**Figure 7.11** In the most general case of linear programming, we have a set $I$ of inequalities and a set $E$ of equalities (a total of $m = |I| + |E|$ constraints) over $n$ variables, of which a subset $N$ are constrained to be nonnegative. The dual has $m = |I| + |E|$ variables, of which only those corresponding to $I$ have nonnegativity constraints.

Primal LP:

$$\begin{aligned} \max\ & c_1x_1 + \cdots + c_nx_n \\ a_{i1}x_1 + \cdots + a_{in}x_n &\le b_i \quad \text{for } i \in I \\ a_{i1}x_1 + \cdots + a_{in}x_n &= b_i \quad \text{for } i \in E \\ x_j &\ge 0 \quad \text{for } j \in N \end{aligned}$$

Dual LP:

$$\begin{aligned} \min\ & b_1y_1 + \cdots + b_my_m \\ a_{1j}y_1 + \cdots + a_{mj}y_m &\ge c_j \quad \text{for } j \in N \\ a_{1j}y_1 + \cdots + a_{mj}y_m &= c_j \quad \text{for } j \notin N \\ y_i &\ge 0 \quad \text{for } i \in I \end{aligned}$$

Two examples of duals for the given primal LPs:

(P) Maximize $z_P = 2x_1 + 3x_2$

subject to

$$\begin{aligned} -x_1 + x_2 &\le 5 \\ x_1 + 3x_2 &\le 35 \\ x_1 &\le 20 \\ x_1 \ge 0,\ x_2 &\ge 0 \end{aligned}$$

(D) Minimize $z_D = 5\pi_1 + 35\pi_2 + 20\pi_3$

subject to

$$\begin{aligned} -\pi_1 + \pi_2 + \pi_3 &\ge 2 \\ \pi_1 + 3\pi_2 &\ge 3 \\ \pi_1 \ge 0,\ \pi_2 \ge 0,\ \pi_3 &\ge 0 \end{aligned}$$

(P) Maximize $z_P = -3x_1 - 2x_2$

subject to

$$\begin{aligned} -x_1 - x_2 &= 8 \\ x_1 + 2x_2 &\ge 13 \\ x_1 \ge 0,\ x_2 & \text{ unrestricted} \end{aligned}$$

(D) Minimize $z_D = 8\pi_1 + 13\pi_2$

subject to

$$\begin{aligned} -\pi_1 + \pi_2 &\ge -3 \\ -\pi_1 + 2\pi_2 &= -2 \\ \pi_1 \text{ unrestricted},\ \pi_2 &\le 0 \end{aligned}$$

Examples from Operations Research Models and Methods
Paul A. Jensen and Jonathan F. Bard