Problem 1.

Let \( P(x) \) be the statement \( x = x^2 \); the domain is all integers. Determine the truth values of the following. (10 points)

(a) \( P(0) \)
(b) \( P(1) \)
(c) \( P(2) \)
(d) \( \exists x \ P(x) \)
(e) \( \forall x \ P(x) \)

Problem 2.

Let \( P(x) \) be the statement \( \cdots \)x has visited France\'; the domain is the students in the school. Express the following sentences in English.

(a) \( \exists x \ P(x) \) (5 points)
(b) \( \forall x \ P(x) \) (5 points)
(c) \( \exists x \ \neg P(x) \) (5 points)
(d) \( \forall x \ \neg P(x) \) (5 points)

Problem 3.

Express the following statements using quantifiers; clearly define your predicate function. (20 points)

(a) A Passenger on an airline qualifies for an elite flyer if the passenger flies more than 25k miles in a year or takes more than 25 flights during that year.

(b) There is a student who has taken more than 21 credit hours in a semester and receive all A’s

Problem 4.

Determine whether the following quantifiers are equivalent or not, and justify it.

(a) \( \forall x \ (P(x) \rightarrow Q(x)) \) and \( \forall x \ P(x) \rightarrow \forall x \ Q(x) \) (15 points)
(b) \( \forall x \ P(x) \lor \forall x \ Q(x) \) and \( \forall x \ (P(x) \lor Q(x)) \) (15 points)
(c) \( \forall x \ (P(x) \rightarrow A) \) and \( \exists x \ P(x) \rightarrow A \) (20 points)