Homework 10/21/2013

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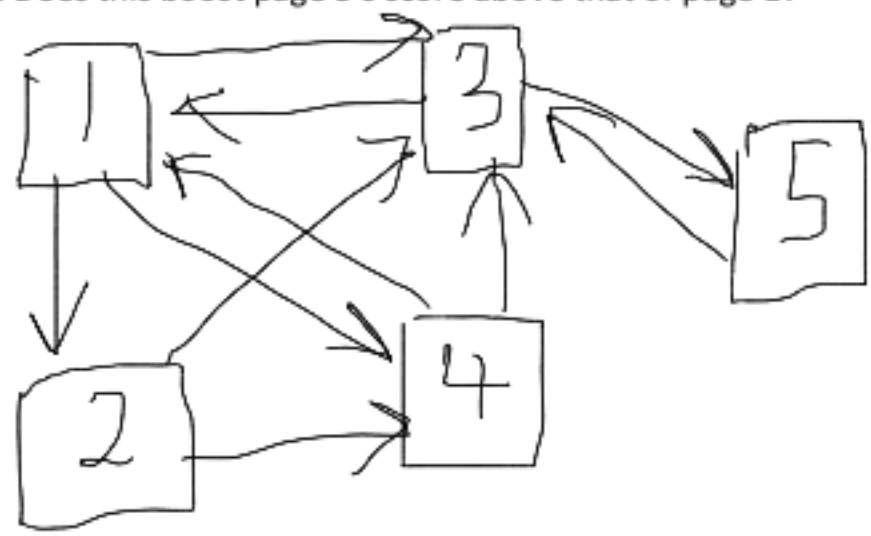
Exercise 1. Suppose the people who own page 3 in the web of Figure 1 are infuriated by the fact that its importance score, computed using formula (2.1), is lower than the score of page 1. In an attempt to boost page 3's score, they create a page 5 that links to page 3; page 3 also links to page 5. Does this boost page 3's score above that of page 1?

$$x1 = \frac{x3}{2} + \frac{x4}{2}$$
$$x2 = \frac{x1}{3}$$
$$x3 = \frac{x1}{3} + \frac{x2}{2} + \frac{x4}{2} + \frac{x5}{1}$$
$$x4 = \frac{x1}{3} + \frac{x2}{2}$$
$$x5 = \frac{x3}{2}$$



$$\begin{bmatrix} 0 & 0 & 1/2 & 1/2 & 0 \\ 1/3 & 0 & 0 & 0 & 0 \\ 1/3 & 1/2 & 0 & 1/2 & 1 \\ 1/3 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 0 \end{bmatrix}$$

Eigenvector with eigenvalue 1:

[0.489 0. 163 0.734 0.244 0.367]

x1=0.489/1.997=24%
x2=0.163/1.997=8%
x3=0.734/1.997=36%
x4=0.244/1.997=12%
x5=0.367/1.997=18%

online eigenvector calculator:
http://www.math.ubc.ca/~israel/applet/mcalc/matcalc.html

Page 3 now has 36% rating, and Page 1 became 24%.
Before page 3 had 29%, and page 1 was 38%.

So, yes, page 3 was able to boost its score above page 1.

Exercise 7: Prove that if A is an n × n column-stochastic matrix and $0 \le m \le 1$, then $M = (1 - m)A + mS$ is also a column-stochastic matrix.

All entries of S is 1/n, $n \neq 0$.

All entries of each column of A will add up to 1.

S is n x n matrix with all elements 1/n. So, all entries of each column of S will also add up to 1.

The following equation must equal 1 when $0 \ge m \ge 1$:
$(1 - m) * 1 + m * 1 = 1$

So, M must consist of columns, each adding up to 1, and hence M must be stochastic.