

# P<sup>3</sup>: Joint Optimization of Charger Placement and Power Allocation for Wireless Power Transfer

Sheng Zhang<sup>†</sup>, Zhuzhong Qian<sup>†</sup>, Fanyu Kong<sup>†</sup>, Jie Wu<sup>‡</sup>, and Sanglu Lu<sup>†</sup>

<sup>†</sup>State Key Lab. for Novel Software Technology, Nanjing University, P.R. China

<sup>‡</sup>Department of Computer and Information Sciences, Temple University, USA

Emails: {sheng,qzz,sanglu}@nju.edu.cn, kongfy@dislab.nju.edu.cn, jiewu@temple.edu

**Abstract**—Wireless power transfer is a promising technology to extend the lifetime of, and thus enhance the usability of, the energy-hungry battery-powered devices. It enables energy to be wirelessly transmitted from power chargers to energy receiving devices. Existing studies have mainly focused on maximizing network lifetime, optimizing charging efficiency, minimizing charging delay, etc. Different from these works, our objective is to optimize charging quality in a 2-D target area. Specifically, we consider the following charger Placement and Power allocation Problem (P<sup>3</sup>): Given a set of candidate locations for placing chargers, find a charger placement and a corresponding power allocation to maximize the charging quality, subject to a power budget. We prove that P<sup>3</sup> is NP-complete. We first study P<sup>3</sup> with fixed power levels, for which we propose a  $(1-1/e)$ -approximation algorithm; we then design an approximation algorithm of factor  $\frac{1-1/e}{2L}$  for P<sup>3</sup>, where  $e$  is the base of the natural logarithm, and  $L$  is the maximum power level of a charger. We also show how to extend P<sup>3</sup> in a cycle. Extensive simulations demonstrate that, the gap between our design and the optimal algorithm is within 4.5%, validating our theoretical results.

**Index Terms**—Wireless power transfer, power allocation, sub-modularity, approximation algorithm

## I. INTRODUCTION

We have witnessed the increasing potential of wireless devices to improve the quality of our lives in the last few years. To extend the lifetime of, and thus enhance the usability of, these battery-powered devices, solutions from different perspectives have been proposed, including energy harvesting [1], energy conservation [2], and battery replacement [3]. However, they remain limited due to various reasons.

Recent breakthroughs in wireless power transfer [4, 5] provide a promising alternative that has attracted significant attention from both academia and industry. With this technology, energy can be wirelessly transmitted from power chargers to energy receiving devices such as RFID tags, sensors, smartphones, and Tesla cars [6]. Existing studies regarding this issue have mainly focused on maximizing network lifetime [7], optimizing charging efficiency [8], energy provisioning [9], collaboration between chargers [10], minimizing charging delay [11], minimizing maximum radiation point [12], etc.

Different from existing works, we consider the following scenario. A service provider decides to offer a wireless power charging service in an area of interest, *e.g.*, a campus or park. Based on historical data analysis and market investigation, it could predict the location information of future potential customers (*i.e.*, devices) and then preselect a certain number

of candidate locations for placing wireless power chargers (chargers for short in the sequel). The power of each charger is adjustable within an appropriate range. The maximum cover distance of a charger is determined by its power and the environment. The power received by a device from multiple chargers is assumed to be additive [9]. Given a power budget, the wireless charging service provider wants to maximize its revenue, which is proportional to the charging quality defined later in the paper. In order to maximize the charging quality, a limited number of chargers with appropriate power levels must be strategically placed at a subset of the candidate locations.

This charger Placement and Power allocation Problem (P<sup>3</sup>) can be briefly stated as follows: *Given a set of candidate locations for placing chargers, how to find a charger placement and a corresponding power allocation to maximize the charging quality, subject to a power budget.* In this paper, we prove that the P<sup>3</sup> problem is NP-complete by reduction from the set cover problem [13]. To gain a better understanding, we first consider the P<sup>3</sup> problem with fixed power levels, where the power level of every candidate location is fixed, for which we propose a  $(1-1/e)$ -approximation algorithm. Then, based on the acquired insights, we design an approximation algorithm of factor  $\frac{1-1/e}{2L}$  for the P<sup>3</sup> problem, where  $e$  is the base of the natural logarithm, and  $L$  is the maximum power level.

We also discuss an extension of P<sup>3</sup>. When the power consumption rates of devices exhibit cyclic patterns, how do we decide a subset of the candidate locations and corresponding power levels for each time slot in a cycle? We show that solving this problem is not equivalent to solving multiple consecutive P<sup>3</sup> problems, and we propose a  $\frac{1-1/e}{2L}$ -approximation algorithm for this problem.

The contributions of this paper are three-fold:

- To the best of our knowledge, we are the first to study the joint optimization of charger placement and power allocation problem. We present a formal problem statement and prove that the problem is NP-complete.
- We develop two approximation algorithms for P<sup>3</sup> with and without fixed power levels, respectively. Evaluations confirm the effectiveness of the proposed algorithms.
- We discuss how to extend P<sup>3</sup> in a cycle and propose a  $\frac{1-1/e}{2L}$ -approximation algorithm for this problem.

The rest of the paper is organized as follows. We discuss related work in Section II. We introduce the problem in Section III. We present our solution to P<sup>3</sup> in Section IV. Before

we conclude the paper in Section VI, we evaluate our design in Section V.

## II. RELATED WORK

Kurs *et al.* experimentally demonstrated that energy can be efficiently transmitted between magnetically resonant objects without any interconnecting conductors [4]. Intel developed the wireless identification and sensing platform (WISP) for battery-free programmable monitoring [14]. Motivated by these enabling technologies, most of prior studies envisioned employing mobile vehicles equipped with wireless chargers to deliver energy to sensor nodes.

