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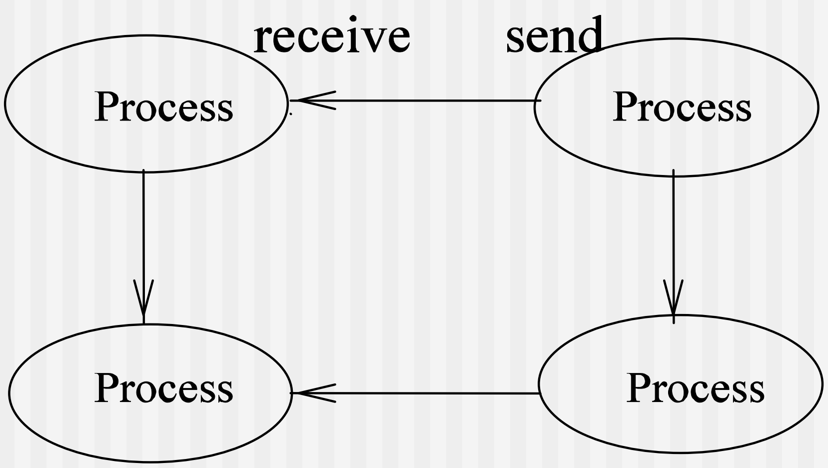
State Model

- A **process** executes three types of events: **internal** actions, **send** actions, and **receive** actions.
- A **global state (**also **configuration)**: a collection of local states and the state of all the communication channels.

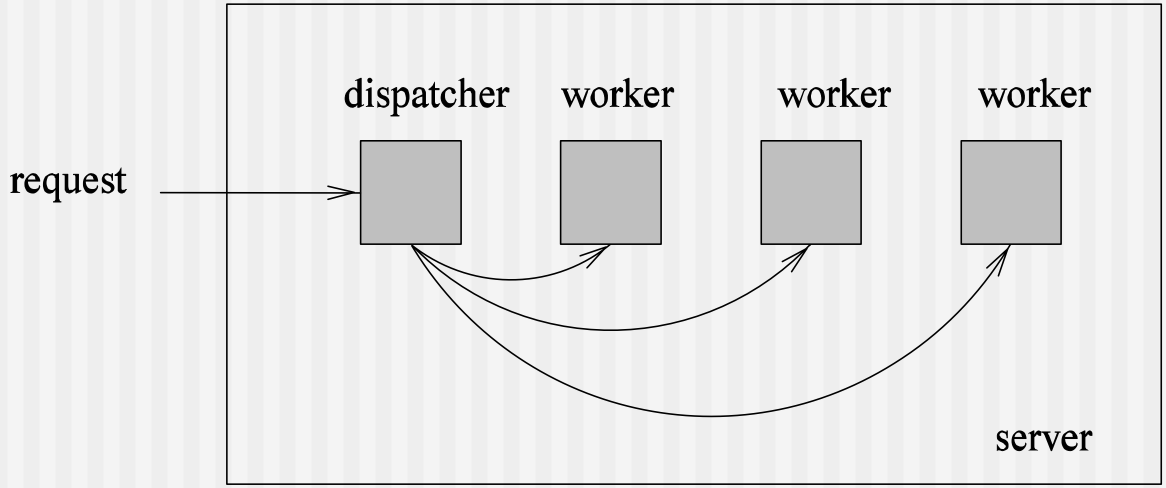
- Global state evolves by means of **transitions**
- **Initiator**: first event
- **Distributed algorithm**: multiple initiators



System structure from logical point of view.

Thread

- lightweight process (maintain minimum information in its context)
- multiple threads of control per process
- multithreaded servers (vs. single-threaded process)



A multithreaded server in a dispatcher/worker model.

Preliminary

Assertions: a predicate on the configurations of an algorithm

Invariant, such as loop invariant, is an assertion

e.g., {I} **while** c body {¬c ∧ I} (under Floyd-Hoare logic)

calculate sum: 1+2+…+n, two assertions I: 1+2+…+k and c: k < n

Safety property: if it is true in each reachable configuration

i.e., something bad will never happen (e.g., absence of deadlock, mutual exclusion, partial correctness)

Liveness property: if executions, from some point on, contain a configuration in which the assertion holds

i.e., something good will eventually happen (e.g., fairness, termination)

Fair: if every event that can happen in infinitely many times is performed infinitely often

Complexity: time, space, message (bit) complexity

Happened-Before Relation

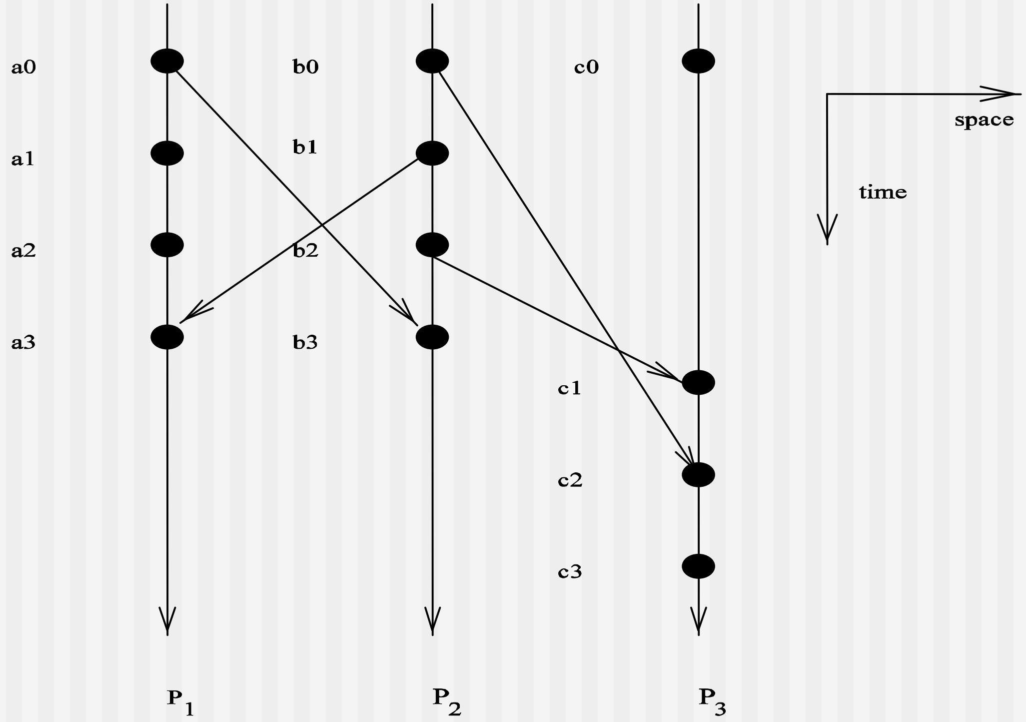
The **happened-before relation** (denoted by $\rightarrow$) is defined as follows:

