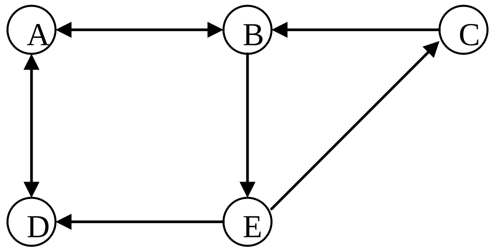
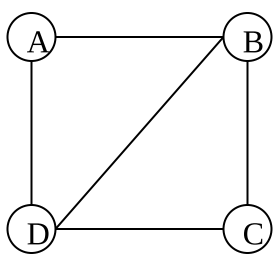
1. (a) Let a directed graph G_1 be given.



Does each of the following list of vertices form a path in G_1? If yes, determine (by circling) if the path is simple, if it is a circuit, and give its length.

a, b, e, c, b	Yes [simple circuit length ☐] No
a, d, a, d, a	Yes [simple circuit length ☐] No
a, d, e, b, a	Yes [simple circuit length ☐] No
a, b, e, c, b, a	Yes [simple circuit length ☐] No

(b) For the simple graph G_2



Find M^2, where M is the adjacency matrix of G_2

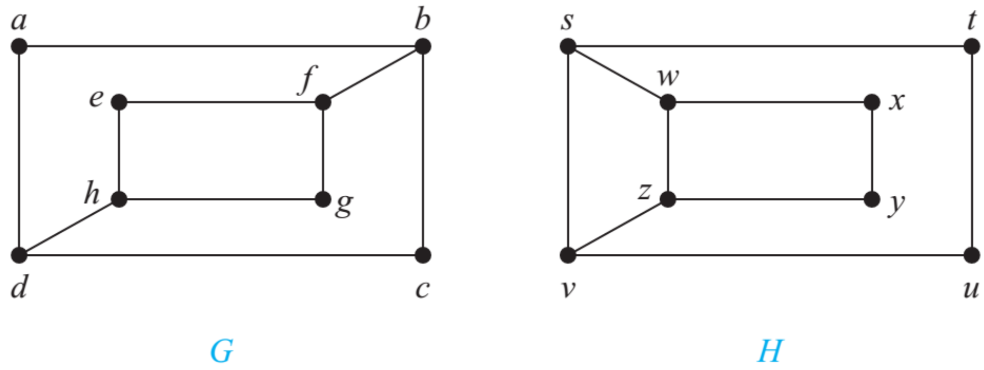
$M^2 =$

☐	☐	☐	☐
☐	☐	☐	☐
☐	☐	☐	☐
☐	☐	☐	☐

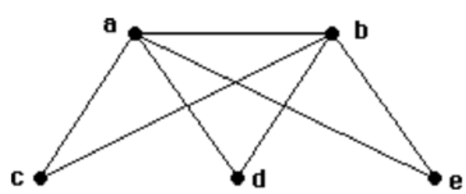
Find the number of paths from A to D in G_2 of length 2. ☐

2. Let $f(n) = 1 + 2 + 3 + \ldots + n$. Show that $f(n)$ is $O(n^2)$. Be sure to specify the values of the witnesses C and k.

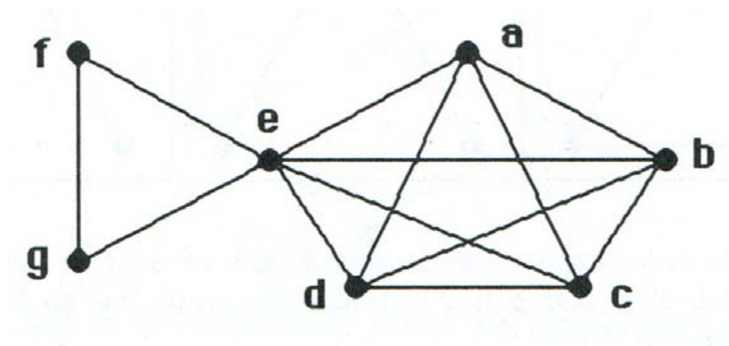
3. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.



4. Is the following graph planar? If so draw it without any edges crossing. If it is not, prove that it is not planar.

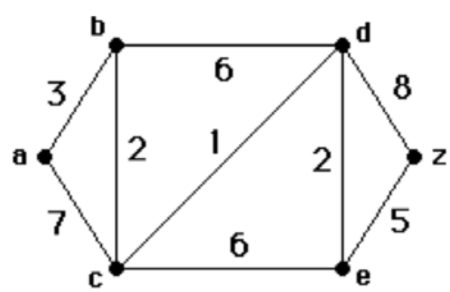


5.
(a) Is there an Euler circuit in the following graph? If so, find such a circuit. If not, explain why no such circuit exists.
(b) Is there a Hamilton circuit in the following graph? If so, find such a circuit. If not, prove why no such circuit exists.



6. Let $f(n) = 3n^2 + 8n + 7$. Show that $f(n)$ is $O(n^2)$. Be sure to specify the values of the witnesses C and k.

7. Use Dijkstra's algorithm to find the length of the shortest path between the vertices *a* and *z* in the following weighted graph.



a	*b*	*c*	*d*	*e*	*z*	*S*
0	∞	∞	∞	∞	∞	*a*
X						
X						
X						
X						
X						
X						
X						
X						

8. How many vertices and how many edges does each of the following graphs have? What is the chromatic number of each of the graphs? Explain your answers.

(a) K_5

(b) C_4

(c) W_5

(d) $K_{2,5}$

9. Describe an algorithm for finding the second largest integer in a sequence of distinct integers. Give a big-O estimate of the number of comparisons used by your algorithm.

10. Give a recursive algorithm for finding the string w^i , the concatenation of *i* copies of *w*, when *w* is a bit string.

11. Give a recursive definition of
a) the set of odd positive integers.
b) the set of positive integer powers of 3.
c) the set of polynomials with integer coefficients.

12. Let S be the subset of the set of ordered pairs of integers defined recursively by
Basis step: $(0, 0) \in S$.
Recursive step: If $(a, b) \in S$, then $(a + 2, b + 3) \in S$ and $(a + 3, b + 2) \in S$.
a) List the elements of S produced by the first two applications of the recursive definition.
b) Use structural induction to show that $5 \mid a + b$ when $(a, b) \in S$.