

Prove the validity of the duality expressions $(A \bullet B)^c = (A^c \circ \hat{B})$ and $(A \circ B)^c = (A^c \bullet \hat{B})$.

Prove: first, from duality of erosion and dilation we know that:

$$(A \ominus B)^c = (A^c \oplus \hat{B}) \quad (1)$$

$$(A \oplus B)^c = (A^c \ominus \hat{B}) \quad (2)$$

Therefore

$$\begin{aligned} (A \bullet B)^c &= ((A \oplus B) \ominus B)^c && \text{definition of close} \\ &= (A \oplus B)^c \oplus \hat{B} && \text{duality of erosion} \\ &= (A^c \ominus \hat{B}) \oplus \hat{B} && \text{duality of dilation} \\ &= (A^c \circ \hat{B}) && \text{definition of open} \end{aligned} \quad (3)$$

$$\begin{aligned} (A \circ B)^c &= ((A \ominus B) \oplus B)^c && \text{definition of open} \\ &= (A \ominus B)^c \ominus \hat{B} && \text{duality of dilation} \\ &= (A^c \oplus \hat{B}) \ominus \hat{B} && \text{duality of erosion} \\ &= (A^c \bullet \hat{B}) && \text{definition of close} \end{aligned} \quad (4)$$