- Rule 1 : If a and b are events in the same process and a was executed before b, then $a \rightarrow b$.
- Rule 2 : If a is the event of sending a message by one process and b is the event of receiving that message by another process, then $a \rightarrow b$.
- Rule 3 : If $a \rightarrow b$ and $b \rightarrow c$, then $a \rightarrow c$.

Relationship Between Two Events

- Two events *a* and *b* are **causally related** if $a \rightarrow b$ or $b \rightarrow a$.
- Two distinct events *a* and *b* are said to be **concurrent** if $a \not\rightarrow b$ and $b \not\rightarrow a$ (denoted as $a \parallel b$).

Example 2



A time-space view of a distributed system.

Example 2 (Cont'd.)

- Rule 1:

  $a_0 \rightarrow a_1 \rightarrow a_2 \rightarrow a_3$

  $b_0 \rightarrow b_1 \rightarrow b_2 \rightarrow b_3$

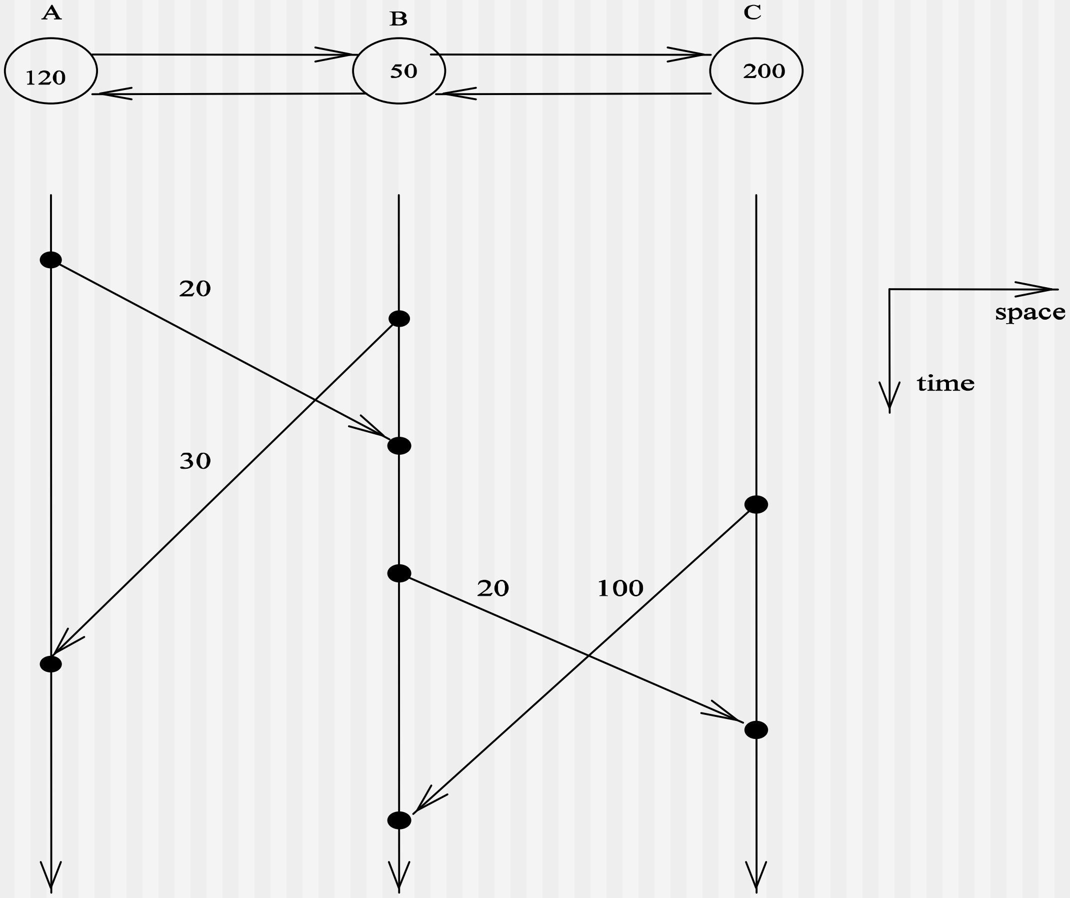
  $c_0 \rightarrow c_1 \rightarrow c_2 \rightarrow c_3$

- Rule 2:

