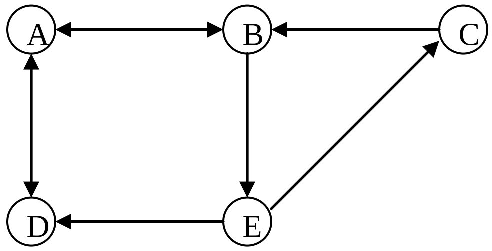
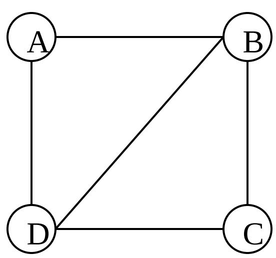
**1**. (a) Let a directed graph $G_1$ be given.




Does each of the following list of vertices form a path in $G_1$? If yes, determine (by circling) if the path is simple, if it is a circuit, and give its length.

| | |
|---|---|
| a, b, e, c, b | Yes [ simple circuit length $\square$ ] No |
| a, d, a, d, a | Yes [ simple circuit length $\square$ ] No |
| a, d, e, b, a | Yes [ simple circuit length $\square$ ] No |
| a, b, e, c, b, a | Yes [ simple circuit length $\square$ ] No |

(b) For the simple graph $G_2$



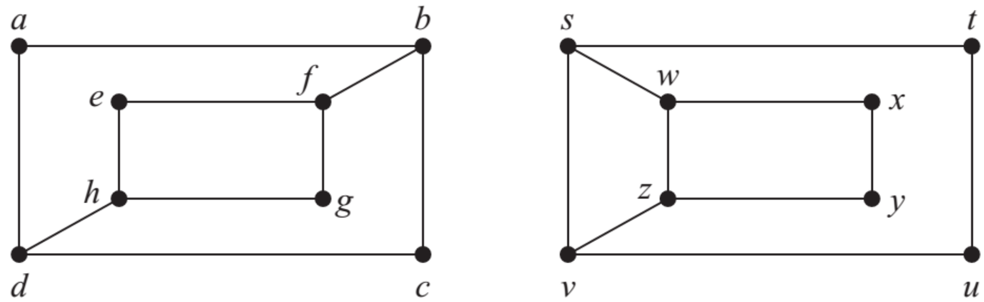

Find $M^2$, where M is the adjacency matrix of $G_2$

$$M^2 = \begin{Bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{Bmatrix}$$

Find the number of paths from A to D in $G_2$ of length 2. $\square$

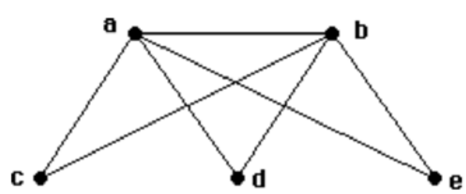
**2.** Let $f(n) = 1 + 2 + 3 + \ldots + n$. Show that $f(n)$ is $O(n^2)$. Be sure to specify the values of the witnesses $C$ and $k$.

3. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

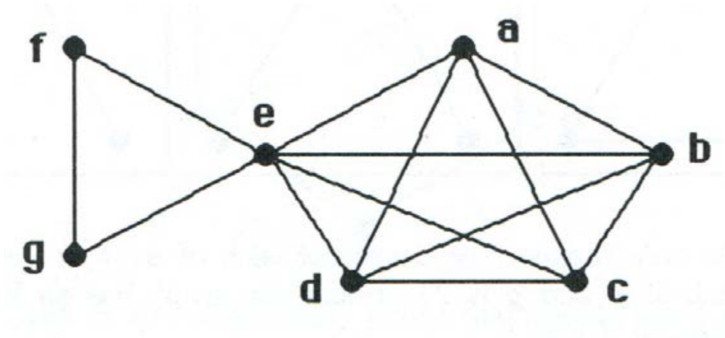



*G* *H*

4. Is the following graph planar? If so draw it without any edges crossing. If it is not, prove that it is not planar.

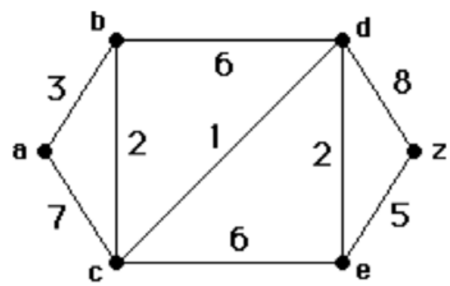


**5**.
(a) Is there an Euler circuit in the following graph? If so, find such a circuit. If not, explain why no such circuit exists.
(b) Is there a Hamilton circuit in the following graph? If so, find such a circuit. If not, prove why no such circuit exists.



**6.** Let $f(n) = 2n^2 + 5n\log(n) + 8n + 7$. Show that $f(n)$ is $O(n^2)$. Be sure to specify the values of the witnesses C and k.

**7**. Use Dijkstra's algorithm to find the length of the shortest path between the vertices *a* and *z* in the following weighted graph.




| *a* | *b* | *c* | *d* | *e* | *z* | *S* |
|---|---|---|---|---|---|---|
| 0 | ∞ | ∞ | ∞ | ∞ | ∞ | *a* |
| X | | | | | | |
| X | | | | | | |
| X | | | | | | |
| X | | | | | | |
| X | | | | | | |
| X | | | | | | |
| X | | | | | | |
| X | | | | | | |

**8**. How many vertices and how many edges does each of the following graphs have? What is the chromatic number of each of the graphs? Explain your answers.

(a) $K_5$

(b) $C_4$

(c) $W_5$

(d) $K_{2,5}$

**9.** Describe an algorithm for finding the second largest integer in a sequence of distinct integers. Give a big-O estimate of the number of comparisons used by your algorithm.

**10**. Give a recursive algorithm for finding the string $w^i$ , the concatenation of $i$ copies of $w$, when $w$ is a bit string.

**11**. Give a recursive definition of
a) the set of odd positive integers.
b) the set of positive integer powers of 3.
c) the set of polynomials with integer coefficients.

**12**. Let $S$ be the subset of the set of ordered pairs of integers defined recursively by
*Basis step:* $(0, 0) \in S$.
*Recursive step:* If $(a, b) \in S$, then $(a + 2, b + 3) \in S$ and $(a + 3, b + 2) \in S$.
**a)** List the elements of $S$ produced by the first two applications of the recursive definition.
**b)** Use structural induction to show that $5 \mid a + b$ when $(a, b) \in S$.