FINAL EXAM

CIS 5515 Design and Analysis of Algorithms (Spring 2021)

Note: For answer to each question, please explain your answer **in plain English** first. There are a total of 100 pts plus 10 bonus points. It is your responsibility to make sure that you have all the pages!

NAME:

(**Problem 1**. 20 points, 4 points each) **True** or **False**?
For each of the statements below, determine whether it is **True** or **False**. Briefly explain your answer or provide a counterexample for each of them.

(a) (1) If Y $\in$ NP-COMPLETE, X $\in$ NP, and X $\leq_P$ Y, then X in NP-COMPLETE.

(2) Based on the results shown in Chapter 8, 3-SAT can be easily reduced to SET-COVER.

(b) (1) Is it possible that both P = NP and NP = EXP?

(2) Is it possible that NP = PSPACE?

(c) (1) For some NP-hard problems, a constant approximation solution is not possible.

(2) For some NP-hard problems, they can be approximated arbitrarily close to the optimum.

(d) The relationship between maximum independent set and minimum vertex cover is like the relationship between maximum flow and minimum cut.

(e) In a tournament, there always exists one or more Hamiltonian paths. In such a path, each player always beats its immediate successor.

(**Problem 2**. 20 points) Consider a *matching market* with three buyers (x, y, z) and three sellers (a, b, c). Among buyers, the valuations for sellers (a, b, c) are (1, 3, 10) for x, (1, 4, 6) for y, and (5, 6, 7) for z.

1. (10 points) Show step-by-step matching process, the corresponding preferred-seller graph at each step, and finally a set of market-clearing prices.

2. (10 points) Is it possible to have multiple *constricted* sets in a step? Will the selection order of constricted sets matter in the final outcome? Briefly explain your conclusions.

(**Problem 3**. 20 points) In a *circulation problem* with two demands (D_1: 5 and D_2: 2) and two supplies (S_1: -5 and S_2: -2). The edge capacities are the following: (S_1, D_1): 5, (S_2, D_1): 1, (S_1, D_2): 1, and (S_2, D_2): 2. There is lower bound for edge (i.e., the minimum amount of flows that have to flow through the edge) (S_1, D_1) which is 2.

1. Show the network flow graph for the circulation problem with the lower bound and then without the lower bound.

2. Solve the circulation problem step-by-step using the augmenting path solution after converting the problem to a general network flow graph.

3. If there is more than one solution, show *all integer solutions*. Transfer these solutions back to the original graph.

(**Problem 4**. 20 points) Reconsider Problem 11.1 (assigning containers to trucks) as follows:

1. Suppose the largest weight of a container $w \leq K/2$, where K is the maximum weight for a truck to hold.
2. Suppose the largest weight $w \leq K/3$.
3. Suppose containers arrive in the sequence of decreasing order of weight.
4. Suppose containers arrive in the sequence of increasing order of weight.

Discuss how the approximation ratio would change. Justify your answer for each case.

(**Problem 5**. 20 points) Revise the *quick sort* algorithm to solve the sequence of kings.

1. Describe the algorithm in plain English first.
2. Provide a pesudo code and illustrate using the example of slide 11 (Adversary Augments slides) with node v_3 as a *pivot*.
3. Discuss the complexity of the algorithm.

(**Bonus problem**, 10 points) Find an optimal algorithm that finds the *third-largest-key* in n elements. Illustrate that your bound is tight. Again, explain your solutions in plain English first, followed by a formal proof.