

Matrix A and vector b are given below.

1. Solve the equation $Ax = 0$, where 0 is a zero vector.

2. Solve the equation $Ax = b$.

In each of the above, be sure to write your answer in vector format. Show your work.

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 5 & 5 & -3 \end{bmatrix}, \vec{b} = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$Ax = 0$

$$\begin{bmatrix} 2 & 2 & 2 & 0 \\ 5 & 5 & -3 & 0 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1 \rightarrow R_1} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 5 & 5 & -3 & 0 \end{bmatrix} \xrightarrow{-5R_1 + R_2 \rightarrow R_2} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & -8 & 0 \end{bmatrix} \xrightarrow{-\frac{1}{8}R_2 \rightarrow R_2} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{-R_2 + R_1 \rightarrow R_1} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$\Rightarrow$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow \begin{cases} x_1 + x_2 = 0 \\ x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = -x_2 \\ x_3 = 0 \end{cases}$$

x_2 is free

$$\vec{x} = \begin{bmatrix} -x_2 \\ x_2 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$Ax = b$

$$\begin{bmatrix} 2 & 2 & 2 & 3 \\ 5 & 5 & -3 & -3 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1 \rightarrow R_1} \begin{bmatrix} 1 & 1 & 1 & \frac{3}{2} \\ 5 & 5 & -3 & -3 \end{bmatrix} \xrightarrow{-5R_1 + R_2 \rightarrow R_2} \begin{bmatrix} 1 & 1 & 1 & \frac{3}{2} \\ 0 & 0 & -8 & -\frac{21}{2} \end{bmatrix} \xrightarrow{-\frac{1}{8}R_2 \rightarrow R_2} \begin{bmatrix} 1 & 1 & 1 & \frac{3}{2} \\ 0 & 0 & 1 & \frac{21}{16} \end{bmatrix} \xrightarrow{-R_2 + R_1 \rightarrow R_1} \begin{bmatrix} 1 & 1 & 0 & \frac{3}{16} \\ 0 & 0 & 1 & \frac{21}{16} \end{bmatrix}$$

$\Rightarrow$

$$\begin{bmatrix} 1 & 1 & 0 & \frac{3}{16} \\ 0 & 0 & 1 & \frac{21}{16} \end{bmatrix} \Rightarrow \begin{cases} x_1 + x_2 = \frac{3}{16} \\ x_3 = \frac{21}{16} \end{cases} \Rightarrow \begin{cases} x_1 = -x_2 + \frac{3}{16} \\ x_3 = \frac{21}{16} \end{cases}$$

x_2 is free

$$\vec{x} = \begin{bmatrix} -x_2 + \frac{3}{16} \\ x_2 \\ \frac{21}{16} \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{3}{16} \\ 0 \\ \frac{21}{16} \end{bmatrix}$$