Peng *et al.* optimized the charging sequence for network lifetime maximization [7]. Li *et al.* proposed routing and charging strategies for the same objective [15]. Shi *et al.* investigated the problem of periodically charging sensors to maximize the ratio of the charger's vacation time (time spent at the home service station) over the cycle time [8, 16]. Tong *et al.* evaluated the performance of multi-node simultaneous charging [17]. To minimize the total charging delay, Fu *et al.* proposed an approx. algorithm for determining the mobile charger stop locations and the corresponding stop durations via discretizing charging power [11]. He *et al.* [9] investigated the energy provision problem of finding the minimum number of RFID readers to cover a given network. Wang *et al.* designed efficient energy monitoring and reporting protocols based on NDN-related mechanisms [18]. Zhang *et al.* leveraged collaboration between mobile chargers to optimize energy usage effectiveness [10, 19]. Dai *et al.* proposed a near optimal solution for determining the maximum electromagnetic radiation point in a given plane [12]. Different from them, our work jointly determines charger placement and power allocation to improve the charging quality, subject to a budget constraint.

## III. PROBLEM FORMULATION

### A. Network Model

We consider a set of  $M$  stationary rechargeable devices  $\mathcal{S} = \{s_1, s_2, \dots, s_M\}$  distributed in a two-dimensional plane. The preselected candidate locations for placing stationary wireless power chargers is denoted by a set  $\mathcal{C} = \{c_1, c_2, \dots, c_N\}$ . We also use  $c_i$  to denote the charger placed at a candidate location  $c_i$  if no confusion is caused. A charger placement is denoted by  $\mathcal{C}'$ , which is a subset of  $\mathcal{C}$ .

The location of a device  $s_j$  can be localized using techniques in [20] and represented as  $(x[s_j], y[s_j])$ . The location of  $c_i$  is  $(x[c_i], y[c_i])$ . The distance function  $d : (\mathcal{C} \cup \mathcal{S}, \mathcal{C} \cup \mathcal{S}) \rightarrow \mathbf{R}$  gives the Euclidean distance between two objects (chargers or devices), *e.g.*, the distance between charger  $c_i$  and device  $s_j$  is defined as

$$d(c_i, s_j) = \sqrt{(x[c_i] - x[s_j])^2 + (y[c_i] - y[s_j])^2}.$$

Fig. 1 is an illustration of some basic concepts. There are 2 candidate locations and 3 devices in the example.

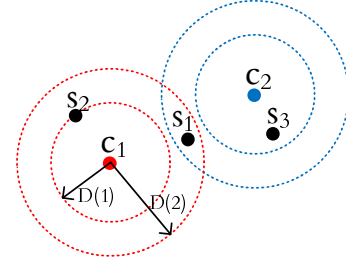


Fig. 1: Illustration of basic concepts. The maximum cover distance of a power level is indicated by the radius of a dashed circle. If we set  $\mathcal{C}' = \{c_1, c_2\}$  and  $\mathbf{H} = (2, 2)$ , we have  $p_{\mathcal{C}'}(s_1) = p(c_1, s_1) + p(c_2, s_1)$ , and  $p_{\mathcal{C}'}(s_3) = p(c_2, s_3)$ .

### B. Charging Model

We assume that the power of each charger is adjustable. Each charger can be operated at  $L$  different power levels. Denote the power of charger  $c_i$  by  $p_i$ ; without loss of generality, we define:

$$p_i = p(h_i) = h_i \cdot p_{\min}$$

where  $h_i \in \{1, 2, \dots, L\}$  is the power level of  $c_i$ , and  $p_{\min}$  is the minimum power of a charger. Note that, this kind of discretization is for simplicity; in fact, as long as the number of allowable power levels of each charger is limited, the proposed solutions are still valid. A power allocation can be denoted by a vector  $\mathbf{H} = (h_1, h_2, \dots, h_N)$ .

According to the profiling experiments in [9], the power  $p(c_i, s_j)$  received by device  $s_j$  from charger  $c_i$  can be quantified by an empirical model as follows:

$$p(c_i, s_j) = \begin{cases} \frac{\alpha}{(d(c_i, s_j) + \beta)^2} p(h_i) & d(c_i, s_j) \leq D(h_i) \\ 0 & d(c_i, s_j) > D(h_i) \end{cases} \quad (1)$$

where  $\alpha$  and  $\beta$  are known constants determined by hardware of chargers and devices and the environment, and  $D(h_i)$  is the maximum cover distance of a charger with power level  $h_i$ .

When a device is far away from a charger, the device would receive negligible power that cannot be rectified to useful electrical energy. The threshold of this negligible power is denoted by  $p_{th}$ . By letting

$$\frac{\alpha}{(D(h_i) + \beta)^2} p(h_i) = p_{th},$$

we have

$$D(h_i) = \sqrt{\frac{\alpha}{p_{th}} p(h_i)} - \beta. \quad (2)$$

That is, given constants  $\alpha$ ,  $\beta$ , and  $p_{th}$ , the maximum coverage radius of a charger  $c_i$  is determined by its power  $p_i = p(h_i)$ . In Fig. 1, when we place a charger  $c_1$  with a power level  $h_1$  being 1, its maximum coverage radius is  $D(1)$ , and thus  $c_1$  cannot transfer power to  $s_1$ , which is more than  $D(1)$  distance away from  $c_1$ .

As evidenced by [17], a charger can transfer energy to multiple devices simultaneously without significantly reducing the received power at one device.

Symbol	Meaning
$N$	the number of candidate locations
$C$	the set of candidate locations
$C'$	a charger placement, <i>i.e.</i> , a subset of $C$
$c_i$	a candidate location or a charger
$p_i$	the power of charger $c_i$
$h_i$	the power level of charger $c_i$
$p_{th}$	the threshold of negligible power
$p_{min}$	the minimum power of a charger
$L$	the maximum power level of a charger
$D(h_i)$	the maximum coverage radius with respect to $h_i$
$\mathbf{H}$	a power allocation, <i>i.e.</i> , $(h_1, h_2, \dots, h_N)$
$M$	the number of stationary devices
$\mathcal{S}$	the set of stationary devices
$s_j$	an energy consuming device
$P_j$	the maximum power consumption rate of device $s_j$
$B$	the power budget
$d(c_i, s_j)$	the distance between charger $c_i$ and device $s_j$
$p(c_i, s_j)$	the power received by device $s_j$ from charger $c_i$
$p_{C'}(s_j)$	the total power received by $s_j$ with respect to $C'$
$Q_{C'}(s_j)$	the charging quality of $C'$ on device $s_j$
$Q(C', \mathbf{H})$	the charging quality with respect to $C'$ and $\mathbf{H}$

Fig. 2: Main notations for quick reference.