  $a_0 \rightarrow b_3$

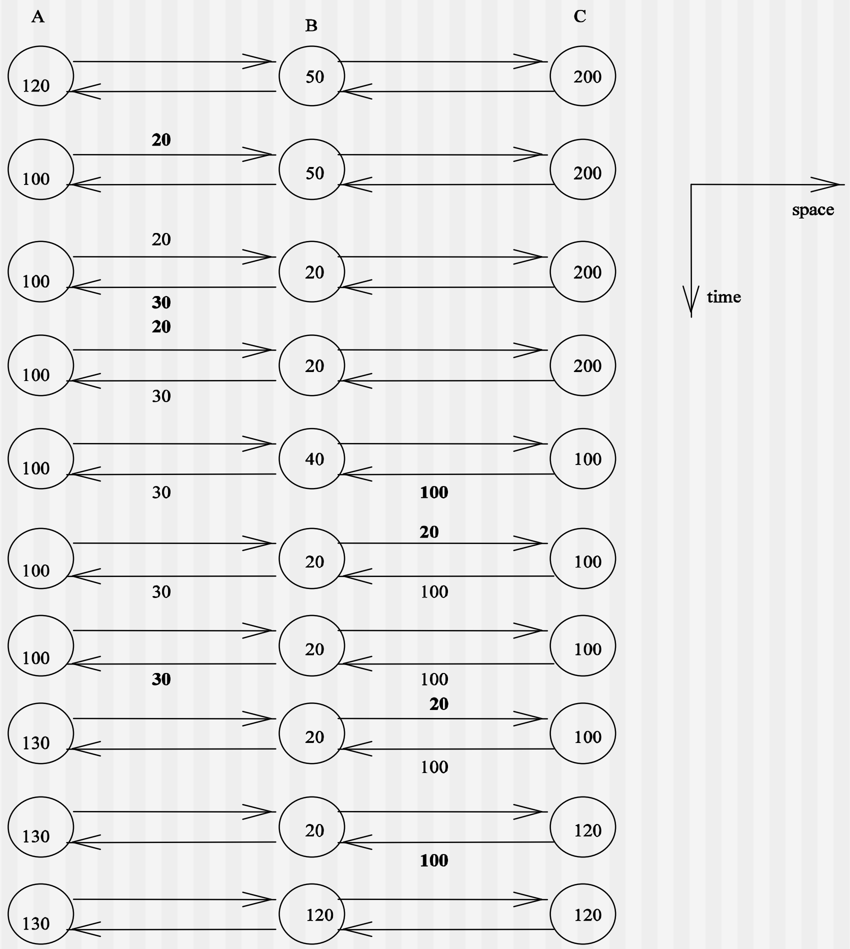
  $b_1 \rightarrow a_3$, $b_2 \rightarrow c_1$, $b_0 \rightarrow c_2$

Example 3



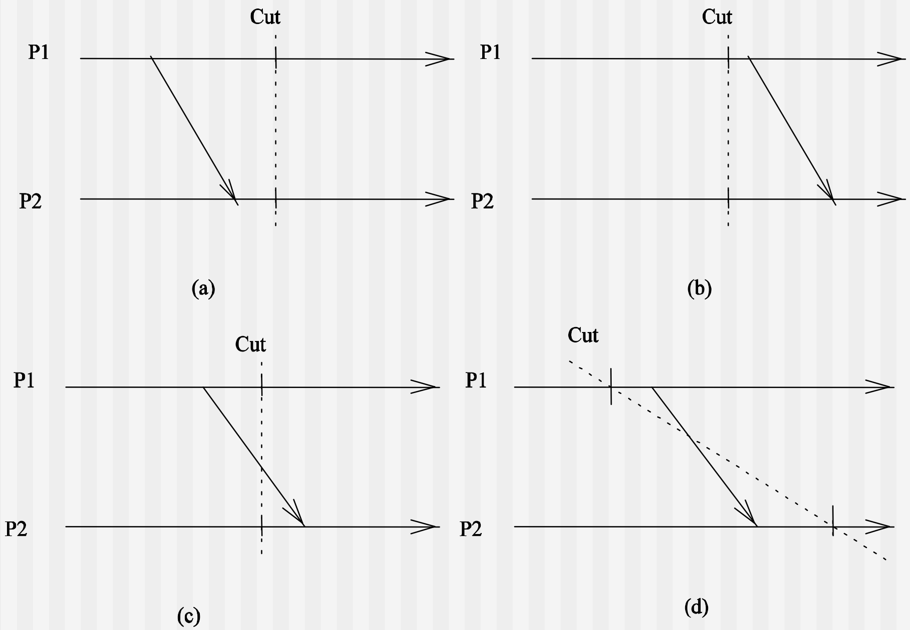
An example of a network of a bank system.

Example 3 (Cont'd.)



A sequence of global states.

Consistent Global State



Four types of cut that cross
a message transmission line.

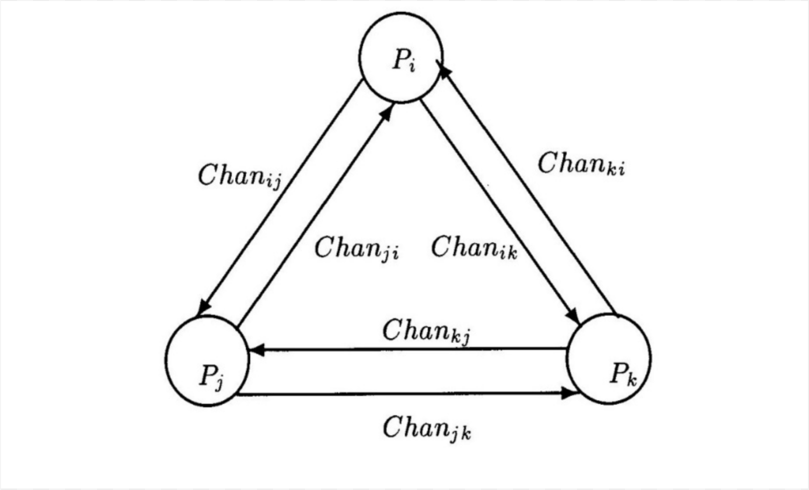
Consistent Global State (Cont'd.)

A **cut** is consistent iff no two cut events are causally related.

- **Strongly consistent**: no (c) and (d).
- **Consistent**: no (d) (orphan message).
- **Inconsistent**: with (d).

Focus 3: Snapshot of Global States

A simple distribute algorithm to capture a consistent global state.



A system with three processes P_i, P_j , and P_k.

Many key concepts: asynchronous computation, global state, information propagation and gathering, …

Chandy and Lamport's Solution

- Rule for sender *P* :

  [*P* records its local state

  ||*P* sends a marker along all the channels on which a marker has not been sent.

  ]

- Rule for receiver *Q*:

  /* on receipt of a marker along a channel *chan* */

  [*Q* has not recorded its state →

  [record the state of *chan* as an empty sequence and follow the "Rule for sender"

  ]

  □ *Q* has recorded its state →

  [record the state of *chan* as the sequence of messages received along *chan* after the latest state recording but before receiving the marker

  ]

  ]

Chandy and Lamport's Solution (Cont'd.)

- It can be applied in any system with FIFO channels (but with variable communication delays).
- The initiator for each process becomes the parent of the process, forming a spanning tree for result collection.
- It can be applied when more than one process initiates the process at the same time.

Chandy and Lamport's Solution (Cont'd.)

