Homework 2, Due day: Feb. 17

All solutions should be typed, using Latex preferably (suggested to use Overleaf software).

(1) Use recursion tree to guess a bound, then proof it using induction. Finally, use master theorem (if applicable) to directly get the bound. Try to make your bounds as tight as possible.

$$T(n) = 2T(n/2) + n$$
$$T(n) = 2T(n-2) + n$$

(2) Repeat the same "best cooking time" in the notes with a quality quadratic function $f = -x^2+20x+1$ (the higher the number, the better the taste).

Peter proposes a 1/3-cut method: the first two cuts are at 1/3 and 2/3 of the original time span to generate three sections, left, middle, and right. Either left or right can be discarded after comparing two tastes (cuts). One more cut is applied to the middle of the longer one of the two remaining sections. Repeat the above process, until the longer section is less than 0.5 unit in length.

Compare Peter's approach with the Golden ratio approach in terms of the number of tastes needed before finding out the best cooking time. Print out all the cooking times and tasting quality values, including the best good time and its value. Repeat the same process for $f' = -x^2 + 30x + 3$.

(3) Chapter 5.2

(4) Chapter 5, 4

(5) Chapter 5, 6

(6) Chapter 5, 7