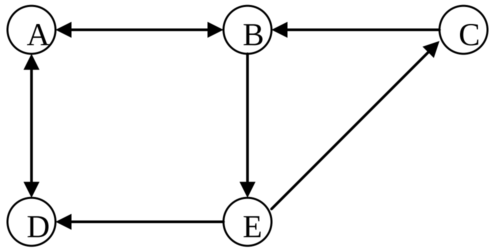
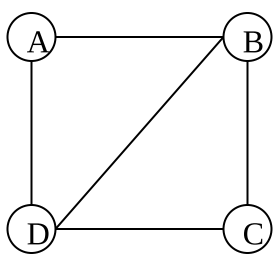
1. (a) Let a directed graph G_1 be given.



Does each of the following list of vertices form a path in G_1? If yes, determine (by circling) if the path is simple, if it is a circuit, and give its length.

a, b, e, c, b	Yes [simple circuit length ☐] No
a, d, a, d, a	Yes [simple circuit length ☐] No
a, d, e, b, a	Yes [simple circuit length ☐] No
a, b, e, c, b, a	Yes [simple circuit length ☐] No

(b) For the simple graph G_2



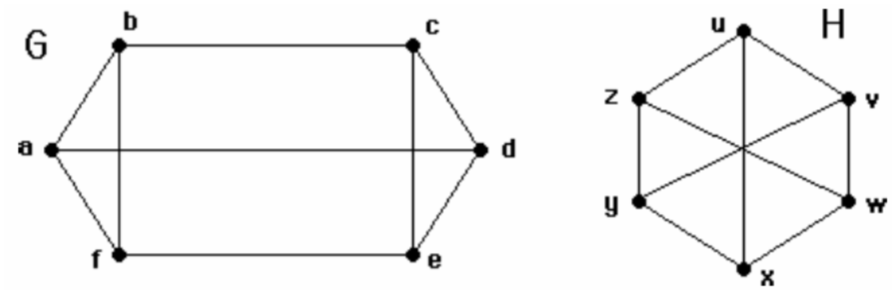
Find M^2, where M is the adjacency matrix of G_2

$$M^2 = \begin{pmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{pmatrix}$$

Find the number of paths from A to D in G_2 of length 2. ☐

2. Provide a pseudo code of an algorithm for finding a closest pair of numbers in a set of *n* real distinct numbers and give a worst-case estimate of the number of comparisons.

3. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.



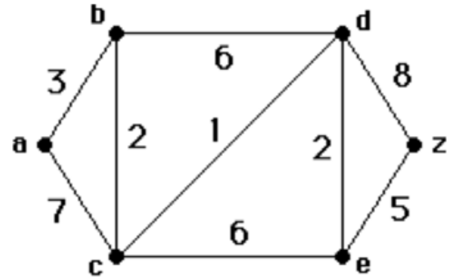
4. Let $a_1 = 2$, $a_2 = 9$, and $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 3$. Show using induction that $a_n \leq 3^n$ for all positive integers n.

5. Prove using induction that for all positive integers n the following formula holds

$$\frac{1}{2}+\frac{1}{2^2}+\ldots+\frac{1}{2^n}=\frac{2^n-1}{2^n}$$

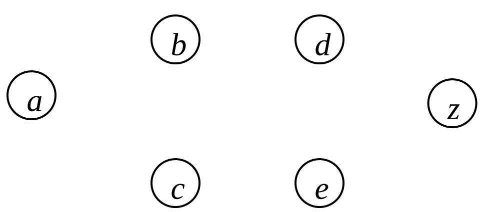
6. Let $f(n) = 5n^2 + 2n\log(n) + 3n + 1$. Show that $f(n)$ is $O(n^2)$. Be sure to specify the values of the witnesses C and k.

7. Use Dijkstra's algorithm to find the length of the shortest path between the vertices *a* and *z* in the following weighted graph. Use the table below to log in your computation.



a	*b*	*c*	*d*	*e*	*z*	*S*
0	∞	∞	∞	∞	∞	*a*
X						
X						
X						
X						
X						
X						
X						
X						

Draw a tree representing the shortest distances from *a* to each of the other vertices. Indicate the distance next to each vertex.



8. How many vertices and how many edges does each of the following graphs have?

(a) K_5

(b) C_4

(c) W_5

(d) $K_{2,5}$

9. Write a pseudocode for an algorithm for evaluating a polynomial of degree *n*,
$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$, at $x = c$.
What is big-O estimate of the time complexity of your algorithm (in terms of the number of multiplications and additions used) as a function of *n*? Explain your answer.

10. Let S be the subset of the set of ordered pairs of integers defined recursively by
Basis step: $(0, 0) \in S$.
Recursive step: If $(a, b) \in S$, then $(a + 2, b + 3) \in S$ and $(a + 3, b + 2) \in S$.
a) List the elements of S produced by the first two applications of the recursive definition.
b) Use structural induction to show that $5 \mid a + b$ when $(a, b) \in S$.

11. For which values of n do these graphs have an Euler circuit?
a) K_n **b)** C_n **c)** W_n **d)** Q_n

12. Show that $\log(\boldsymbol{n}!)$ is $\Theta(\boldsymbol{n}\cdot\log(\boldsymbol{n}))$.