- Distributed algorithm: message-passing
- Distributed snapshot
- Dynamic spanning tree
- Asynchronous systems
- Message dissemination
- Progress termination
- Program debugging
  - Breakpoint
- Simulation
  - Physical and logical processes (event-driven)

Synchronous vs. Asynchronous Systems

Asynchronous Systems:

- Each node is driven by its own (independent) local clock.
- The transmission delay is finite but unpredictable.

Synchronous Systems:

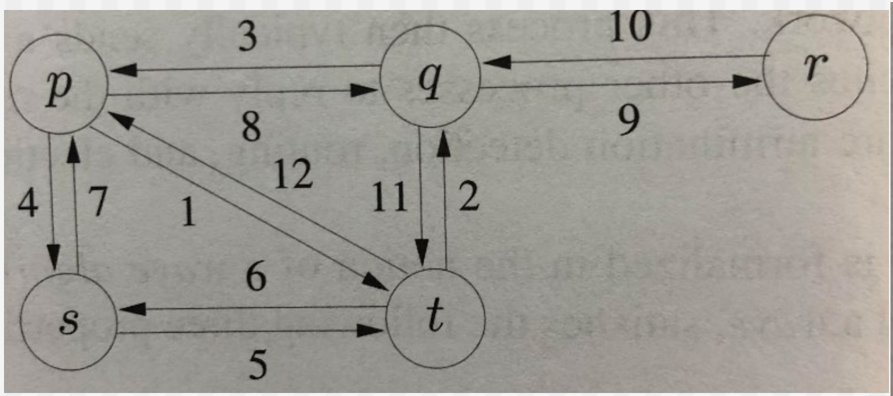
- All nodes are driven by the global clock, which generates intervals (also rounds) of fixed, nonzero duration.
- The transmission delay is nonzero, but strictly less than the duration of an interval.

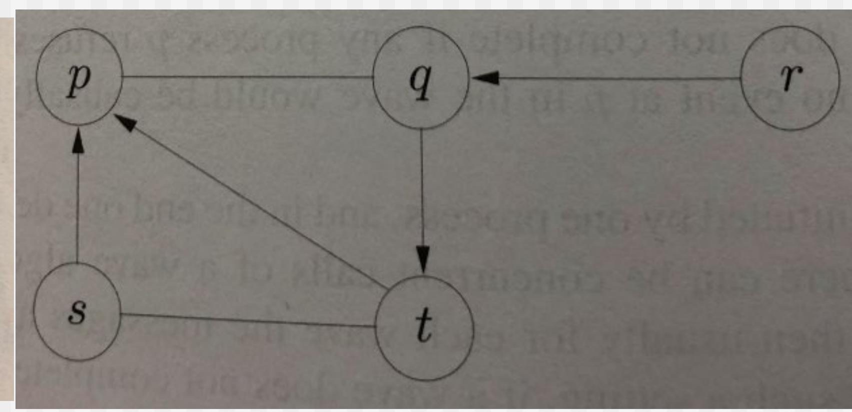
Distributed Algorithms: Traversal

Tarry's algorithm:

- A process forwards the token through the same channel once.
- A process forwards the token to its parent only when there is no other option.

Complexity: 2E messages and at most 2E time units.



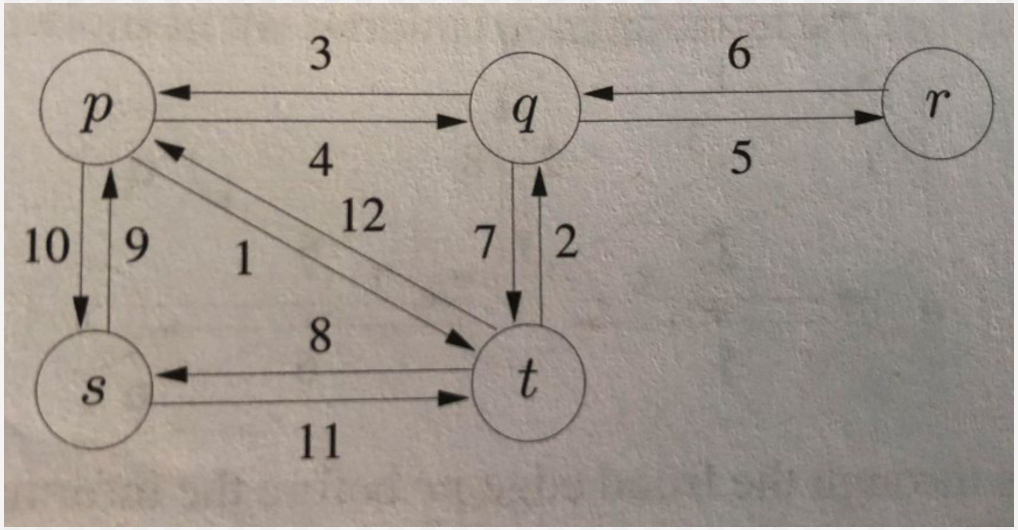


Distributed Algorithms: Traversal (cont'd)

Depth-first search (DFS) algorithm:

- Same as Tarry's algorithm, with the following constraint
- Whenever the token is forwarded to a process has hold the token before, it is sent back to its sender.

Complexity: same as Tarry's algorithm



Distributed Algorithms: Traversal (cont'd)

Extensions to avoid visited nodes:

- Include the IDs of visited nodes

  Complexity: 2(N-1) in time and in messages, but O(N log N) in bit complexity

- **Awerbuch**'s extension: the first-time process with the token informs its neighbors

  Complexity: 4N-2 in time and 4E in messages

- **Cidon**'s extension: improves on Awerbuch's extension

  Complexity: 2(N-1) in time and 4E in messages

Distributed Algorithms: Wave-and-Echo

Wave-and-Echo algorithm (also for counting connected nodes)

- **Initiator** starts by sending a token to all its neighbors.
- When a node receives a token for the first time, it makes the sender its parent, and sends the token to all its neighbors.
- When a node has received messages from all its neighbors, it sends a message to its parent.
- When the **initiator** has received messages from all its neighbors, it stops.

