CIS 2166 Fall 2013 Homework 2 on Matrix Algebra

1. Find a condition on b1, b2, b3 so that these systems are solvable

(a) $$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

(b) $$\begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

2.
Consider the space F spanned by the 4 vectors $v_1 = (4,2,4,2)$, $v_2 = (-1,4,5,10)$, $v_3 = (-5,2,1,8)$ and $v_4 = (6,6,10,10)$.

(a) Are the v_i's linearly independent?
(b) Give a basis of F.
(c) What is the dimension of F?
(d) Are $v_1 + 2v_2 + 3v_3$, $v_1 - v_2$ and v_4 linearly independent?

3.

Write the product $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} \pi \\ e \end{bmatrix}$ in two ways:
(a) as dot products of the rows with the column vector
(b) as a linear combination of the columns.

4.

(a) What matrix A takes $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ to $A\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ to $A\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$?

(b) What is $A\begin{bmatrix} 1 \\ 2 \end{bmatrix}$?