CIS 2166
HW 3 for Matrix Algebra part 2

1. Find the determinants, eigenvalues and eigenvectors of all the matrices below. Check if the determinant equals the product of its eigenvalues and if its trace equals the sum of its eigenvalues.

A = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, C = $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$, D = $\begin{bmatrix} 2 & 0 & 0 \\ 4 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}$,

E = $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$, F = $\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$, G = $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$, H = $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

2. The following are characteristic polynomials of some matrices. Construct a matrix that corresponds to each characteristic polynomial. What are the eigenvalues of those matrices?

$\lambda^2 - 4\lambda + 4$
$\lambda^2 + 2\lambda - 3$
$\lambda^2 - 4$
$\lambda^2 + 4$

3. Determinant of matrix A which is of size 10-by-10 is 5. What is the determinant of this matrix after each of the following transformations:

- First, its first 2 rows are multiplied by 4
- Second, its first column is added to columns 3 and 4
- Third, its 3rd row is switched with its 6th row
- Fourth, its 8th row is replaced with its 1st row.

4. If matrix A is orthogonal (dot product of each pair of its columns is zero; also dot product of each pair of its rows is 0; dot product of each row or column with itself is 1), show that $A^T = A^{-1}$. Hint: calculate A^TA and AA^T

5. Show that for a square matrix A of size n, $\det(cA) = c^n\det(A)$, where c is any scalar. Hint: use property that multiplying one row of a matrix by c also multiplies the determinant of the matrix by c.