Semi-supervised Learning

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CS5350/6350: Machine Learning

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- Supervised Learning models require labeled data
- Learning a reliable model usually requires plenty of labeled data
- Labeled Data: Expensive and Scarce
- Unlabeled Data: Abundant and Free/Cheap
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- **Semi-supervised Learning:** Devising ways of utilizing unlabeled data with labeled data to learn better models

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- **This class:** Semi-supervised Learning (SSL) will refer to Semi-supervised Classification/Regression

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- Transductive Learning: The set $\mathcal{U}$ is the test data and is available at the training time

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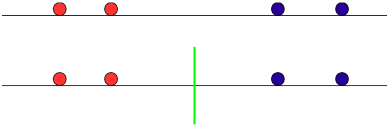

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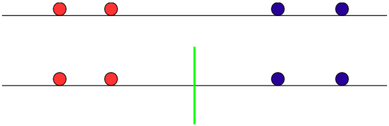

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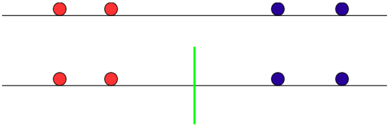

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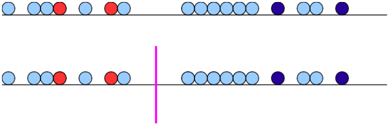

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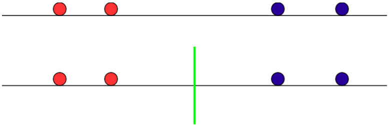

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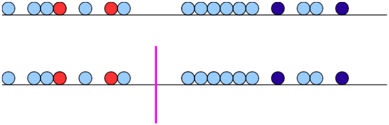

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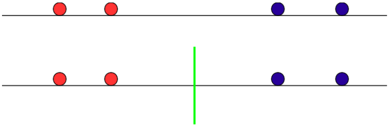

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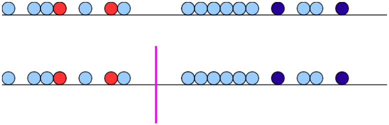

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- Unlabeled data can give a better sense of the class separation boundary

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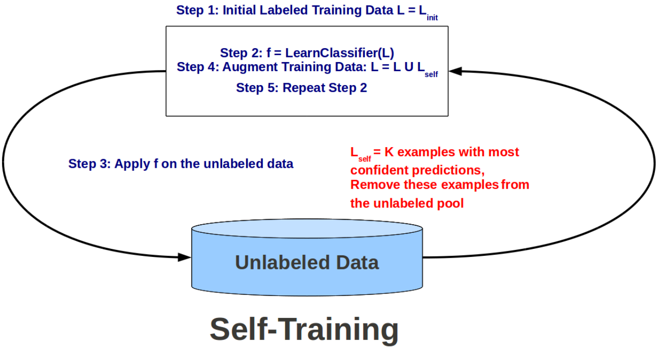

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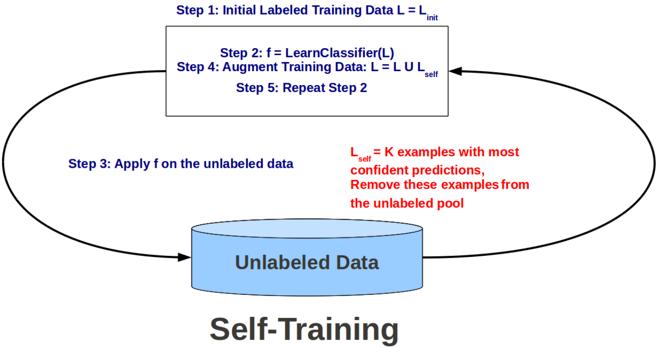

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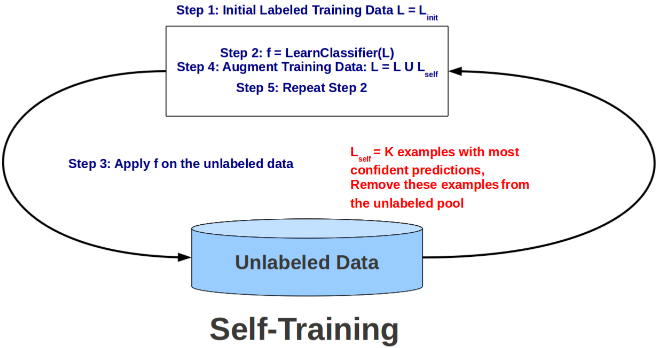

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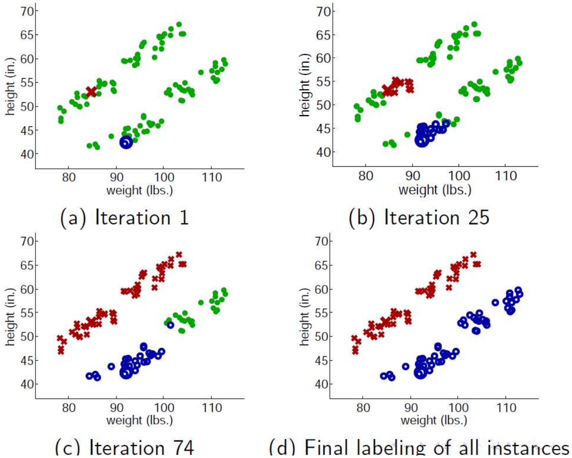
- **Caution:** Prediction mistake can reinforce itself

