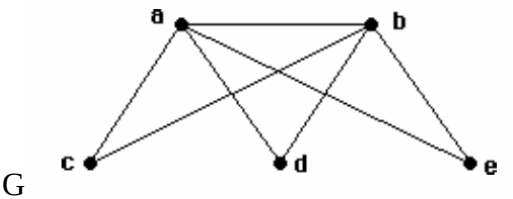
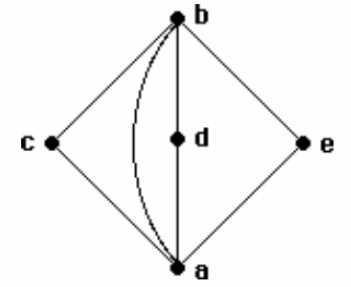
1. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

G 

H 

2. Let $a_1 = 2$, $a_2 = 9$, and $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 3$. Show that $a_n \leq 3^n$ for all positive integers n.

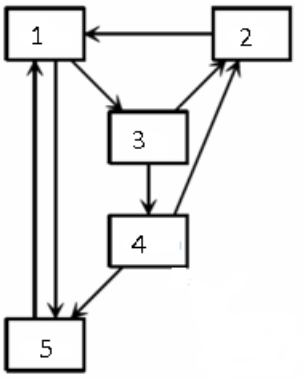
3. (a) How many vertices and how many edges are in this graph?
(b) Is this graph planar? Justify your answer.
(c) Does this graph have an Euler circuit? Justify your answer.
(d) What is the chromatic number of this graph?

G1: K_5

G2: C_4

G3 $K_{5,5}$

4. For the web graph shown below write the link matrix A that expresses the system of PageRank linear equations in the form Ax = x, where $x = [x_1\ x_2\ x_3\ x_4\ x_5]^T$.
Is the matrix M = (1 − m)A + mS for m=0.25 column-stochastic? Justify your answer.



5.
(a) (5pts) Construct a graph for the finite state automation $M=(S,\Sigma,\delta,s0,F)$,
where $S=\{s0, s1, s2, s3\}$, $\Sigma=\{0,1\}$, $F=\{s2\}$ and the transition function δ is given by the table.
(b) (5pts) Describe the language recognized by this finite state automation. You can describe it as a regular expression, a set, or in natural language.

state	Input: 0	Input: 1
s0	s0	s1
s1	s1	s2
s2	s2	s3
s3	s3	s3

6. What is the language generated by the grammar with productions $S \rightarrow SA$, $S \rightarrow 0$, $A \rightarrow 1A$, and $A \rightarrow 1$, where S is the start symbol?

7. Find a grammar for the set $\{ 0^{2n}1^{n} \mid n \geq 0 \}$. Is your grammar regular, context free or context sensitive?

8. Construct a finite-state machine with output that produces a 1 if and only if the last three input bits read are all 0s, otherwise it should procure 0s.

9. Construct a deterministic finite-state automaton (with no output) that recognizes the set of all bit strings that end with 10.

10. Use the method of Gaussian elimination to find x for the system of linear equations Ax=b, where A and b are given below. Show your work.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 2 & 6 & 11 \end{bmatrix}, b = \begin{bmatrix} 10 \\ 4 \\ 6 \end{bmatrix}$$

11. Use method of Gaussian elimination to find the determinant of matrix B given below. What is the rank of B? Show your work.

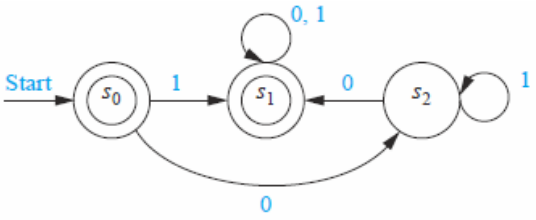
$$B = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 3 \\ -6 & 2 & 1 \end{bmatrix}$$

12. Find the eigenvalues and the eigenvectors of these two matrices. Show your work.

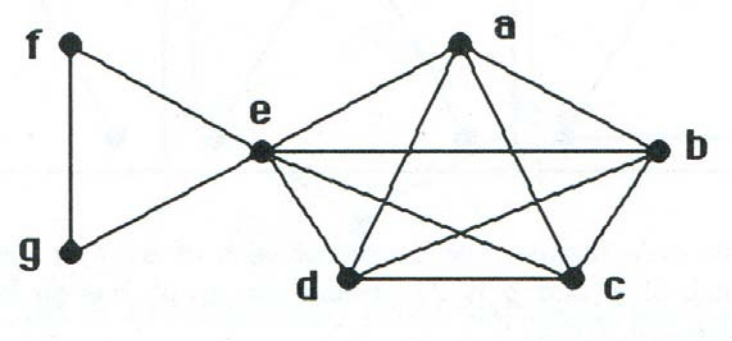
$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \quad \text{and} \quad A + I = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$$

13. Construct a Turing machine that recognizes the set of all bit strings that contain at least two 1s.

14. Find the language recognized by the given deterministic finite-state automaton.



15.
(a) Is there an Euler circuit in the following graph? If so, find such a circuit. If not, explain why no such circuit exists.
(b) Is there a Hamilton circuit in the following graph? If so, find such a circuit. If not, prove why no such circuit exists.



16. Write a pseudocode for an algorithm for evaluating a polynomial of degree n,
$p(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, at $x = c$.
What is big-O estimate of the time complexity of your algorithm (in terms of the number of multiplications and additions used) as a function of n? Explain your answer.