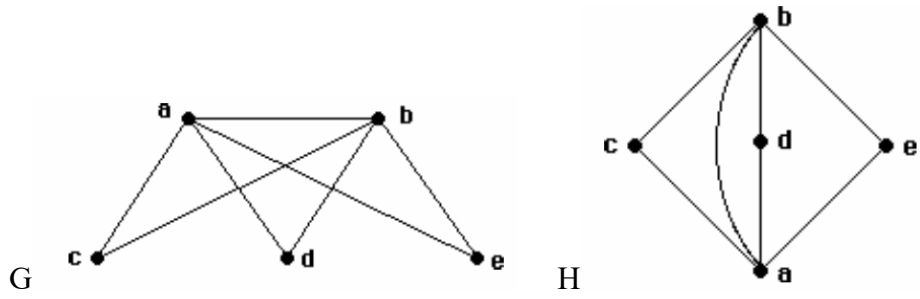


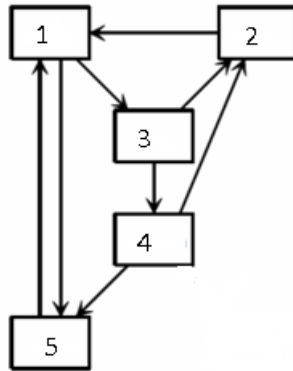
1. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.



2. Prove by induction that $n^2 - 7n + 12$ is nonnegative whenever n is an integer with $n \geq 3$.

3. (a) How many vertices and how many edges are in K_n for $n > 2$?
- (b) For which n does K_n (for $n > 2$) have a Euler circuit? Justify your answer.
- (c) For which n does K_n (for $n > 2$) have a Hamilton circuit? Justify your answer.
- (d) How many vertices and how many edges are in complete bipartite graph $K_{m,n}$ for $m, n > 1$?
- (e) For which n does complete bipartite graph $K_{m,n}$ (for $m, n > 1$) have a Euler circuit? Justify your answer.
- (f) For which n does complete bipartite graph $K_{m,n}$ (for $m, n > 1$) have a Hamilton circuit? Justify your answer.

4. For the web graph shown below write the link matrix A that expresses the system of PageRank linear equations in the form $Ax = x$, where $x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T$. Is the matrix $M = (1 - m)A + mS$ for $m=0.25$ column-stochastic? Justify your answer.



5. Use the method of Gaussian elimination to find x for the system of linear equations $Ax=b$, where A and b are given below. Show your work.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 2 & 6 & 11 \end{bmatrix}, b = \begin{bmatrix} 10 \\ 4 \\ 6 \end{bmatrix}$$

6. Use method of Gaussian elimination to find the determinant of matrix **B** given below. Show your work.

$$\begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 3 \\ 2 & -2 & 0 \end{bmatrix}$$

7. Find the eigenvalues and the eigenvectors of these two matrices. Show your work.

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \quad \text{and} \quad A + I = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$$

8. Find the eigenvalues and the eigenvectors of matrix A. Show your work.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 3 & 3 \end{bmatrix}.$$

9. Find the matrix A that performs those transformations, in order, on the Cartesian plane. To which point is the point $(-2, 1)$ mapped by this transformation.
- (a) horizontal stretch by a factor of 3
 - (b) reflection across the line $y = x$

10. Find the standard matrix \mathbf{A} for the given linear transformation T .

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 - 3x_3 \\ 0 \\ x_1 + 4x_3 \\ 5x_2 + x_3 \end{bmatrix}$$

11. Provide a pseudo code of an algorithm that takes a list of n integers ($n > 1$) and finds the average value of the largest integer and the smallest integer in the list. What is its worst case time complexity in the terms of the number of comparisons? Justify your answer.

12. Let $f(n) = 2n\log(n^2+3) + 7n + 1$. What is big-O estimate of $f(n)$? Be sure to specify the values of the witnesses C and k .