We assume the power received by one device from multiple chargers is additive [9]. That is, given a charger placement  $C'$ , the total power  $p_{C'}(s_j)$  received by device  $s_j$  is

$$p_{C'}(s_j) = \sum_{c_i \in C'} p(c_i, s_j). \quad (3)$$

For example, in Fig. 1, if we set  $C' = \{c_1, c_2\}$  and  $\mathbf{H} = (2, 2)$ , we have  $p_{C'}(s_1) = p(c_1, s_1) + p(c_2, s_1)$ ,  $p_{C'}(s_2) = p(c_1, s_2)$ , and  $p_{C'}(s_3) = p(c_2, s_3)$ .

### C. Problem Definition

The maximum power consumption rate of device  $s_j$  is represented by  $P_j$ . If the total power  $p_{C'}(s_j)$  received by device  $s_j$  is larger than  $P_j$ , the over-received power, *i.e.*,  $p_{C'}(s_j) - P_j$ , would be useless. Therefore, we define the charging quality  $Q_{C'}(s_j)$  of  $C'$  on device  $s_j$  as:

$$Q_{C'}(s_j) = \min\{p_{C'}(s_j), P_j\}. \quad (4)$$

Main notations are summarized in Fig. 2 for quick reference.

We define our objective function as follows:

**Definition 1: (Charging Quality)** Given a charger placement  $C'$  and a power allocation  $\mathbf{H}$ , the charging quality, denoted as  $Q(C', \mathbf{H})$ , is defined as the sum of the charging qualities of  $C'$  and  $\mathbf{H}$  on all devices, *i.e.*,  $Q(C', \mathbf{H}) = \sum_{j=1}^M Q_{C'}(s_j)$

The main problem studied in this paper is:

**Problem 1: (Charger Placement and Power Allocation Problem,  $P^3$ )** Given a set  $C$  of candidate locations, a set  $\mathcal{S}$  of devices, and a power budget  $B$ ,  $P^3$  is to find a charger placement  $C'$  and a power allocation  $\mathbf{H}$  to maximize  $Q(C', \mathbf{H})$ , subject to the power budget constraint, *i.e.*,  $\sum_{c_i \in C'} p_i \leq B$ .

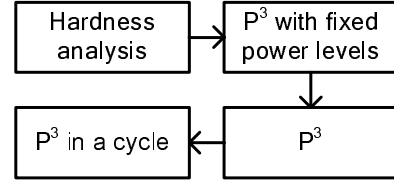


Fig. 3: Flowchart of our solution.

## IV. SOLUTION TO $P^3$

Fig. 3 shows the flowchart of our solution. In this section, we first show that  $P^3$  is NP-complete, then we propose an approximation algorithm for a simplified case of the  $P^3$  problem, where the power level of each candidate location is fixed, and finally we present a  $\frac{1-1/e}{2L}$ -approximation algorithm for the  $P^3$  problem. We also discuss an extension of  $P^3$  and propose a provably good solution to it.

### A. Hardness Analysis

**Theorem 1:** The  $P^3$  problem is NP-complete.

*Proof:* We prove this theorem by reduction from the Set Cover problem (SC) [13], which is NP-complete. The decision version of the SC problem is as follows: Given a universe  $\mathcal{U} = \{e_1, e_2, \dots, e_m\}$  of  $m$  elements and an integer  $y$ , a collection of subsets of  $\mathcal{U}$ ,  $\mathcal{R} = \{\mathcal{R}_1, \mathcal{R}_2, \dots, \mathcal{R}_k\}$ , does there exist a sub-collection of  $\mathcal{R}$  of size  $y$  that covers all elements of  $\mathcal{U}$ ?

Given an instance of the decision version of the SC problem, we construct an instance of the  $P^3$  problem as follows. We let  $L = 1$ , *i.e.*, every charger can only operated at the fixed power  $p_{min}$ . For each element  $e_j$  in  $\mathcal{U}$ , we construct a device  $s_j$  in  $P^3$ . We assume that all devices have the same maximum power consumption rate, *i.e.*,  $P_1 = P_2 = \dots = P_m = P$ . For each  $\mathcal{R}_i \in \mathcal{R}$ , we add a candidate location  $c_i$  to  $P^3$ . For each element  $e_j$  in  $\mathcal{R}_i$ , we move  $s_j$  into the coverage of  $c_i$ . We also make sure that, as long as a device  $s_j$  is within the coverage of a location  $c_i$ ,  $p(c_i, s_j) \geq P$ ; this can be achieved by setting  $p_{min}$  to a sufficiently large value.

Combining these together, we get the following special case of the decision version of the  $P^3$  problem: Given a candidate location set  $C$  of size  $k$ , and a device set  $\mathcal{S}$  of size  $m$ , does there exist a charger placement  $C'$  of size  $\lfloor \frac{B}{p_{min}} \rfloor$ , such that  $Q(C', (1, 1, \dots, 1)) \geq mP$ ?

It is not hard to see that the construction can be finished in polynomial time; thus, we reduce solving the NP-complete SC problem to solving a special case of the  $P^3$  problem, implying that  $P^3$  is NP-hard. It is easy to verify that  $P^3$  is in NP; the theorem follows immediately. ■

### B. Approximation Algorithm for $P^3$ with Fixed Power Levels

In this subsection, we study the  $P^3$  problem where the charger at each location can only work at a fixed power level, *i.e.*,  $h_i$  is constant for all candidate locations. The approximation algorithms designed here will serve as basics of the algorithm for  $P^3$  proposed in the next subsection.

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**Algorithm 1** The Greedy Placement Algorithm (GPA)

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**Input:**  $C, S, B$ , and  $p$ 
**Output:**  $C'$ 

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1:  $C' \leftarrow \emptyset$ 
2: while  $|C'| < \lfloor \frac{B}{p} \rfloor$  do
3:    $c \leftarrow \arg \max_{c \in C \setminus C'} (Q(C' \cup \{c\}) - Q(C'))$ 
4:    $C' \leftarrow C' \cup \{c\}$ 
5: end while
6: return  $C'$ 

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1) *Uniform Case:* We first look at the uniform case of power levels, *i.e.*,  $h_1 = h_2 = \dots = h_N = h$ . In other words,  $p_1 = p_2 = \dots = p_N = p(h) = h \cdot p_{\min}$ . For convenience, denote the power of each charger by  $p$  in this subsection.

