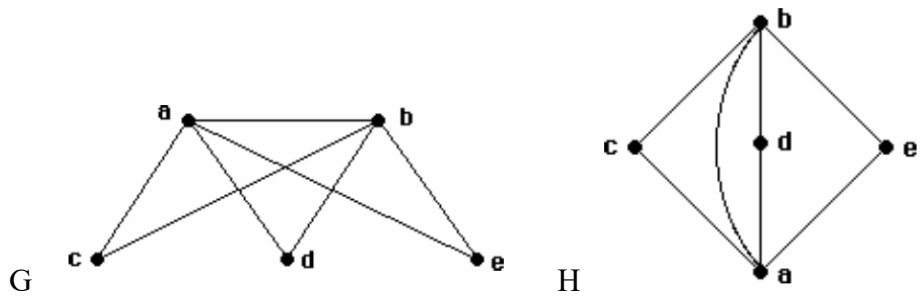


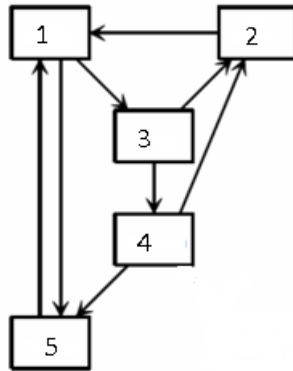
1. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.



2. Prove by induction that  $n^2 - 7n + 12$  is nonnegative whenever  $n$  is an integer with  $n \geq 3$ .

3. (a) How many vertices and how many edges are in  $K_n$  for  $n > 2$ ?
- (b) For which  $n$  does  $K_n$  (for  $n > 2$ ) have a Euler circuit? Justify your answer.
- (c) For which  $n$  does  $K_n$  (for  $n > 2$ ) have a Hamilton circuit? Justify your answer.
- (d) How many vertices and how many edges are in complete bipartite graph  $K_{m,n}$  for  $m, n > 1$ ?
- (e) For which  $n$  does complete bipartite graph  $K_{m,n}$  (for  $m, n > 1$ ) have a Euler circuit? Justify your answer.
- (f) For which  $n$  does complete bipartite graph  $K_{m,n}$  (for  $m, n > 1$ ) have a Hamilton circuit? Justify your answer.

4. For the web graph shown below write the link matrix  $A$  that expresses the system of PageRank linear equations in the form  $Ax = x$ , where  $x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T$ .  
Is the matrix  $M = (1 - m)A + mS$  for  $m=0.25$  column-stochastic? Justify your answer.



5. Use the method of Gaussian elimination to find  $x$  for the system of linear equations  $Ax=b$ , where  $A$  and  $b$  are given below. Show your work.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 2 & 6 & 11 \end{bmatrix}, b = \begin{bmatrix} 10 \\ 4 \\ 6 \end{bmatrix}$$

6. Use method of Gaussian elimination to find the determinant of matrix **B** given below. Show your work.

$$\begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 3 \\ 2 & -2 & 0 \end{bmatrix}$$

7. Find the eigenvalues and the eigenvectors of these two matrices. Show your work.

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \quad \text{and} \quad A + I = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$$

8. Find the eigenvalues and the eigenvectors of matrix A. Show your work.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 3 & 3 \end{bmatrix}.$$



9. Find the matrix  $A$  that performs those transformations, in order, on the Cartesian plane. To which point is the point  $(-2, 1)$  mapped by this transformation.
- (a) horizontal stretch by a factor of 3
  - (b) reflection across the line  $y = x$

10. Find the standard matrix **A** for the given linear transformation **T**.

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 - 3x_3 \\ 0 \\ x_1 + 4x_3 \\ 5x_2 + x_3 \end{bmatrix}$$

**11.** Provide a pseudo code of an algorithm for finding the second largest number in a sequence of  $n$  distinct integers ( $n > 1$ ) distinct integers. What is its worst-case time complexity in the terms of the number of comparisons? Justify your answer.

**12.** Let  $f(n) = 2n\log(n^2+3) + 7n + 5$ . What is big-O estimate of  $f(n)$ ? Be sure to specify the values of the witnesses  $C$  and  $k$ .