Problem 1.

Let \( S(x) \) be the predicate “\( x \) is a student,” \( F(x) \) the predicate “\( x \) is a faculty member,” and \( A(x,y) \) the predicate “\( x \) has asked \( y \) a question,” where the domain consists of all people associated with your school.

Use quantifiers to express each of these statements.

a) Lois has asked Professor Michaels a question.

b) Every student has asked Professor Gross a question.

c) Every faculty member has either asked Professor Miller a question or been asked a question by Professor Miller.

d) Some student has not asked any faculty member a question.

e) There is a faculty member who has never been asked a question by a student.

f) Some student has asked every faculty member a question.

g) There is a faculty member who has asked every other faculty member a question. h) Some student has never been asked a question by a faculty member.

Problem 2.

Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

(a) \( \neg \exists y \exists x P(x, y) \)

(b) \( \neg \exists y (Q(y) \land \forall x \neg R(x, y)) \)

(c) \( \neg \exists y (\exists x R(x, y) \lor \forall x S(x, y)) \)

(d) \( \forall x \exists y (P(x, y) \rightarrow Q(x, y)) \)

Problem 3.

Translate each of these nested quantifiers into an English statement that expresses a mathematical fact.

The domain in each case consists of all real numbers.

(a) \( \forall x \forall y \exists z (x + y = z) \)

(b) \( \forall x \forall y ((x \neq 0) \land (y \neq 0) \iff (xy \neq 0)) \)