General **wave (-and-echo)** algorithm (also for information propagation)

- A process often needs to gather information from all other processes.
- Usually the process starts with an initiator and ends with the same imitator (after collecting all data/results from all other processes).
- When the wave algorithm is issued at multiple nodes. Many waves, except one, will fail

Distributed Algorithms: Termination

Dijkstra-Scholten (tree-based):

- The initiator of the root of the tree.
- Upon receiving a message:
  - If the receiving process is currently not in the tree: the process joins the tree by becoming a child of the sender.
  - If the receiving process is already in the computation: the process immediately sends an acknowledgment message to the sender.
- When a process has no more children and has become idle, the process detaches itself from the tree by sending an acknowledgment to its tree parent.
- Termination occurs when the initiator has no children and has become idle.

Example: global snapshot (with one king)

Distributed Algorithms: Termination (cont'd)

Shavit-Francez (forest-based):

- Same as Dijkstra-Scholten, except with multiple initiators.
- Each non-initiator joining one tree.
- Termination detection initiated by multiple initiators through a wave algorithm

Example: global snapshot (with multiple kings)

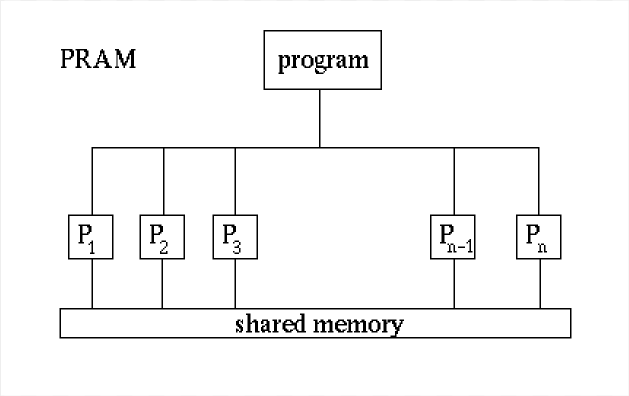
Other termination algorithms:

- **Weight-throwing** algorithm: dividing a fixed weight over the active processes
- **Rana**'s algorithm: waves tagged with logical clocks
- **Safra**'s algorithm: token-based traversal

Other Algorithms: Parallel Algorithms

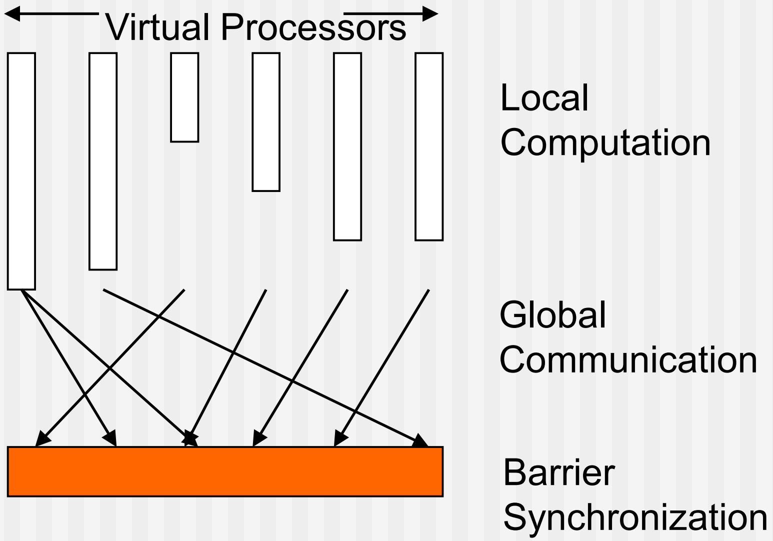
PRAM model

- Parallel random access memory
- EREW, ERCW, CREW, CRCW models
- Chap. 2 of JaJa's
  "an introduction to parallel algorithms"



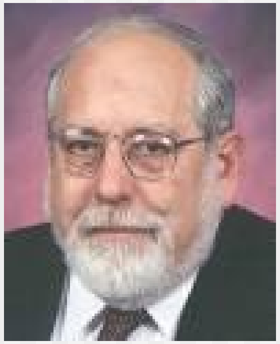
BSP model by L. Valiant (1990)

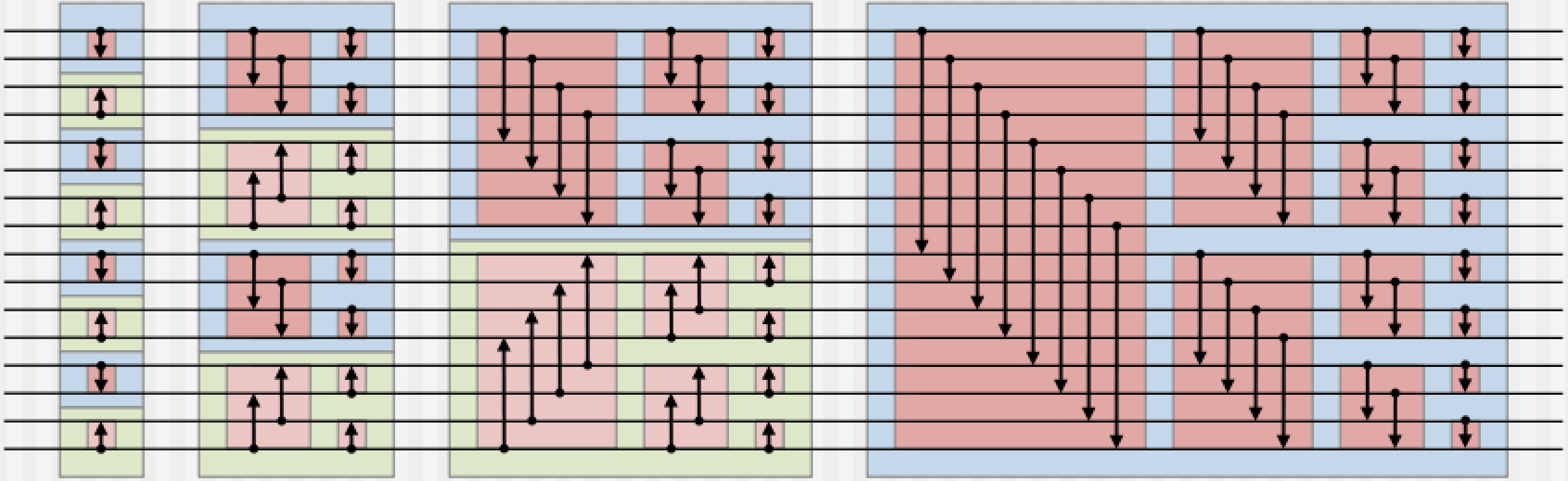
- Bulk synchronous parallel (BSP)
- Sequential composition of "supersteps"
  - Local computation
  - Process communication
  - Barrier synchronization



