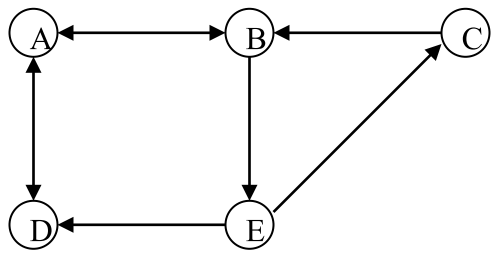
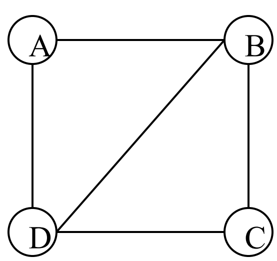
1. (a) Let a directed graph G_1 be given.



Does each of the following list of vertices form a path in G_1? If yes, determine (by circling) if the path is simple, if it is a circuit, and give its length.

a, b, e, c, b	Yes [simple circuit length $\boxed{}$] No
a, d, a, d, a	Yes [simple circuit length $\boxed{}$] No
a, d, e, b, a	Yes [simple circuit length $\boxed{}$] No
a, b, e, c, b, a	Yes [simple circuit length $\boxed{}$] No

(b) For the simple graph G_2



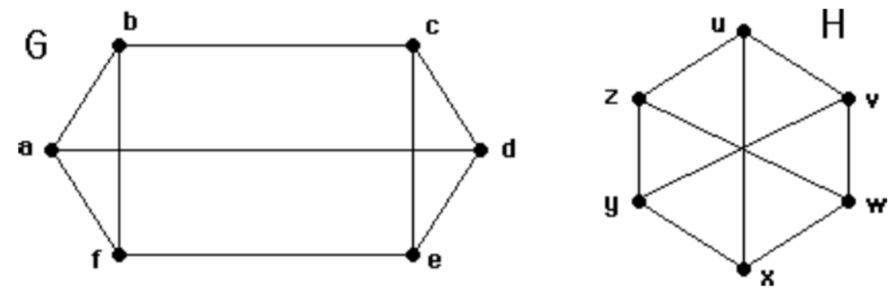
Find M^2, where M is the adjacency matrix of G_2

$$M^2 = \left\{ \begin{matrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{matrix} \right\}$$

Find the number of paths from A to D in G_2 of length 2. $\boxed{}$

2. Provide a pseudo code of an algorithm for finding a closest pair of numbers in a set of n real distinct numbers and give a worst-case estimate of the number of comparisons.

3. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

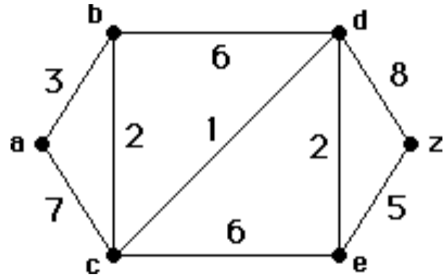


4. Let $a_1 = 2$, $a_2 = 9$, and $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 3$. Show using induction that $a_n \leq 3^n$ for all positive integers n.

5. Use mathematical induction to show that $\sum_{j=0}^{n}(j+1) = \frac{(n+1)(n+2)}{2}$ whenever n is a nonnegative integer.

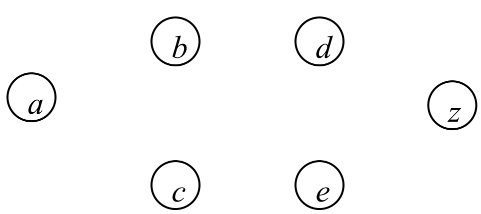
6. Let $f(n) = 2n\log(n^2+5) + 3n + 1$. What is big-O estimate of $f(n)$? Be sure to specify the values of the witnesses C and k.

7. Use Dijkstra’s algorithm to find the length of the shortest path between the vertices *a* and *z* in the following weighted graph. Use the table below to log in your computation.



a	*b*	*c*	*d*	*e*	*z*	*S*
0	∞	∞	∞	∞	∞	*a*
X						
X						
X						
X						
X						
X						
X						
X						

Draw a tree representing the shortest distances from *a* to each of the other vertices. Indicate the distance next to each vertex.



8. How many vertices and how many edges does each of the following graphs have?
(a) K_5

(b) C_4

(c) W_5

(d) $K_{2,5}$

9. Write a pseudocode for an algorithm for evaluating a polynomial of degree n,
$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$, at $x = c$.
What is big-O estimate of the time complexity of your algorithm (in terms of the number of multiplications and additions used) as a function of n? Explain your answer.

10. For which values of n do these graphs have a Euler circuit?
a) K_n **b)** C_n **c)** W_n **d)** Q_n

11. What is the effect in the time required to solve a problem when you double the size of the input from n to $2n$? Express your answer in the simplest form possible, either as a ratio or a difference. Explain the meaning of your answer.
a) $\log n$
b) $100n$
c) n^2

12. Give a recursive algorithm for finding the maximum of a finite set of integers, the recursion should make use of the fact that the maximum of n integers is the larger of the last integer in the list and the maximum of the first $n - 1$ integers in the list.