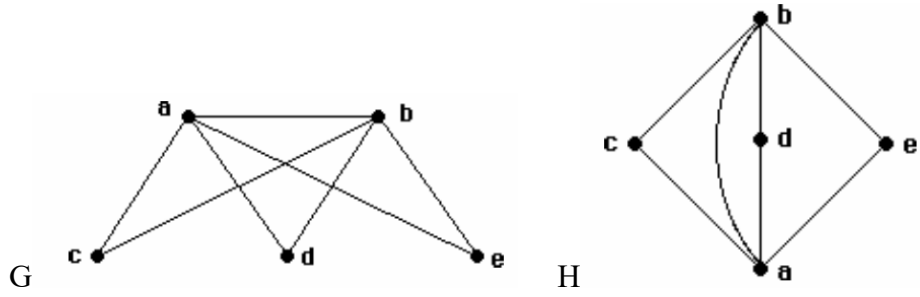


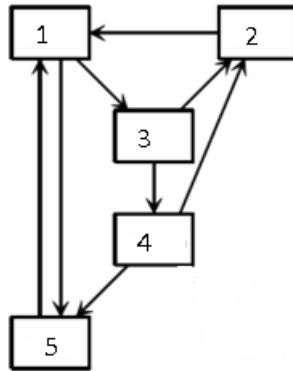
1. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.



2. Use induction to prove that every amount of postage of n cents greater than 5 cents can be formed from 3-cent and 4-cent stamps.

3. (a) For which $m, n > 1$ does complete bipartite graph $K_{m,n}$ have an Euler circuit? Justify your answer.
- (b) For which $m, n > 1$ does complete bipartite graph $K_{m,n}$ have a Hamilton circuit? Justify your answer.

4. For the web graph shown below write the link matrix A that expresses the system of PageRank linear equations in the form $A\mathbf{x} = \mathbf{x}$, where $\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T$. Is the matrix $\mathbf{M} = (1 - m)\mathbf{A} + m\mathbf{S}$ for $m=0.25$ column-stochastic? Justify your answer.



5. Use the method of Gaussian elimination to find \mathbf{x} for the system of linear equations $\mathbf{Ax}=\mathbf{b}$, where \mathbf{A} and \mathbf{b} are given below. Show your work.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 2 & 6 & 11 \end{bmatrix}, b = \begin{bmatrix} 10 \\ 4 \\ 6 \end{bmatrix}$$

6. Use the method of Gaussian elimination to find the determinant of matrix **B** given below. Show your work.

$$\begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 3 \\ 2 & -2 & 0 \end{bmatrix}$$

7. Find the eigenvalues and the eigenvectors of these two matrices. Show your work.

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \quad \text{and} \quad A + I = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$$

8. Find the eigenvalues and the eigenvectors of matrix A. Show your work.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 3 & 3 \end{bmatrix}.$$

9. Find the matrix A that performs those transformations, in order, on the Cartesian plane. To which point is the point $(-4, 1)$ mapped by this transformation?
- (a) horizontal stretch by a factor of 2
 - (b) reflection across the y -axis.

10. Find the standard matrix \mathbf{A} for the given linear transformation T .

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 - 3x_3 \\ 0 \\ x_1 + 4x_3 \\ 5x_2 + x_3 \end{bmatrix}$$

11. Provide a pseudo-code of an algorithm for finding the second largest number in a sequence of n distinct integers ($n > 1$) distinct integers. What is its worst-case time complexity in terms of the number of comparisons? Justify your answer.

12. Consider a single perceptron with input vector \mathbf{x} and weight vector \mathbf{w} . The activation function is ReLU $g(z) = \max\{0, z\}$. Compute the output value for \mathbf{x} and weight vector \mathbf{w} given below. Show your work.

$$\mathbf{x} = [1, 0, 1], \mathbf{w} = [2, -1.6, -2.5].$$