1. (a) Let a directed graph  $G_1$  be given.



Does each of the following list of vertices form a path in  $G_1$ ? If yes, determine (by circling) if the path is simple, if it is a circuit, and give its length.

a, b, e, c, b	Yes [ simple circuit	length	]	No
a, d, a, d, a	Yes [ simple circuit	length	]	No
a, d, e, b, a	Yes [ simple circuit	length	]	No
a, b, e, c, b, a	Yes [ simple circuit	length	]	No

(b) For the simple graph  $G_2$ 



Find  $M^2$ , where M is the adjacency matrix of  $G_2$ 



Find the number of paths from A to D in  $G_2$  of length 2.

**2.** List all the comparison steps used to search for 9 in the sequence 1, 3, 4, 5, 6, 8, 9, 11 using a) a linear search. b) a binary search.

3. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.



4.

(a) Is there an Euler circuit in the following graph? If so, find such a circuit. If not, explain why no such circuit exists.

(b) Is there a Hamilton circuit in the following graph? If so, find such a circuit. If not, prove why no such circuit exists.



**5.** Let  $f(n) = 3n^2 + 8n + 7$ . Show that f(n) is  $O(n^2)$ . Be sure to specify the values of the witnesses C and k.

6. How many vertices and how many edges does each of the following graphs have? (a)  $\ensuremath{K_5}$ 

(b) C<sub>4</sub>

(c) W<sub>5</sub>

(d) K<sub>2,5</sub>

**7.** Describe an algorithm for finding the second largest integer in a sequence of distinct integers. Give a big-O estimate of the number of comparisons used by your algorithm.