Parallel Algorithm: Bitonic sorter by K. Batcher

- Sorting network based on **Bitonic sequence**
  - Up-then-Down or Down-then-Up
  - $O(n \log^2(n))$ comparators
  - $O(\log^2(n))$ latency
- Also Batcher's **odd-even sort** (small -> large)

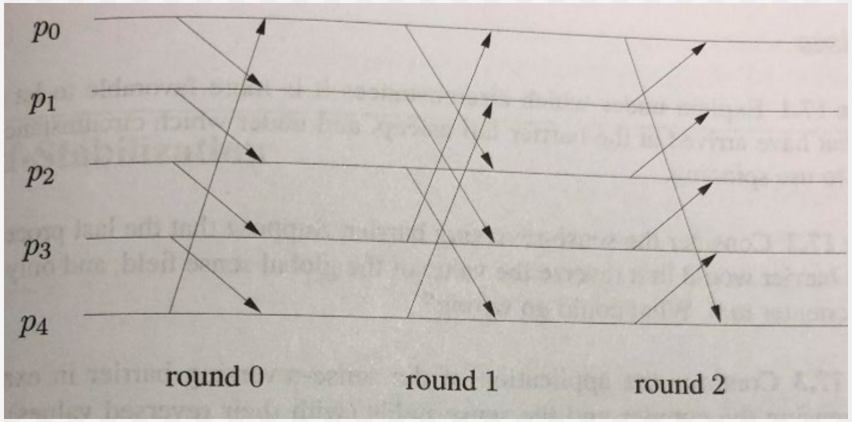




Barrier Synchronization

- **Sequential**: One process p (leader, through leader election if needed)
  - Process p issues wave-and-echo to all nodes
  - Process p indicates next round to all nodes

- **Parallel**: Processes p_0, p_1, …, p_{N-1}, n starts from 0 until $\log_2 N$ -1
  - Notifies process $p_{(i+2^n)}$ mod N,
  - Waits for notification by process $P_{(i-2^n)}$ mod N, and
  - Processes to round n+1



Focus 4: Lamport's Logical Clocks

Based on a "**happen-before**" relation that defines a **partial order** on events

- *Rule*$_1$. Before producing an event (an external send or internal event), we update *LC* :

  $LC_{\mathrm{i}} = LC_{\mathrm{i}} + d \qquad (d > 0)$

  (d can have a different value at each application of *Rule*$_1$)
- *Rule*$_2$. When it receives the time-stamped message (m, LC_j, j), P_{i} executes the update

  $LC_i = \max\{Lc_i, LC_j\} + d \quad (d > 0)$

Focus 4 (Cont'd.)

A **total order** based on the partial order derived from the happen-before relation

$$a \text{ (in } P_i \text{)} \Rightarrow b \text{ (in } P_j \text{)}$$

iff

(1) $LC(a) < LC(b)$ or (2) $LC(a) = LC(b)$ and $P_i < P_j$ where < is an arbitrary total ordering of the process set, e.g., <can be defined as $P_i < P_j$ iff $i < j$.

A total order of events in the table for Example 2:

$$a_0\ b_0\ c_0\ a_1\ b_1\ a_2\ b_2\ \mathrm{a}_3\ b_3\ c_1\ \mathrm{c}_2\ c_3$$

Vector and Matrix Logical Clock

Linear clock: if $a \rightarrow b$ then $LC_a < LC_b$

Vector clock: $a \rightarrow$ b iff $LC_a < LC_b$

Each P_i is associated with a vector $LC_i[1..n]$, where

- $LC_i[i]$ describes the progress of P_i, i.e., its own process.
- LC_i $[j]$ represents P_i's knowledge of P_j's progress.
- The $LC_i[1..n]$ constitutes P_i's local view of the logical global time.

Vector and Matrix Logical Clock (Cont'd.)

When $d = 1$ and $init = 0$

- $LC_i[i]$ counts the number of internal events
- $LC_i[j]$ corresponds to the number of events produced by P_j that causally precede the current event at P_i.

Knowledge and implicitly knowledge

Vector and Matrix Logical Clock (Cont'd.)

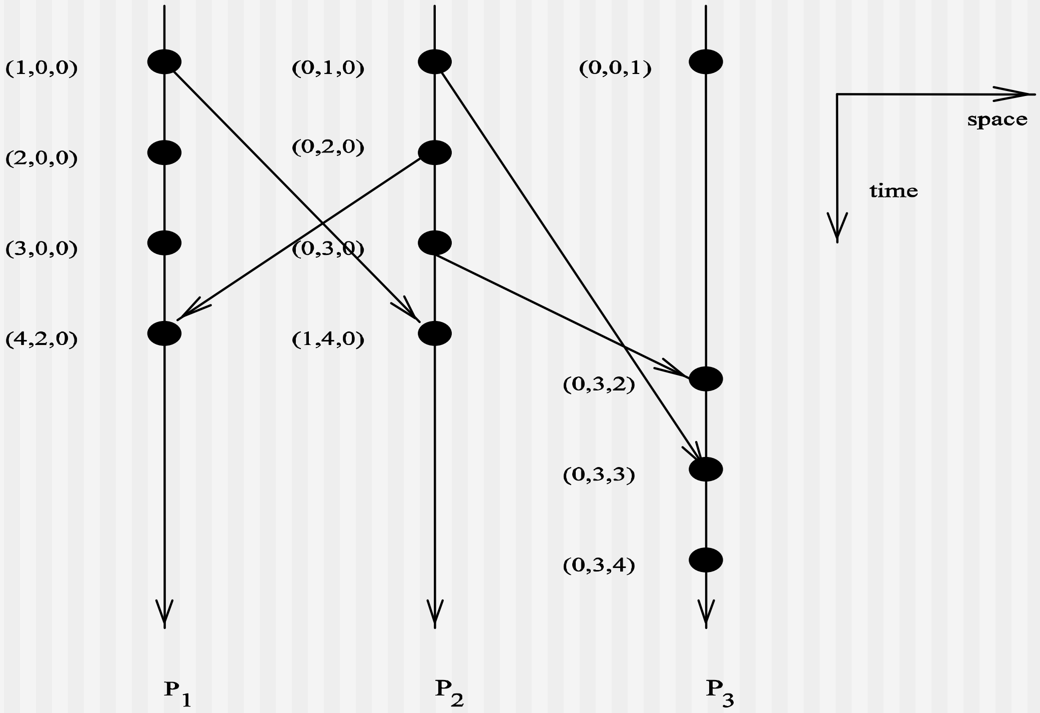
- *Rule*$_1$. Before producing an event (an external send or internal event), we update $LC_i[i]$:

