3.4
\( P(B \mid T) = P(T \mid B) \frac{P(B)}{P(T)}. \)

\[
P(T) = P(T \mid B) \cdot P(B) + P(T \mid B') \cdot P(B')
\]
\[
0.1000061 = 0.7 \times 0.000013 + 0.1 \times (1 - 0.000013)
\]
\[
P(B \mid T) = 0.7 \times 0.000013 / 0.1000061 = 0.000090994
\]
\[
P(B \mid T') = P(T \mid B') \frac{P(B')}{P(T)}
\]
\[
0.000086994 = 0.1 \times (1 - 0.000013) / 0.1000061
\]

3.9
\( P(A) = \frac{3}{4} \quad P(B) = \frac{2}{5} \quad P(A \cup B) = \frac{4}{5} \)
\( P(A \cap B) = P(A) + P(B) - P(A \cup B) \)
\[
\frac{7}{20} = \frac{3}{4} + \frac{2}{5} - \frac{4}{5}
\]
\( P(A \cap B) = P(B \mid A) \cdot P(A) \) and so
\( P(B \mid A) = \frac{P(A) \cdot P(B)}{P(A \cap B)} \)
\[
\frac{3}{14} = \frac{3}{4} / \left(\frac{7}{20}\right)
\]

3.11
a. The probability that the driver’s blood level does not exceed the legal limit given that the driver tested positive.

b. We are given that \( P(A \mid B) = P(A' \mid B') = p = 0.95 \)
\( P(B) = 0.05, \) so \( P(B') = 1.0 - 0.05 = 0.95 \)

Observe first that \( (A \cap B') \) and \( (A' \cap B') \) are disjoint, and that the union of these two sets is \( B' \). It follows that \( P(B') = P((A \cap B') \cup (A' \cap B')) = P(A \cap B') + P(A' \cap B') \)
Rearranging terms gives
\( P(A \cap B') = P(B') - P(A' \cap B') \)
It follows that
\( P(A \mid B') \cdot P(B') = P(B') - P(A' \mid B') \cdot P(B') \). So long as \( P(B') > 0 \), we can divide through by \( P(B') \) giving
\( P(A \mid B') = 1 - P(A' \mid B') \).
We wish to find \( P(B' \mid A) = P(A \mid B') P(B') / P(A) \)
\( P(A) = P(A \mid B) \cdot P(B) + P(A \mid B') \cdot P(B') \)
\( P(A) = p \times 0.05 + (1-p) \times (1 - 0.05) \)
Then \( P(B' \mid A) = (1-p) \times (1 - 0.05) / (p \times 0.05 + (1-p) \times 1 - 0.05) \)
For p = .95, this gives

\[(.05 \times .95) / (.95 \times .05 + .05 \times .95) = \frac{1}{2}\]

c. We want \( P(B|A) = P(A|B) \times P(B) / P(A) \)

\[= p \times 0.5 / (p \times 0.05 + (1-p) \times 0.95) = 0.9 \]

Solving for p:

\[0.05p = 0.045p + 0.855 - 0.855p \]

\[0.86p = 0.855 \]

\[p = 99.42\% \]

3.16

a:

\[P(T|D) = 0.98 \]
\[P(T'|D') = 0.95 \]
\[P(D) = 0.01 \]
\[P(D') = 0.99 \]

\[P(D|T) = P(T|D) \times P(D) / P(T) \]

\[P(T) = P(T|D) \times P(D) + P(T|D') \times P(D') \]

\[0.0593 = 0.98 \times 0.01 + (1 - 0.95) \times 0.99 \]

\[P(D|T) = 0.98 \times 0.01 / 0.0593 = 0.165 \]

b:

\[P(D|(S \cap T)) = P(S \cap T|D) \times P(D) / P(S \cap T) \]

\[P(S \cap T) = P(S \cap T|D) \times P(D) + P(S \cap T|D') \times P(D') \]

\[= P(S|D) \times P(D) + P(T|D) \times P(D) + P(S|D') \times P(D') \times P(D) \]

\[= 0.98 \times 0.01 \times 0.98 \times 0.01 + 0.05 \times 0.99 \times 0.05 \times 0.99 = 0.00254629 \]

\[P(D|(S \cap T)) = 0.00009604 / 0.00254629 = 0.28 \]

3.18

We are given that 0 < P(A) < 1 and 0 < P(B) < 1.

a) If A and B are disjoint, then \( A \cap B = \emptyset \) so 0 = P(A \cap B) = P(A) * P(B \| A).

We are given that P(A) > 0. Thus P(B \| A) must be 0.

But P(B) > 0. Hence, P(B \| A) \neq P(B) and A and B are not independent.
b) If A and B are independent, then \( P(A \cap B) = P(A) \times P(B) > 0 \) since both A and B are non-zero. But this means that \( A \cap B \) is not empty (otherwise the probability of their intersection would be 0.) Hence, A and B are not disjoint.

c) Suppose A is a subset of B. Then \( A \cap B = A \). Thus \( P(A) = P(A \cap B) = P(A) \times P(B \mid A) \) and we see that \( P(B \mid A) = 1 \neq P(B) \) since \( P(B) > 1 \). Thus A and B are not independent.

d) A is a subset of \( A \cup B \), and by part c) above cannot in any event be independent.