In this case, the objective function  $Q(C', \mathbf{H})$  degenerates into  $Q(C')$ , and the  $P^3$  problem can be reformulated as follows: Given a set  $C$  of candidate locations, a set  $S$  of devices, a power budget  $B$ , and a fixed power  $p$ ,  $P^3$  is to find a charger placement  $C'$  to maximize  $Q(C')$ , subject to the budget constraint, *i.e.*,  $|C'| \leq \lfloor \frac{B}{p} \rfloor$ .

In the following, we prove that the objective function  $Q(C')$  has three tractable properties: nonnegativity, monotonicity, and submodularity, which enable us to propose a  $(1 - 1/e)$ -approximation algorithm shown in Alg. 1.

**Definition 2: (Nonnegativity, Monotonicity, and Submodularity)** Given a non-empty finite set  $\mathcal{U}$ , and a function  $f$  defined on the power set  $2^{\mathcal{U}}$  of  $\mathcal{U}$  with real values,  $f$  is called *nonnegative* if  $f(\mathcal{A}) \geq 0$  for all  $\mathcal{A} \subseteq \mathcal{U}$ ;  $f$  is called *monotone* if  $f(\mathcal{A}) \leq f(\mathcal{A}')$  for all  $\mathcal{A} \subseteq \mathcal{A}' \subseteq \mathcal{U}$ ;  $f$  is called *submodular* if  $f(\mathcal{A} \cup \{e\}) - f(\mathcal{A}) \geq f(\mathcal{A}' \cup \{e\}) - f(\mathcal{A}')$  for all  $\mathcal{A} \subseteq \mathcal{A}' \subseteq \mathcal{U}$ .

We have the following theorem:

**Theorem 2:** The objective function  $Q(C')$  is nonnegative, monotone, and submodular.

*Proof:* According to Def. 1,  $Q(C')$  is nonnegative. For all  $C' \subseteq C'' \subseteq C$ , we have

$$Q(C') = \sum_{j=1}^M Q_{C'}(s_j) \leq \sum_{j=1}^M Q_{C''}(s_j) = Q(C''),$$

implying that,  $Q(C')$  is monotone. For all  $C' \subseteq C'' \subseteq C$ , we need to prove  $Q(C' \cup \{c\}) - Q(C') \geq Q(C'' \cup \{c\}) - Q(C'')$ . It is sufficient to show that for any  $s_j \in S$ , we have

$$Q_{(C' \cup \{c\})}(s_j) - Q_{C'}(s_j) \geq Q_{(C'' \cup \{c\})}(s_j) - Q_{C''}(s_j).$$

Based on Equ. (3) and Equ. (4), we prove the above inequation in three cases:

- $P_j \leq P_{C'}(s_j)$ : since  $P_{C''}(s_j) \geq P_{C'}(s_j)$ , we have

$$Q_{(C' \cup \{c\})}(s_j) - Q_{C'}(s_j) = 0 = Q_{(C'' \cup \{c\})}(s_j) - Q_{C''}(s_j).$$

- $P_{C'}(s_j) < P_j < P_{C''}(s_j)$ : in this case,  $Q_{(C' \cup \{c\})}(s_j) - Q_{C'}(s_j) = 0$ , and we have

$$\begin{aligned} & Q_{(C' \cup \{c\})}(s_j) - Q_{C'}(s_j) \\ &= \min\{P_j - P_{C'}(s_j), P_{(C' \cup \{c\})}(s_j) - P_{C'}(s_j)\} \\ &= \min\{P_j - P_{C'}(s_j), p(c, s_j)\} \\ &\geq 0 = Q_{(C'' \cup \{c\})}(s_j) - Q_{C''}(s_j). \end{aligned}$$

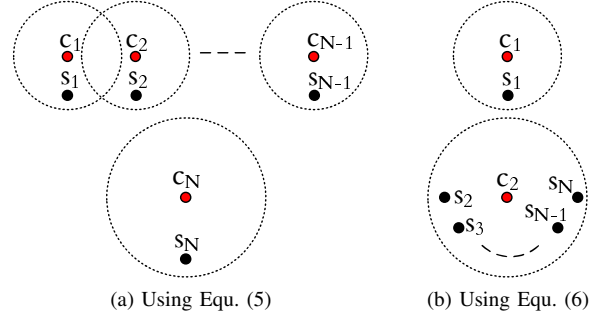


Fig. 4: Motivational examples show that, directly applying Equ. (5) or Equ. (6) to the non-uniform case may perform very bad.

- $P_{C''}(s_j) \leq P_j$ : in this case, we have

$$\begin{aligned} Q_{(C' \cup \{c\})}(s_j) - Q_{C'}(s_j) &= \min\{P_j - P_{C'}(s_j), p(c, s_j)\}, \\ Q_{(C'' \cup \{c\})}(s_j) - Q_{C''}(s_j) &= \min\{P_j - P_{C''}(s_j), p(c, s_j)\}. \end{aligned}$$

If  $p(c, s_j) \leq P_j - P_{C''}(s_j)$ , then

$$Q_{(C' \cup \{c\})}(s_j) - Q_{C'}(s_j) = p(c, s_j) = Q_{(C'' \cup \{c\})}(s_j) - Q_{C''}(s_j).$$

If  $P_j - P_{C''}(s_j) < p(c, s_j) < P_j - P_{C'}(s_j)$ , then

$$\begin{aligned} Q_{(C' \cup \{c\})}(s_j) - Q_{C'}(s_j) &= p(c, s_j) \\ &> P_j - P_{C''}(s_j) = Q_{(C'' \cup \{c\})}(s_j) - Q_{C''}(s_j). \end{aligned}$$

If  $P_j - P_{C'}(s_j) \leq p(c, s_j)$ , then

$$\begin{aligned} Q_{(C' \cup \{c\})}(s_j) - Q_{C'}(s_j) &= P_j - P_{C'}(s_j) \\ &\geq P_j - P_{C''}(s_j) = Q_{(C'' \cup \{c\})}(s_j) - Q_{C''}(s_j). \end{aligned}$$

Therefore, we proved that  $Q(C')$  is submodular.  $\blacksquare$

According to the results in [21, 22], we have a  $(1 - 1/e)$ -approx. algorithm shown in Alg. 1, which starts with an empty set  $C'$ , in each iteration, we add the location that maximizes the marginal gain of the objective function into  $C'$ , *i.e.*,

$$c \leftarrow \arg \max_{c \in C \setminus C'} (Q(C' \cup \{c\}) - Q(C')). \quad (5)$$

There are at most  $N$  iterations in Alg. 1; in each iteration, we need to check at most  $N$  locations to find the location that maximizes the marginal gain. It takes  $O(MN)$  time to compute  $Q(C')$ , thus, the time complexity of Alg. 1 is  $O(MN^3)$ .

