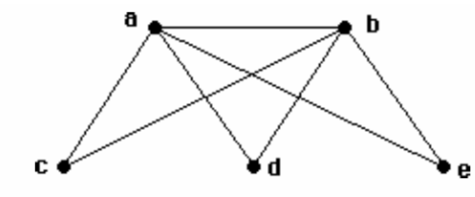
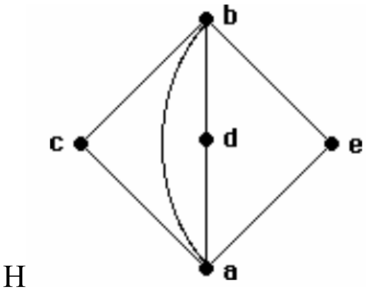
1. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

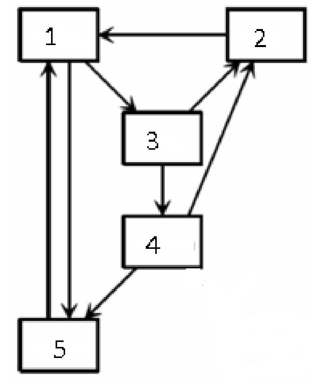
G 

H 

2. Use induction to prove that every amount of postage of n cents greater than 5 cents can be formed from 3-cent and 4-cent stamps.

3. (a) For which n does complete bipartite graph $K_{m,n}$ (for m,n>1) have an Euler circuit? Justify your answer.
(b) For which n does complete bipartite graph $K_{m,n}$ (for m,n>1) have a Hamilton circuit? Justify your answer.

4. For the web graph shown below write the link matrix A that expresses the system of PageRank linear equations in the form $\mathbf{A}\mathbf{x} = \mathbf{x}$, where $\mathbf{x} = [x_1\ x_2\ x_3\ x_4\ x_5]^T$.
Is the matrix $\mathbf{M} = (1 - m)\mathbf{A} + m\mathbf{S}$ for m=0.25 column-stochastic? Justify your answer.



5. Use the method of Gaussian elimination to find **x** for the system of linear equations **Ax**=**b**, where **A** and **b** are given below. Show your work.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 2 & 6 & 11 \end{bmatrix}, b = \begin{bmatrix} 10 \\ 4 \\ 6 \end{bmatrix}$$

6. Use the method of Gaussian elimination to find the determinant of matrix **B** given below. Show your work.

$$\begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 3 \\ 2 & -2 & 0 \end{bmatrix}$$

7. Find the eigenvalues and the eigenvectors of these two matrices. Show your work.

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \quad \text{and} \quad A + I = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$$

8. Find the eigenvalues and the eigenvectors of matrix A. Show your work.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 3 & 3 \end{bmatrix}.$$

9. Find the matrix **A** that performs those transformations, in order, on the Cartesian plane. To which point is the point (-4, 1) mapped by this transformation?
(a) horizontal stretch by a factor of 2
(b) reflection across the y-axis.

10. Find the standard matrix **A** for the given linear transformation T.

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 - 3x_3 \\ 0 \\ x_1 + 4x_3 \\ 5x_2 + x_3 \end{bmatrix}$$

11. Provide a pseudo-code of an algorithm for finding the second largest number in a sequence of n distinct integers ($n > 1$) distinct integers. What is its worst-case time complexity in terms of the number of comparisons? Justify your answer.

12. Suppose we store numbers in the nodes of a full binary tree. The numbers obey the **heap property** if, for every node *u* in the tree, the value V(*u*) is at least as big as the value in each of *u*'s children. Let T be a bull binary tree with the heap property. Prove with structural induction that the value in the root of T is at least as large as the value in any other node of the tree. Hint. In the inductive step, split up tree T at its root, producing two smaller subtrees T_L and T_R.