$$LC_i[i] := LC_i[i] + \mathrm{d} \quad (d > 0)$$

- *Rule*$_2$. Each message piggybacks the vector clock of the sender at sending time. When receiving a message (m, LC_j , j), P_i executes the update.

$$LC_i[k] := \max\,(LC_i[k]; LC_j[k]),\ 1 \le \mathrm{k} \le n$$
$$LC_i[i] := LC_i[i] + d$$

Example 4



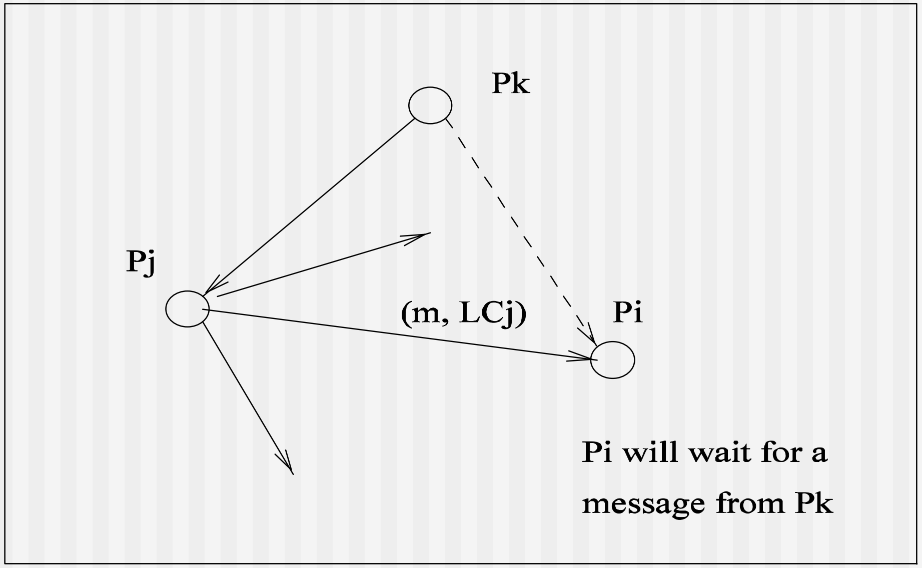
An example of vector clocks.

Example 5: Totally-Ordered Multicasting

- Two copies of the account at A and B (with balance of $10,000).
- Update 1: add $1,000 at A.
- Update 2: add interests (based on 1% interest rate) at B.
- Update 1 followed by Update 2: $11,110.
- Update 2 followed by Update 1: $11,100.

Example 6: Application of Vector Clock

Internet electronic bulletin board service



Network News.

When receiving *m* with vector clock LC_j from process *j*, P_i inspects timestamp LC_j and will postpone delivery until all messages that causally precede *m* have been received.

Matrix Logical Clock

Each P_i is associated with a matrix $LC_i[1..n, 1..n]$ where

- $LC_i[i, i]$ is the local logical clock.
- $LC_i[k, l]$ represents the view (or knowledge) P_i has about P_k's knowledge about the local logical clock of P_l.

If

$$\min(LC_i[k, i]) \geq t$$

then P_i knows that every other process knows its progress until its local time t.

Physical Clock

- Correct rate condition:

  $\forall_i \; |dPC_i(t)/\, dt - 1\,| < \alpha$

- Clock synchronization condition:

  $\forall_i \; \forall_j \; |PC_i(t) - PC_j(t)| < \beta$

Lamport's Logical Clock Rules for Physical Clock

- For each i, if P_i does not receive a message at physical time t, then PC_i is differentiable at t and $dPC(t)/dt > 0$.
- If P_i sends a message m at physical time t, then m contains $PC_i(t)$.
- Upon receiving a message (m, PC_j) at time t, process P_i sets PC_i to maximum $(PC_i(t - 0), PC_j + \mu_m)$ where μ_m is a predetermined minimum delay to send message m from one process to another process.