2) *Non-uniform Case:* We then study the non-uniform case of power levels, in this case,  $P^3$  can be reformulated as follows: Given a set  $C$  of candidate locations, a set  $S$  of devices, a power budget  $B$ , and a power allocation  $\mathbf{H} = (h_1, h_2, \dots, h_N)$ ,  $P^3$  is to find a charger placement  $C'$  to maximize  $Q(C')$ , subject to the budget constraint, *i.e.*,  $\sum_{c_j \in C'} p_i \leq B$ .

To solve this problem, an intuitive method is using the same greedy idea as in Equ. (5). However, we show in Fig. 4(a) that, this method may perform very bad. In Fig. 4(a), there are  $N = L + 1$  candidate locations and  $M = N$  devices;  $h_1 = h_2 = \dots = h_{N-1} = 1$ , and  $h_N = L$ ; the radii of dashed circles indicate the maximum coverage distance of each charger;  $p(c_1, s_1) = p(c_2, s_2) = \dots = p(c_{N-1}, s_{N-1}) = p(c_N, s_N) - \epsilon$ , where  $\epsilon$  satisfies  $0 < \epsilon < p(c_N, s_N)$ . Given a power budget  $B = L \cdot p_{\min}$ , using

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**Algorithm 2** Approx. Alg. for  $P^3$  with Known Power Levels

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**Input:**  $C, S, B$ , and  $\mathbf{H}$ **Output:**  $C'$ 

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1:  $C_1 \leftarrow \emptyset, C_2 \leftarrow \emptyset$ 
2: while  $B \geq \sum_{c_i \in C_1} p_i + \min_{c_i \in C \setminus C_1} p_i$  do
3:    $c \leftarrow \arg \max_{c \in C \setminus C_1, \sum_{c_i \in C_1 \cup \{c\}} p_i \leq B} (Q(C_1 \cup \{c\}) - Q(C_1))$ 
4:    $C_1 \leftarrow C_1 \cup \{c\}$ 
5: end while
6: while  $B \geq \sum_{c_i \in C_2} p_i + \min_{c_i \in C \setminus C_2} p_i$  do
7:    $c_x \leftarrow \arg \max_{c_x \in C \setminus C_2, \sum_{c_i \in C_2 \cup \{c_x\}} p_i \leq B} \frac{Q(C_2 \cup \{c_x\}) - Q(C_2)}{p_x}$ 
8:    $C_2 \leftarrow C_2 \cup \{c_x\}$ 
9: end while
10: return  $\arg \max_{C' \in \{C_1, C_2\}} Q(C')$ 
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Equ. (5), the charger  $c_N$  would be picked. However, if we pick  $c_1, c_2, \dots$ , and  $c_{N-1}$ , the charging quality would be  $L \cdot p(c_1, s_1)$  instead of  $p(c_1, s_1) + \epsilon$ . When  $\epsilon$  is approaching zero, using Equ. (5) would generate approximately  $1/L$  of the charging quality returned by the optimal solution.

Another intuitive method is that, in each iteration, we pick the location that maximize the marginal ratio of objective gain to power cost, *i.e.*,

$$c_x \leftarrow \arg \max_{c_x \in C \setminus C', \sum_{c_i \in C' \cup \{c_x\}} p_i \leq B} \frac{Q(C' \cup \{c_x\}) - Q(C')}{p_x}. \quad (6)$$

However, we show in Fig. 4(b) that, this method may also perform badly. In Fig. 4(b), there are 2 candidate locations and  $M = L + 1$  devices;  $h_1 = 1$ , and  $h_2 = L$ ;  $p(c_1, s_1) = p(c_2, s_2) = p(c_2, s_3) \dots = p(c_2, s_{M-1}) = p(c_2, s_M) + \epsilon$ , where  $\epsilon$  satisfies  $0 < \epsilon < p(c_1, s_1)$ . Given a power budget  $B = L \cdot p_{\min}$ , using Equ. (6), the charger  $c_1$  would be picked. However, if we pick  $c_2$ , the charging quality would be  $(L - 1) \cdot p(c_1, s_1) + p(c_2, s_M)$  instead of  $p(c_1, s_1)$ . When  $\epsilon$  is approaching zero, this method would also generate approximately  $1/L$  of the charging quality returned by the optimal solution.

Surprisingly enough, if we simultaneously apply the above two methods to the non-uniform case of  $P^3$  with fixed power levels, and return the better one of the two results, we will get an approx. algorithm (Alg. 2) of factor  $\frac{1}{2}(1 - \frac{1}{e})$  according to [23]. The time complexity of Alg. 2 is also  $O(MN^3)$ .

### C. Approximation Algorithm for $P^3$

We design a probably good approximation algorithm, namely TCA, in Alg. 3 that simultaneously determines  $C'$  and  $\mathbf{H}$ .

Denote the set  $\{1, 2, \dots, L\}$  by  $\mathcal{H}$ ; denote by  $\mathbf{I}_i$  a row vector where the  $i$ -th element is 1 and all of the other elements are zeros, *i.e.*,  $\mathbf{I}_i = (0, 0, \dots, 1, \dots, 0)$ . We use  $\mathbf{H}[c_i]$  to represent the power level of a candidate location  $c_i$ .

