CIS 2166

Name:

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TUN (last 4 digits):

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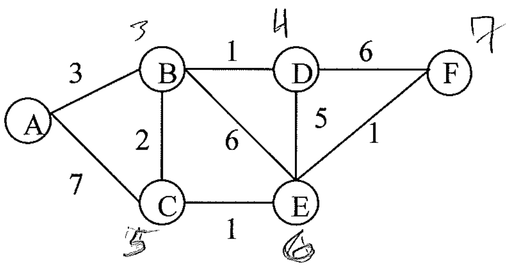
Midterm Exam, Spring 2016

This exam consists of 9 questions. You must do questions 1 and 2.
You need to answer 5 out of the remaining 7 questions, i.e., delete two questions from 3 —9.
Hence you should answer the total of 7 questions.
Good luck.

Question	Points	Out of
1		10
2		10
Subtotal		
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total		70

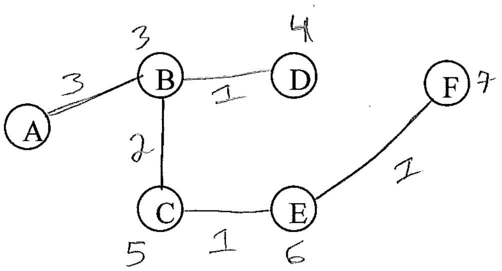
1. For the given graph below complete the table with Dijkstra's algorithm for finding the shortest path from A to all other vertices. Extend the table if needed.



Finish the table

A	B	C	D	E	F	S
0	∞	∞	∞	∞	∞	A
X	3(A)	7(A)	×	×	×	B
X	×	5(B)	4(B)	9(B)	×	D
X	×	5(B)	×	9(D)	10(D)	C
X	×	×	×	6(C)	×	E
X	×	×	×	×	7(E)	F
X						
X						
X						

Draw a tree representing the shortest distances from A to each of the other vertices. Indicate the distance next to each vertex.

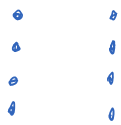


2. Determine whether each of following is possible for a simple graph G = (V, E) with |V| = 8, |E| = 13, (G can be different for each question). Justify your answers.

(a) has a component isomorphic to K_5 possible impossible

K_5 has 5 vertices & 10 edges

(b) is a bipartite possible impossible



(c) has a simple path of length 5 possible impossible

(d) has a simple circuit of length 14 possible impossible

because, we can have only 13 edges

3. Provide a pseudo code of an algorithm that takes a list of n integers ($n > 1$) and finds the average of the largest and smallest integers in the list. What is its worst case time complexity in the terms of the number of comparisons? Justify your answer.

procedure minmax $(a_1, a_2, \ldots, a_n)$

min $= a_1$; max $= a_1$;

for $i = 2$ to n

if min $< a_i$ then min $= a_i$;

if max $> a_i$ then max $= a_i$;

endfor

average $= \dfrac{\text{min} + \text{max}}{2}$;

return average

$O(n)$

4. Prove that the function $f(x) = (x + 2)\log(x^2 + 1)$ is $O(x \log x)$.

If $f_1(x)$ is $O(g_1(x))$ & $f_2(x)$ is $O(g_2(x))$,

then $f_1(x) * f_2(x)$ is $O(g_1(x) * g_2(x))$

We show that $x+2$ is $O(x)$

$x+2 \leq 2x$ for $x \geq k=2$, $C=2, k=2$

Then $x+2$ is $O(x)$

We show that $\log(x^2+1)$ is $O(\log x)$

$x^2+1 \leq 2x^2$ for $x \geq 2$

$\log(x^2+1) \leq \log(2x^2) = \log 2 + \log(x^2) =$

$= 1 + 2\log x \leq \log x + 2\log x = 3\log x$ for $x \geq 2$

we showed $\log(x^2+1) \leq 3\log x$ for $x \geq k=2$

$k=2, C=3$

Hence $\log(x^2+1)$ is $O(\log x)$

5. Provide a pseudo code for the algorithm to compute M^2 (the regular matrix power of 2) for a matrix M of size n x n. Determine the worst case time complexity of this algorithm in the terms of the number of additions and multiplications.

```
procedure power2 (matrix M)
  for i = 1 to n
     for j = 1 to n
        C(i,j) = 0;
        for k = 1 to n
           C(i,j) = C(i,j) + M(i,k) * M(k,j);
        endfor
     endfor
  endf
  return C
```

$O(n^3)$ since we have 3 nested for loops

6. Give a recursive algorithm for finding the string w^i , the concatenation of i copies of w, when w is a bit string.

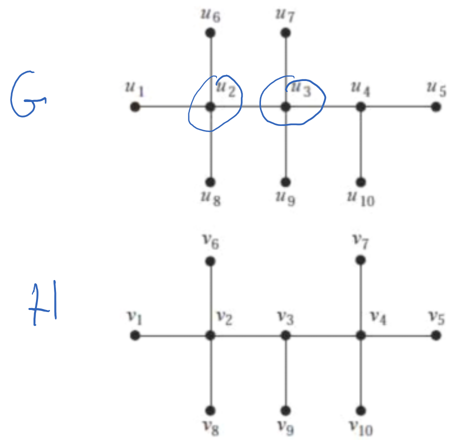
procedure power (w, i)

if $i = 0$, then return λ;

if $i > 0$ then

return $w \circ power(w, i-1)$;

7. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.



They are not isomorphic, because u_2 & u_3 in G have both deg 4 and are adjacent,
There is no such pair of nodes in H.

8. Consider the graphs K_5 , $K_{2,3}$, and W_5 .
(a) Which of these graphs has an Euler circuit? Justify your answers.
(b) Which has an Euler path? Justify your answers.

a) K_5 yes, since every vertex has an even deg.

$K_{2,3}$ & W_5 no, since they have some vertices of odd deg.

b) K_5 yes

$K_{2,3}$ yes, because it has exactly two nodes of odd deg.

W_5 no, because more than 2 vertices of odd deg

9. (a) Show that $2^n < n!$ whenever n is an integer with $n \geq 4$.
(b) Give a recursive definition of the set of positive integer powers of 5.

a) Basis Step: for $k=4$ $\quad 2^4 = 16 < 4! = 24$

Inductive step:

Inductive hypothesis $\quad 2^k < k!$

we need to show that $2^{k+1} < (k+1)!$

$$2^{k+1} = 2 \cdot 2^k \overset{IH}{<} \underset{\sim}{2} \cdot k! < \underline{(k+1)}\, k! = (k+1)!$$

b) Basis Step: $5 \in S$

Recursive Step: If $n \in S$, then $5n \in S$