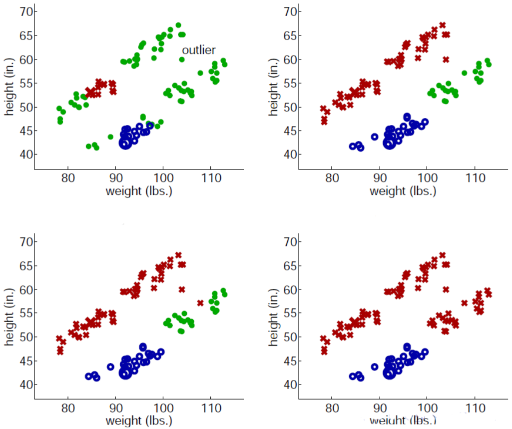
Self-Training: A Good Case

- Base learner: KNN classifier



Self-Training: A Bad Case

- Base learner: KNN classifier
- Things can go wrong if there are **outliers**. Mistakes get reinforced



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- Technical Condition: Views should be conditionally independent
  - Intuitively, we don't want redundancy between the views

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- Finally, use a voting or averaging to make predictions on the test data

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  - E.g., voting, consensus, etc.
- Backed by a number of theoretical results

Cluster-and-Label Approach

Input: $(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_l, y_l)$, $\mathbf{x}_{l+1}, \ldots, \mathbf{x}_{l+u}$,
a clustering algorithm $\mathcal{A}$, a supervised learning algorithm $\mathcal{L}$

1. Cluster $\mathbf{x}_1, \ldots, \mathbf{x}_{l+u}$ using $\mathcal{A}$.
2. For each cluster, let S be the labeled instances in it:
3. Learn a supervised predictor from S: $f_S = \mathcal{L}(S)$.
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- Assumption: Clusters coincide with decision boundaries
  - Poor results if this assumption is wrong

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  - Use this model to "guess" the label of each $\mathbf{x}_j \in \mathcal{U}$ (compute expected label). Assuming binary labels (+1/-1), we can compute:

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- A general scheme; can be used with any probabilistic learning model
  - E.g., naïve Bayes, logistic regression, linear regression etc.
  - $P(y_j|\hat{\theta}, \mathbf{x}_j)$ would have to be defined accordingly

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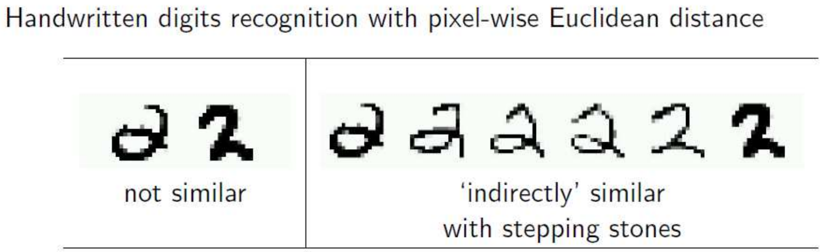
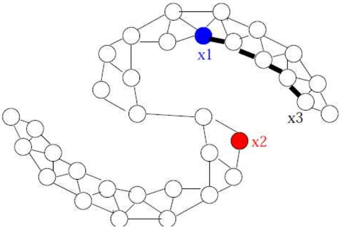


Graph Regularization

- Nodes: $X_l \cup X_u$
- Edges: similarity weights computed from features, e.g.,
  - k-nearest-neighbor graph, unweighted (0, 1 weights)
  - fully connected graph, weight decays with distance $w = \exp\left(-\|x_i - x_j\|^2/\sigma^2\right)$
  - ϵ-radius graph
- Assumption Instances connected by heavy edge tend to have the same label.



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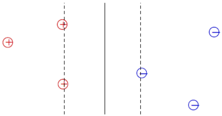
- Several variants and ways to solve the above problem (refer to the SSL survey paper's section on graph based methods)

Transductive SVM: Avoiding Dense Regions

SVMs



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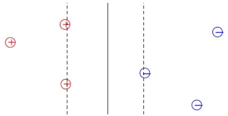
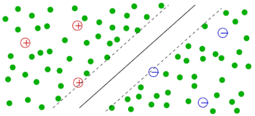


Semi-supervised SVMs (S3VMs) = Transductive SVMs (TSVMs)



- Unlabeled data from different classes are separated by large margin
- Idea: The decision boundary shouldn't lie in the regions of high density
- For details, refer to the SSL survey paper's section on transductive SVMs

Concluding Thoughts

- Unlabeled data can help if the model assumptions are appropriate
- Incorrect assumptions can hurt (so be careful)
- SSL is also motivated by human learning
- There exists several other ways of learning with labeled data scarcity. E.g.,
  - Active Learning (next class)
  - Crowd-sourcing (free-of-cost labels)

Luis von Ahn: Games with a purpose (ReCaptcha)

Word rejected by OCR
(Optical Character Recogintion)

You provide a free label!