Before giving some explanations about TCA, we first consider the following variant of the  $P^3$  problem ( $VP^3$ ): *For each candidate location  $c_i$ , we are given  $L$  chargers with constant but exactly different power levels, *i.e.*, the power levels of these*

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**Algorithm 3** Two-Choice-based Approx. Alg. for  $P^3$  (TCA)

---

**Input:**  $C, S$ , and  $B$ **Output:**  $C'$  and  $\mathbf{H}$ 

```
1:  $\mathcal{Z} \leftarrow C \times \mathcal{H}$ 
2:  $\mathcal{Z}_1 \leftarrow \emptyset, \mathbf{H}_1 \leftarrow \mathbf{0}$ 
3: while  $B \geq \min_{(c_i, h_k) \in \mathcal{Z} \setminus \mathcal{Z}_1} p(h_k) + \sum_{(c_i, h_k) \in \mathcal{Z}_1} p(h_k)$  do
4:    $(c, h) \leftarrow \arg \max_{(c, h) \in \mathcal{Z} \setminus \mathcal{Z}_1, \sum_{(c_i, h_k) \in \mathcal{Z}_1 \cup \{(c, h)\}} p(h_k) \leq B} (Q(\mathcal{Z}_1 \cup \{(c, h)\}) - Q(\mathcal{Z}_1))$ 
5:    $\mathcal{Z}_1 \leftarrow \mathcal{Z}_1 \cup \{(c, h)\}$ 
6:   if  $\mathbf{H}_1[c] < h$  then  $\mathbf{H}_1[c] \leftarrow h$ 
7: end while
8:  $(C'_1, \mathbf{H}_1) \leftarrow \text{RemoveDuplicationAndUtilize}(C, S, B, \mathbf{H}_1)$ 
9:  $\mathcal{Z}_2 \leftarrow \emptyset, \mathbf{H}_2 \leftarrow \mathbf{0}$ 
10: while  $B \geq \min_{(c_i, h_k) \in \mathcal{Z}_2} p(h_k) + \sum_{(c_i, h_k) \in \mathcal{Z}_2} p(h_k)$  do
11:    $(c, h) \leftarrow \arg \max_{(c, h) \in \mathcal{Z} \setminus \mathcal{Z}_2, \sum_{(c_i, h_k) \in \mathcal{Z}_2 \cup \{(c, h)\}} p(h_k) \leq B} \frac{Q(\mathcal{Z}_2 \cup \{(c, h)\}) - Q(\mathcal{Z}_2)}{p(h)}$ 
12:    $\mathcal{Z}_2 \leftarrow \mathcal{Z}_2 \cup \{(c, h)\}$ 
13:   if  $\mathbf{H}_2[c] < h$  then  $\mathbf{H}_2[c] \leftarrow h$ 
14: end while
15:  $(C'_2, \mathbf{H}_2) \leftarrow \text{RemoveDuplicationAndUtilize}(C, S, B, \mathbf{H}_2)$ 
16: return  $\arg \max_{(C', \mathbf{H}) \in ((C'_1, \mathbf{H}_1), (C'_2, \mathbf{H}_2))} Q(C', \mathbf{H})$ 
17:
18: Sub-procedure: RemoveDuplicationAndUtilize
19: Input:  $C, S, B$ , and  $\mathbf{H}$ 
20: Output:  $C'$  and  $\mathbf{H}$ 
21:  $C' \leftarrow \emptyset, B' \leftarrow 0$ 
22: for all  $\mathbf{H}[c_i] > 0$  do
23:    $C' \leftarrow C' \cup \{c_i\}, B' \leftarrow B' + p(\mathbf{H}[c_i])$ 
24: end for
25: while  $B - B' \geq p_{\min}$  do
26:    $c_i \leftarrow \arg \max_{\mathbf{H}[c_i] + 1 \leq L} Q(C' \cup \{c_i\}, \mathbf{H} + \mathbf{I}_i) - Q(C', \mathbf{H})$ 
27:    $C' \leftarrow C' \cup \{c_i\}, \mathbf{H}[c_i] \leftarrow \mathbf{H}[c_i] + 1, B' \leftarrow B' + p_{\min}$ 
28: end while
29: return  $(C', \mathbf{H})$ 
```

---

*chargers are 1, 2, ..., and L.* We use  $(c_i, h_k)$  to denote the charger of a constant power level  $h_k$  that can only be placed at  $c_i$ . Note that there are, in total,  $N \cdot L$  chargers. Given a power budget  $B$ , how do we find a charger placement that maximize the objective function defined below?

Let  $\mathcal{Z}$  be the Cartesian product of  $C$  and  $\mathcal{H}$ ; denote by  $\mathcal{Z}'$  a subset of  $\mathcal{Z}$ . We redefine several functions for the  $VP^3$  problem via overloading. The power  $p(c_i, s_j, h_k)$  received by device  $s_j$  from  $c_i$  (its power level is  $h_k$ ) is

$$p(c_i, s_j, h_k) = \begin{cases} \frac{\alpha}{(d(c_i, s_j) + \beta)^2} p(h_k) & d(c_i, s_j) \leq D(h_k), \\ 0 & d(c_i, s_j) > D(h_k). \end{cases} \quad (7)$$

Correspondingly, given a charger placement  $\mathcal{Z}'$ , the total power  $p_{\mathcal{Z}'}(s_j)$  received by device  $s_j$  is  $p_{\mathcal{Z}'}(s_j) = \sum_{(c_i, h_k) \in \mathcal{Z}'} p(c_i, s_j, h_k)$ . The charging quality of  $\mathcal{Z}'$  on  $s_j$  is

$$Q_{\mathcal{Z}'}(s_j) = \min\{p_{\mathcal{Z}'}(s_j), P_j\}. \quad (8)$$

The objective function is

$$Q(\mathcal{Z}') = \sum_{j=1}^M Q_{\mathcal{Z}'}(s_j).$$

The main idea of TCA is as follows. We use the two greedy heuristics (*i.e.*, Equ. (5) and Equ. (6)) to solve the VP<sup>3</sup> problem defined above, and get two results  $\mathcal{Z}_1$  and  $\mathcal{Z}_2$ , respectively. Note that, there may be more than one charger placed at a candidate location in  $\mathcal{Z}_1$  or  $\mathcal{Z}_2$ . We then invoke the RemoveDuplicationAndUtilize sub-procedure to transform  $\mathcal{Z}_1$  and  $\mathcal{Z}_2$  into two solutions to P<sup>3</sup>, respectively.

