MIDTERM EXAM

CIS 5515 Design and Analysis of Algorithms (Spring 2022)

Note: For answer to each question, please explain your answer **in plain English** first. There are a total of 100 pts plus 10 bonus points. It is your responsibility to make sure that you have all the pages!

NAME:

(**Problem 1**. 20 points, 4 points each) **True** or **False**?
For each of the statements below, determine whether it is **True** or **False**. Briefly explain your answer or provide a counterexample for each of them.

(a) Given $T(n) = 3n^3 + 2n^2 \lg n + n + 2$

  (a) $T(n) = O(n^4)$
  (b) $T(n) = \Omega(n^2 \lg n)$
  (c) $T(n) = \Theta(n^3)$
  (d) $T(n) = O(n^3)$

(b) If $T(n) = 8T(n/2) + \Theta(n^2)$ then $T(n) = \Theta(n^2 \lg n)$.

(c) Let G be an undirected graph on n nodes. G has $n-1$ edges if (1) G is connected and (2) G does not contain a cycle.

(d) In stable marriage, if w is ranked last in m's preference list and m is ranked last in w's list, then (m, w) will not appear in any stable matching.

(e) In HW2 assignment, the golden ratio search can always beat Peter's proposed approach for a given quadratic quality function and a given search range.

(**Problem 2**. 20 points) Given n **girls** and $2n$ **boys**, each with her/his private preference orders,

1. (5 points) Design a girl-initiated extended G-S algorithm such that it is female-optimal.
2. (5 points) Briefly show that your algorithm is still stable.
3. (5 points) Suppose each girl can marry exactly two boys, define a new concept of stability.
4. (5 points) Design an extended G-S algorithm for 3. that terminates and all matches are stable.

(**Problem 3**. 20 points) Extend the Weighted Interval Scheduling as following: each schedule can only include up to k out of total n jobs. The objective is still to find a compatible subset with the maximum total value.

1. (15 points) Provide a recursive solution in plain English first, followed by pseudo code and complexity analysis.

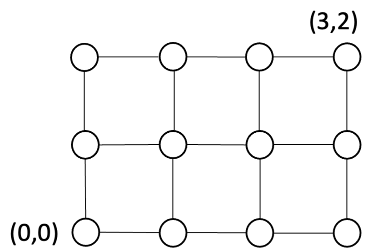
2. (5 points) Use the example in the class notes to illustrate your algorithm with a solution (i.e., selected jobs) for $k = 2$. The weight distribution from job a to job h is 2, 8, 7, 10, 3, 9, 13, and 11, respectively.

(**Problem 4**. 20 points) Extend question 5.7 to a 3-D $n \times n \times n$ grid graph. Two nodes (i, j, k) and (i', j', k') are neighbors if and only if $|i-i'|+|j-j'|+|k-k'| = 1$, i.e., each node has up to six neighbors. A node is local minimum if it has a minimum value among its 1-hop neighborhood. Provide a high-level solution in plain English, followed by pseudo code and complexity analysis.

- (5 points) Find a simple $\Theta(n^3)$ algorithm that finds a local minimum.
- (15 points) Enhance the simple algorithm so that the complexity of the enhanced algorithm is less than $\Theta(n^3)$. Provide a proof of algorithm complexity.

(**Problem 5**. 20 points) Given a 2-D grid graph (like the graph in question 5.7), the distance of two adjacent nodes is 1.

1. (5 points) Find a recursive solution that determines the number of shortest paths from (0, 0) to (m, n).
2. (5 points) Determine the number of shortest paths in terms of m and n. Use $m = 3$ and $n = 2$ as an example to illustrate.
3. (5 points) Provide an iterative solution for the same problem.
4. (5 points) Show and prove the complexity of two solutions.



(**Bonus problem**, 10 pts), Suppose C is a given set of currency denominations $C = \{1, p, 2p^2,$ $3p^3$, ..., $np^n\}$, where $p > 1$ and $n \geq 0$ are integer.

1. (2 points) Show how does the greedy algorithm described in the class notes make changes for 80 when $p = 2$ and $n = 4$.
2. (8 points) Show that the greedy algorithm discussed in the class always finds an optimal solution in terms of minimizing the number of coins for changes.