities

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

$$L_i = -\log P(Y = y_i|X = x_i)$$

	Unnormalized log-probabilities / logits	exp	unnormalized probabilities	normalize	probabilities	compare	Correct probs
	Probabilities must be >= 0				Probabilities must sum to 1		
cat	3.2		24.5		0.13		1.00
car	5.1		164.0		0.87		0.00
frog	-1.7		0.18		0.00		0.00

Cross Entropy

$$H(P, Q) = H(p) + D_{KL}(P\|Q)$$

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as probabilities

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Maximize probability of correct class

$$L_i = -\log P(Y = y_i|X = x_i)$$

Putting it all together:

$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

cat	3.2
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as probabilities

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Maximize probability of correct class

$$L_i = -\log P(Y = y_i|X = x_i)$$

Putting it all together:

$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

cat	3.2
car	5.1
frog	-1.7

Q1: What is the min/max possible softmax loss L_i?

Q2: At initialization all s_j will be approximately equal; what is the softmax loss L_i, assuming C classes?

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as probabilities

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Maximize probability of correct class

$$L_i = -\log P(Y = y_i|X = x_i)$$

Putting it all together:

$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

cat	3.2
car	5.1
frog	-1.7

Q2: At initialization all s will be approximately equal; what is the loss?
A: -log(1/C) = log(C),
If C = 10, then L_i = log(10) ≈ 2.3