The transformation works as follows (take  $\mathcal{Z}_1$  for example): For each location  $c_i$ , we set  $\mathbf{H}_1[c_i]$  to be the maximum value of  $h_k$  among all  $(c_i, h_k) \in \mathcal{Z}_1$ , *i.e.*,  $\mathbf{H}_1[c_i] = \max_{(c_i, h_k) \in \mathcal{Z}_1} h_k$  (line 6 of Alg. 3). For each location  $c_i$ , if  $\mathbf{H}_1[c_i] > 0$ , we add it into  $C'_1$ , which is equivalent to  $C'_1 = \{c_i | (c_i, h_k) \in \mathcal{Z}_1\}$  (lines 22-24). As Alg. 3 shows, we retain only one charger for each location in transforming  $\mathcal{Z}_1$  into  $C'_1$  and  $\mathbf{H}_1$ , which implies that there may be some unused budget for  $C'_1$  and  $\mathbf{H}_1$ . We thus try to improve  $C'_1$  and  $\mathbf{H}_1$  by utilizing the remaining budget  $(B - B'_1)$  (lines 25-28). In each iteration, we allocate a fixed power  $p_{\min}$  to the location that maximizes the marginal objective gain<sup>1</sup>.

We choose the better one of  $(C'_1, \mathbf{H}_1)$  and  $(C'_2, \mathbf{H}_2)$  as the final solution to P<sup>3</sup>. The time complexity of TCA is  $O(MN^3L^3)$ . The following theorem gives the bound on the worst case performance of TCA. Later, we will see in the simulations that, the gap between TCA and the optimal solution is 4.5% at most and 2.0% on average.

**Theorem 3:** TCA is a factor  $\frac{1-1/e}{2L}$  approx. algorithm for P<sup>3</sup>.

*Proof:* Denote by  $(C^*, \mathbf{H}^*)$  the optimal solution to P<sup>3</sup>, and by  $(C', \mathbf{H})$  the solution returned by TCA. We want to prove that,

$$\frac{Q(C', \mathbf{H})}{Q(C^*, \mathbf{H}^*)} \geq \frac{1-1/e}{2L}.$$

Let  $\mathcal{Z}^*$  be the optimal solution to VP<sup>3</sup>, and let

$$\mathcal{Z}' = \arg \max_{\mathcal{Z}' \in \{\mathcal{Z}_1, \mathcal{Z}_2\}} Q(\mathcal{Z}')$$

where  $\mathcal{Z}_1$  and  $\mathcal{Z}_2$  are the solution generated by lines 3-7 and 9-14 of TCA, respectively. According to the results in Section IV-B, we know

$$Q(\mathcal{Z}') \geq \frac{1-1/e}{2} Q(\mathcal{Z}^*).$$

Notice that, if we restrict the number of chargers that can be placed at each candidate location to one, the VP<sup>3</sup> problem is equivalent to P<sup>3</sup>. From this viewpoint, we have

$$Q(\mathcal{Z}^*) \geq Q(C^*, \mathbf{H}^*).$$

In  $\mathcal{Z}_1$  (resp.  $\mathcal{Z}_2$ ), we can place at most  $L$  chargers for one location. When transforming  $\mathcal{Z}_1$  (resp.  $\mathcal{Z}_2$ ) into  $C'_1$  and  $\mathbf{H}_1$

<sup>1</sup>Note that, we can further improve lines 25-28 of TCA by applying both Equ. (5) and Equ. (6) and choosing the better one. However, the improvement would be little. We choose the current form of TCA for brevity.

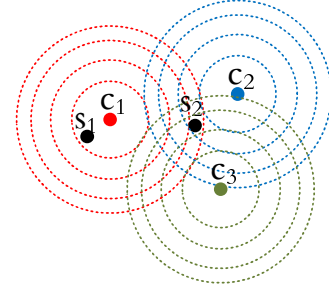


Fig. 5: There are 3 candidate locations and 2 devices. The radii of dashed circles show the maximum cover distances of four different power levels.

(resp.  $C'_2$  and  $\mathbf{H}_2$ ), for each location, we retain the charger that has the largest power level, if any. Therefore, we have

$$Q(C'_1, \mathbf{H}_1) \geq \frac{Q(\mathcal{Z}_1)}{L}, \quad \text{and} \quad Q(C'_2, \mathbf{H}_2) \geq \frac{Q(\mathcal{Z}_2)}{L}.$$

Combining them together, we have

$$\begin{aligned} Q(C', \mathbf{H}) &= \max\{Q(C'_1, \mathbf{H}_1), Q(C'_2, \mathbf{H}_2)\} \geq \frac{\max\{Q(\mathcal{Z}_1), Q(\mathcal{Z}_2)\}}{L} \\ &= \frac{Q(\mathcal{Z}')}{L} \geq \frac{1-1/e}{2L} Q(\mathcal{Z}^*) \geq \frac{1-1/e}{2L} Q(C^*, \mathbf{H}^*). \end{aligned}$$

#### D. A Concrete Example

In this subsection, we provide an example that helps readers better understand TCA. There are 3 candidate locations and 2 devices in the plane, as shown in Fig. 5. Following existing works [9, 11, 12], we assume that,  $p_{\min} = 50$ ,  $L = 4$ ,  $p_{th} = 0.01$ ,  $\alpha = 0.64$ , and  $\beta = 30$ . Given these parameters, we have  $D(1) \approx 27$ ,  $D(2) = 50$ ,  $D(3) \approx 68$ , and  $D(4) \approx 83$ . The radii of dashed circles show the maximum cover distances of four different power levels. We also know that,  $d(c_1, s_1) = 20$ ,  $d(c_1, s_2) = 70$ ,  $d(c_2, s_2) = 40$ , and  $d(c_3, s_2) = 60$ . The maximum power consumption rate of  $s_1$  or  $s_2$  is 0.07, *i.e.*,  $P_1 = P_2 = 0.07$ . Given a power budget  $B = 500$ , we now check how TCA computes the result.

