

**Figure 1.9 RSA.**

Bob chooses his public and secret keys.

- He starts by picking two large ( $n$ -bit) random primes  $p$  and  $q$ .
- His public key is  $(N, e)$  where  $N = pq$  and  $e$  is a  $2n$ -bit number relatively prime to  $(p-1)(q-1)$ . A common choice is  $e = 3$  because it permits fast encoding.
- His secret key is  $d$ , the inverse of  $e$  modulo  $(p-1)(q-1)$ , computed using the extended Euclid algorithm.

Alice wishes to send message  $x$  to Bob.

- She looks up his public key  $(N, e)$  and sends him  $y = (x^e \bmod N)$ , computed using an efficient modular exponentiation algorithm.
- He decodes the message by computing  $y^d \bmod N$ .

$p = 5$  &  $q = 11 \Rightarrow N = 55$ ,  $e = 3$ ,  $(p-1)(q-1) = 40$   
 $\gcd(3, 40) = 1$ , we use extended Euclid to find  $d$

$$40 = 13 \cdot 3 + 1 \Rightarrow 1 = \underline{40} - 13 \cdot \underline{3} \Rightarrow 3^{-1} \equiv -13 \equiv 27 \pmod{40}$$

$$x = 13 \Rightarrow y = 13^3 = 52 \pmod{55}$$

$$y^{27} = 52^{27} = 13 \pmod{55}$$

$$\text{hence } 13 = (13^3)^{27} \pmod{55}$$