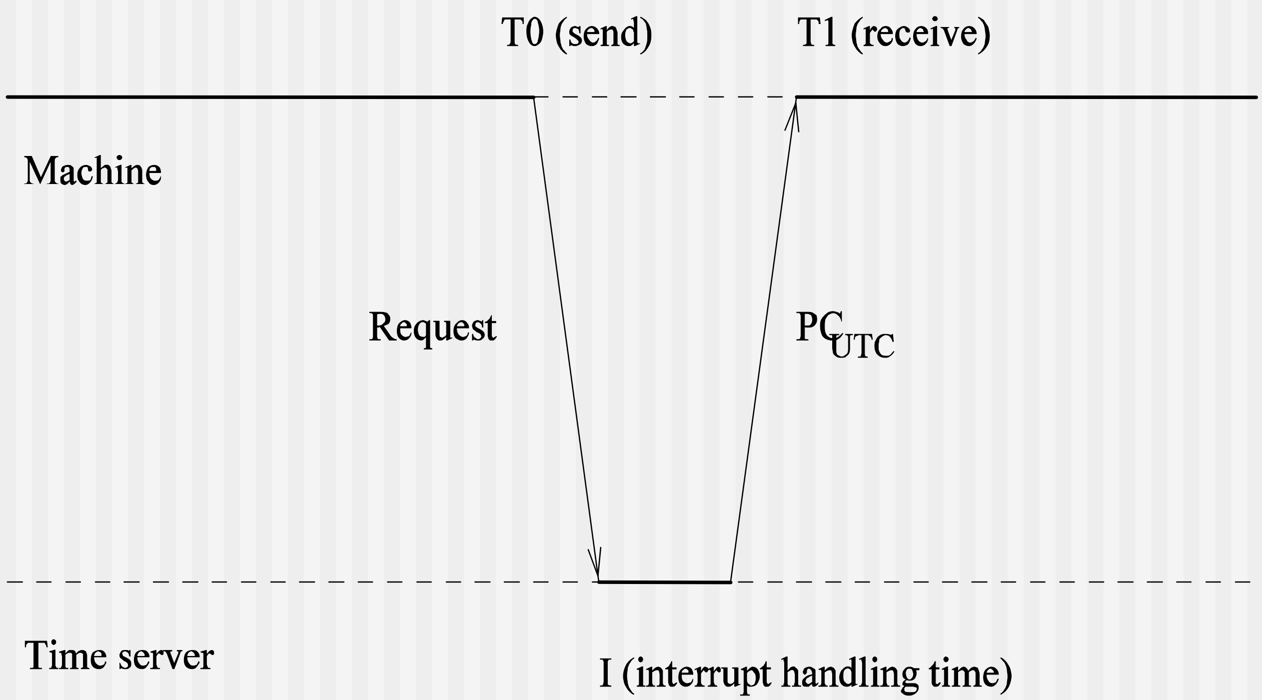
Focus 5: Clock Synchronization

- UNIX **make** program:
  - Re-compile when *file.c*'s time is large than *file.o*'s.
  - Problem occurs when source and object files are generated at different machines with no global agreement on time.
- **Maximum drift rate** ρ : $1-\rho \leq dPC/dt \leq 1+\rho$
  - Two clocks (with opposite drift rate ρ) may be $2\rho\Delta t$ apart at a time Δ after last synchronization.
  - Clocks must be resynchronized at least every $\delta/2\rho$ seconds in order to guarantee that they will be differ by no more than δ.

Cristian's Algorithm

- Each machine sends a request every $\delta/2\rho$ seconds.
- Time server returns its current time PC_{UTC} (UTC: Universal Coordinate Time).
- Each machines changes its clock (normally set forward or slow down its rate).
- Delay estimation: $(T_r - T_s - I)/2$, where T_r is receive time, T_s send time, and I interrupt handling time.

Cristian's Algorithm (Cont'd.)



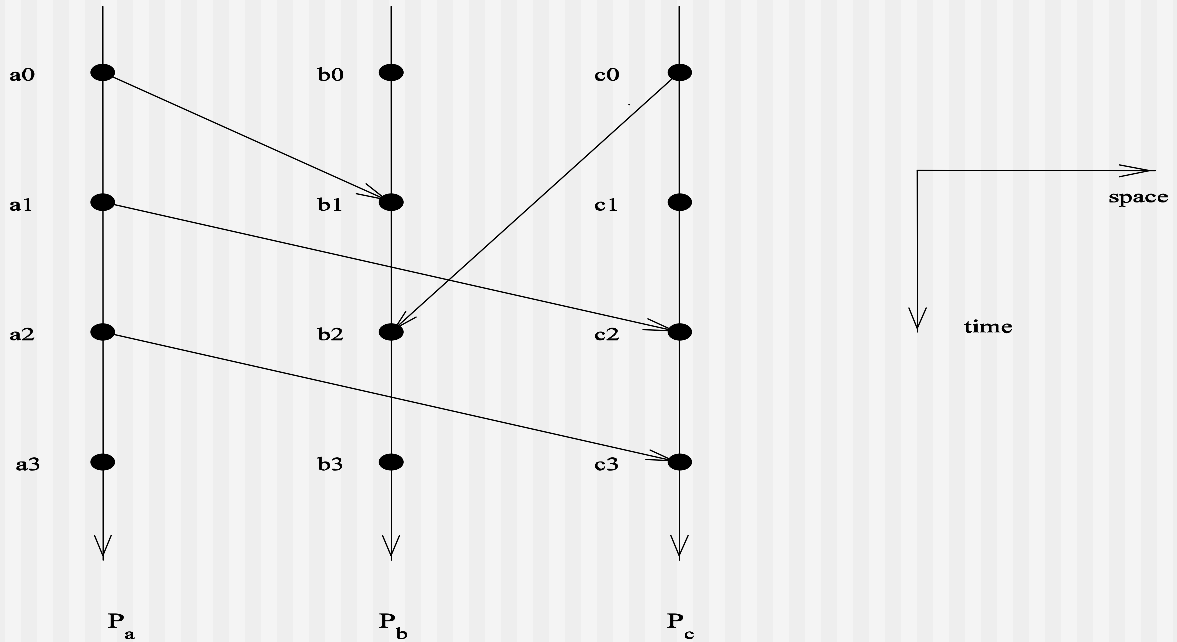
Getting correct time from a time server.

Three Ways to Demonstrate the Properties

- Testing and debugging (run the program and see what happens)
- Operational reasoning (exhaustive case analysis)
- Assertional reasoning (abstract analysis)

Exercise 2

1. Consider a system where processes can be dynamically created or terminated. A process can generate a new process. For example, P_1 generates both P_2 and P_3. Modify the happened-before relation and the linear logical clock scheme for events in such a dynamic set of processes.
2. For the distributed system shown in the figure below.



Exercise 2 (Cont'd)

- Provide all the pairs of events that are related.
- Provide logical time for all the events using
  - linear time, and
  - vector time
  - Assume that each LC_i is initialized to zero and $d = 1$.

3. Provide linear logical clocks for all the events in the system given in Problem 2. Assume that all LC's are initialized to zero and the d's for P_a, P_b, and P_c are 1, 2, 3, respectively. Does condition $a \rightarrow b \Rightarrow LC(a) < LC(b)$ still hold? For any other set of d's? and why?
4. Traversal on graph {(a, b), (b, c), (b, d), (c, e), (d, e), (e, f)} using Terry's solution, DFS solution, and Awerbuch's extension.
5. Show details of sorting (4, 6, 1, 3, 8, 5, 7, 2) and (1, 4, 7, 8, 2, 6, 5, 3) on an 8-input-and-8-output Batcher's Even-Odd sorting network.