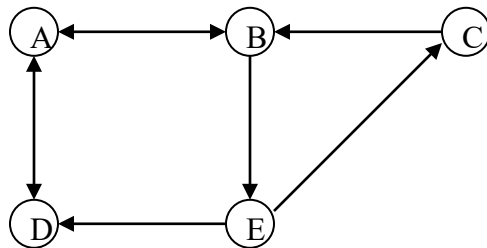


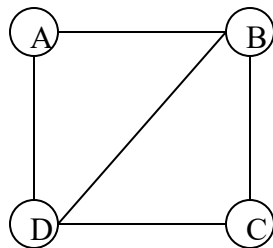
1. (a) Let a directed graph G_1 be given.



Does each of the following list of vertices form a path in G_1 ? If yes, determine (by circling) if the path is simple, if it is a circuit, and give its length.

- | | |
|------------------|---|
| a, b, e, c, b | Yes [simple circuit length <input type="text"/>] No |
| a, d, a, d, a | Yes [simple circuit length <input type="text"/>] No |
| a, d, e, b, a | Yes [simple circuit length <input type="text"/>] No |
| a, b, e, c, b, a | Yes [simple circuit length <input type="text"/>] No |

(b) For the simple graph G_2



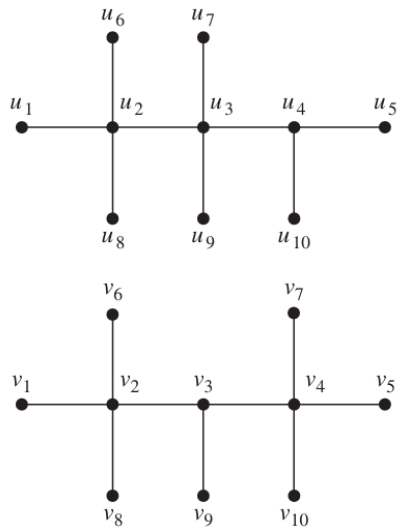
Find M^2 , where M is the adjacency matrix of G_2

$$M^2 = \left\{ \begin{array}{cccc} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{array} \right\}$$

Find the number of paths from A to D in G_2 of length 2.

2. Given are n points on the plane. Provide a pseudo-code of an algorithm for finding the radius of the smallest circle centered at one of the points that contains all the points.

3. Determine whether the given pair of graphs G and H is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

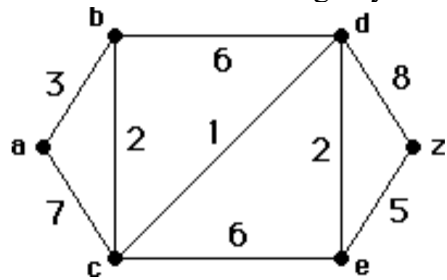


4. Let $a_1 = 2$, $a_2 = 9$, and $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 3$. Show using induction that $a_n \leq 3^n$ for all positive integers n .

5. Use mathematical induction to show that $\sum_{j=0}^n (j+1) = \frac{(n+1)(n+2)}{2}$ whenever n is a nonnegative integer.

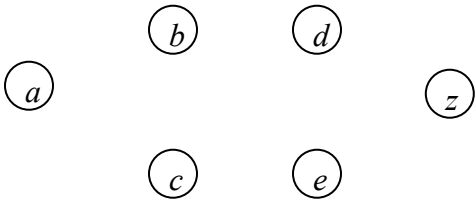
6. Let $f(n) = 2n \log(n^2 + 5) + 3n + 1$. What is big-O estimate of $f(n)$? Be sure to specify the values of the witnesses C and k .

7. Use Dijkstra’s algorithm to find the length of the shortest path between the vertices a and z in the following weighted graph. Use the table below to log in your computation.



a	b	c	d	e	z	S
0	∞	∞	∞	∞	∞	a
X						
X						
X						
X						
X						
X						
X						
X						

Draw a tree representing the shortest distances from a to each of the other vertices. Indicate the distance next to each vertex.



8. How many vertices and how many edges does each of the following graphs have?

(a) K_5

(b) C_4

(c) W_5

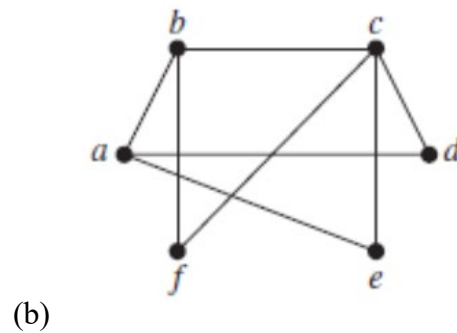
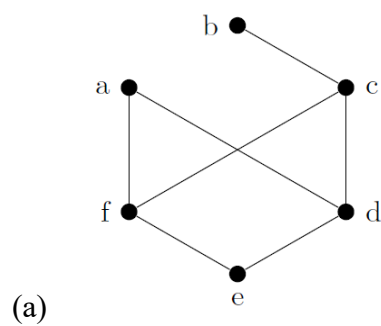
(d) $K_{2,5}$

9. Write a pseudocode for an algorithm for evaluating a polynomial of degree n ,

$p(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$, at $x = c$.

What is big-O estimate of the time complexity of your algorithm (in terms of the number of multiplications and additions used) as a function of n ? Explain your answer.

10. Are the following graphs bipartite? Justify your answers.



11. What is the effect in the time required to solve a problem when you double the size of the input from n to $2n$? Express your answer in the simplest form possible, either as a ratio or a difference. Explain the meaning of your answer.

- a) $\log n$
- b) $100n$
- c) n^2

12. Give a recursive algorithm for determining whether a finite list of distinct integers is sorted in increasing order. The recursion should make use of the fact that the list is sorted if and only if the first $n - 1$ integers are sorted and the $(n-1)$ th integer $<$ nth integer.