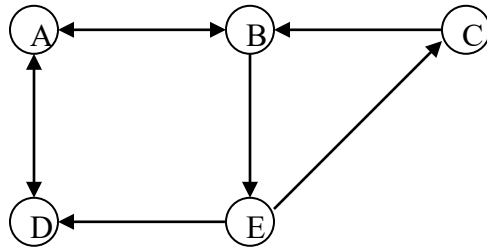


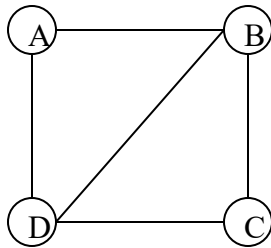
1. (a) Let a directed graph  $G_1$  be given.



Does each of the following list of vertices form a path in  $G_1$ ? If yes, determine (by circling) if the path is simple, if it is a circuit, and give its length.

- |                  |   |
|------------------|---|
| a, b, e, c, b    | Yes [ simple circuit length <input type="text"/> ] No |
| a, d, a, d, a    | Yes [ simple circuit length <input type="text"/> ] No |
| a, d, e, b, a    | Yes [ simple circuit length <input type="text"/> ] No |
| a, b, e, c, b, a | Yes [ simple circuit length <input type="text"/> ] No |

(b) For the simple graph  $G_2$



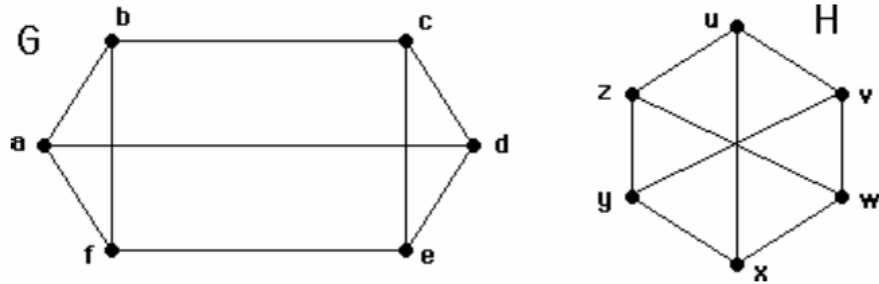
Find  $M^2$ , where  $M$  is the adjacency matrix of  $G_2$

$$M^2 = \left\{ \begin{array}{cccc} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{array} \right\}$$

Find the number of paths from A to D in  $G_2$  of length 2.

2. Provide a pseudo code of an algorithm for finding a closest pair of numbers in a set of  $n$  real distinct numbers and give a worst-case estimate of the number of comparisons.

3. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

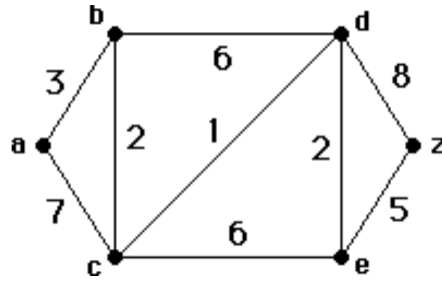


4. Let  $a_1 = 2$ ,  $a_2 = 9$ , and  $a_n = 2a_{n-1} + 3a_{n-2}$  for  $n \geq 3$ . Show using induction that  $a_n \leq 3^n$  for all positive integers  $n$ .

5. Use mathematical induction to show that  $\sum_{j=0}^n (j + 1) = \frac{(n+1)(n+2)}{2}$  whenever  $n$  is a nonnegative integer.

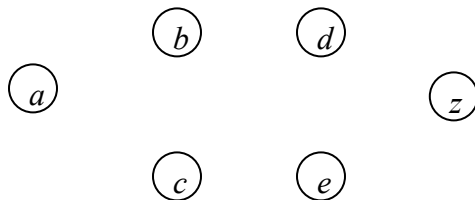
6. Let  $f(n) = 2n \log(n^2 + 5) + 3n + 1$ . What is big-O estimate of  $f(n)$ ? Be sure to specify the values of the witnesses  $C$  and  $k$ .

7. Use Dijkstra's algorithm to find the length of the shortest path between the vertices  $a$  and  $z$  in the following weighted graph. Use the table below to log in your computation.



$a$	$b$	$c$	$d$	$e$	$z$	$S$
0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$a$
X						
X						
X						
X						
X						
X						
X						
X						
X						

Draw a tree representing the shortest distances from  $a$  to each of the other vertices. Indicate the distance next to each vertex.



8. How many vertices and how many edges does each of the following graphs have?

(a)  $K_5$

(b)  $C_4$

(c)  $W_5$

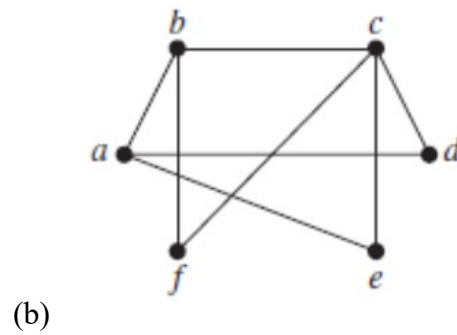
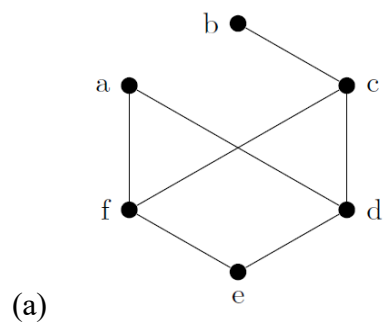
(d)  $K_{2,5}$

9. Write a pseudocode for an algorithm for evaluating a polynomial of degree  $n$ ,

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0, \text{ at } x = c.$$

What is big-O estimate of the time complexity of your algorithm (in terms of the number of multiplications and additions used) as a function of  $n$ ? Explain your answer.

10. Are the following graphs bipartite? Justify your answers.



11. What is the effect in the time required to solve a problem when you double the size of the input from  $n$  to  $2n$ ? Express your answer in the simplest form possible, either as a ratio or a difference. Explain the meaning of your answer.

- a)  $\log n$
- b)  $100n$
- c)  $n^2$

**12.** Give a recursive algorithm for finding the maximum of a finite set of integers, the recursion should make use of the fact that the maximum of  $n$  integers is the larger of the last integer in the list and the maximum of the first  $n - 1$  integers in the list.