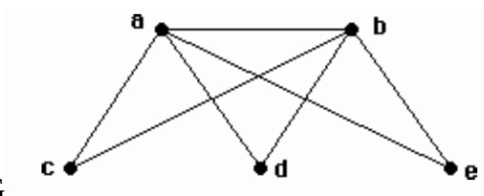
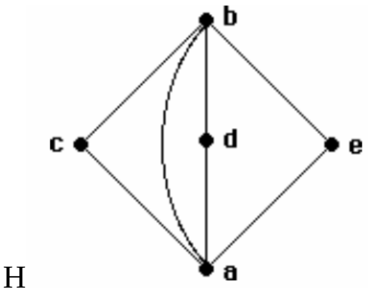
1. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

G 


H 

2. Let $a_1 = 2$, $a_2 = 9$, and $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 3$. Show that $a_n \leq 3^n$ for all positive integers n.

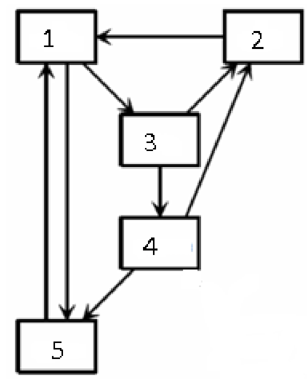
3. (a) How many vertices and how many edges are in this graph?
(b) Is this graph planar? Justify your answer.
(c) Does this graph have an Euler circuit? Justify your answer.
(d) What is the chromatic number of this graph?

G1: $K_5$

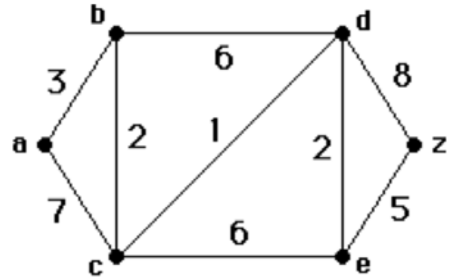
G2: $C_4$

G3 $K_{5,5}$

4. For the web graph shown below write the link matrix A that expresses the system of PageRank linear equations in the form Ax = x, where $x = [x_1\ x_2\ x_3\ x_4\ x_5]^T$.
Write the matrix M = (1 − m)A + mS for m=0.25.
Is the matrix M column-stochastic? Justify your answer.



**5**. Use Dijkstra's algorithm to find the length of the shortest path between the vertices *a* and *z* in the following weighted graph.




| *a* | *b* | *c* | *d* | *e* | *z* | *S* |
|---|---|---|---|---|---|---|
| 0 | ∞ | ∞ | ∞ | ∞ | ∞ | *a* |
| X | | | | | | |
| X | | | | | | |
| X | | | | | | |
| X | | | | | | |
| X | | | | | | |
| X | | | | | | |
| X | | | | | | |
| X | | | | | | |

6. What is the language generated by the grammar with productions
$S \rightarrow SA$, $S \rightarrow 0$, $A \rightarrow 1A$, and $A \rightarrow 1$, where S is the start symbol?

7. Find a grammar for the set $\{ 0^{2n}1^{n} \mid n \geq 0 \}$.

8. Construct a finite-state machine with output that produces a 1 if and only if the last three input bits read are all 0s.

9. Construct a deterministic finite-state automaton (with no output) that recognizes the set of all bit strings that end with 10.

10. Use method of Gaussian elimination to find x for the system of linear equations Ax=b, where A and b are given below. Show your work.

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 3 \\ -6 & -2 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}.$$

11. Use method of Gaussian elimination to find the determinant of matrix B given below. What is the rank of B? Show your work.

$$B = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 3 \\ -6 & 2 & 1 \end{bmatrix}$$

12. Find the determinant, eigenvalues and eigenvectors of the matrix below. Check if the determinant equals the product of its eigenvalues and if its trace equals the sum of its eigenvalues. Show your work.

$$C = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$