In lines 2-7 of Alg. 3, we first check which  $(c, h)$  gives the maximal marginal objective gain. For  $c_1$ , we have  $Q(\{(c_1, 1)\}) = 0.0128$ ,  $Q(\{(c_1, 2)\}) = 0.0256$ ,  $Q(\{(c_1, 3)\}) = 0.0384$ , and  $Q(\{(c_1, 4)\}) = 0.064$ ; for  $c_2$  and  $c_3$ , we can compute these values in a similar way. Thus, in the first iteration of lines 2-8, we add  $(c_1, 4)$  into  $\mathcal{Z}_1$ . It is worth noting that, in the second iteration,  $Q(\{(c_1, 4)\} \cup \{(c_1, 2)\}) - Q(\{(c_1, 4)\}) = \min\{p(c_1, s_1, 2), P_1 - p(c_1, s_1, 4)\} = 0.0188$  (see Equ. (7) and Equ. (8)). The reader can check that,  $\mathcal{Z}_1$  would be  $\{(c_1, 4), (c_2, 4), (c_1, 2)\}$ .

Note that, for each location, we retain only one charger which has the largest power level in  $\mathcal{Z}_1$ . Then we get  $C'_1 = \{c_1, c_2\}$  and  $\mathbf{H}_1 = (4, 4)$ . In lines 25-28, we try to utilize the remaining budget  $500 - (4 + 4) \times 50 = 100$ . We find that, the distance between  $c_3$  and  $s_2$  is larger than  $D(2)$ , thus, there is no need to allocate the remaining 100 units of power to  $c_3$ . And we have  $C'_1 = \{c_1, c_2\}$  and  $\mathbf{H}_1 = (4, 4)$ .



Similarly, after running lines 9-14, we would have  $\mathcal{Z}_2 = \{(c_1, 4), (c_1, 1), (c_2, 2), (c_2, 3)\}$ . Through removing duplications, we have  $\mathcal{C}'_2 = \{c_1, c_2\}$  and  $\mathbf{H}_2 = (4, 3)$ . After utilizing remaining budget, we also have  $\mathcal{C}'_2 = \{c_1, c_2\}$  and  $\mathbf{H}_2 = (4, 4)$ .

The final solution is  $\mathcal{C}' = \{c_1, c_2\}$  and  $\mathbf{H} = (4, 4)$ , yielding an objective of  $Q(\mathcal{C}', \mathbf{H}) = 0.0902$ .

### E. $P^3$ in A Cycle

In this subsection, we provide an extension of  $P^3$  from the time perspective.

Suppose the power consumption rates of devices exhibit cyclic patterns, and a cycle contains  $T$  time slots. We assume that the battery of each device is large enough to store  $(T \cdot \max_{s_j \in \mathcal{S}} P_j)$  energy. In other words, if a device receives  $(T \cdot \max_{s_j \in \mathcal{S}} P_j)$  energy in the 1st time slot of a cycle, it could sustain its operations over the next  $T-1$  time slots. We extend  $P^3$  to  $P^3(T)$ : Given a power budget  $T \cdot B$ , how do we find a charger placement  $\mathcal{C}'_t$  and a power allocation  $\mathbf{H}_t$  for each time slot  $t \in \{1, 2, \dots, T\}$  to maximize the total charging quality?

It is easy to see that,  $P^3(T)$  becomes  $P^3$  when  $T = 1$ . However, solving  $P^3(T)$  is not equivalent to solving  $T$  consecutive  $P^3$  problems. The following example shows that solving  $T$  consecutive  $P^3$  problems may perform badly for  $P^3(T)$ .

Assume that there are  $T$  locations and  $T$  devices. The maximum power consumption rate of  $s_T$  is larger than that of any other device, *i.e.*,  $P_1 = P_2 = \dots = P_{T-1} < P_T$ . For  $1 \leq i \leq T$ ,  $d(c_i, s_i) \leq D(1)$ ; for any  $i \neq j$ ,  $d(c_i, s_j) > D(1)$ . We further assume that, for  $1 \leq i \leq T$ ,  $p(c_i, s_i) \geq T \cdot P_i$ , even when the power level of  $c_i$  is 1.

Given a power budget  $T \cdot p_{\min}$ , if we run TCA with budget  $p_{\min}$  in each time slot independently, since we can only place only one charger with a power of  $p_{\min}$ , and  $P_T > P_1 = P_2 = \dots = P_{T-1}$ , in each time slot TCA would allocate  $p_{\min}$  to  $c_T$ . And the final charging quality is  $T \cdot P_T$ . However, the optimal solution is to allocate  $p_{\min}$  to  $c_{(T+1-i)}$  in the  $i$ -th time slot, leading to a total charging quality of  $\frac{T(T-1)}{2} P_1 + T \cdot P_T$ . When  $P_T$  approaches  $P_1$ , the optimal solution is  $O(T)$  times as large as the result generated by solving  $T$  consecutive  $P^3$  problems.

We propose to solve  $P^3(T)$  in a similar way as TCA. We first solve a variant of this problem, and then transform the solution to the variant into the solution to  $P^3(T)$ . The variant problem is: For each candidate location *in the  $k$ -th time slot*, we are given  $L$  chargers with constant but exactly different power levels. We use  $(c_i, h_i, t_k)$  to denote the charger of a constant power level  $h_i$  that can only be placed at  $c_i$  in the  $k$ -th time slot. Note that there are, in total,  $T \cdot N \cdot L$  chargers. Given a power budget  $T \cdot B$ , how do we find a charger placement that maximize the total charging quality? The solution transformation process is similar to that in Alg. 3. We can also prove that this algorithm is a factor  $\frac{1-1/e}{2L}$  approx. algorithm for  $P^3(T)$ . We omit the details due to space limitations.

As a final note, we are also thinking about offering wireless charging services in some scenarios without any electric wires, *e.g.*, disaster areas (no infrastructure), and temporal conferences. A natural method to cope with this no/low-infrastructure challenge is relying on collaboration, *i.e.*, charg-

ers form an ad-hoc network in which they deliver energy amongst themselves.

## V. PERFORMANCE EVALUATION

In this section, we conduct extensive simulations to evaluate our design under different network settings and reveal insights of the proposed design performance.

### A. Baseline Setup

Since currently there is no algorithm available for the  $P^3$  problem, we introduce three algorithms for comparison.

*Optimal Algorithm (OPT)*:  $P^3$  in general is NP-complete. We simply use brute force to find the optimal solution. Due to its high time complexity  $O(ML^N)$ , it is only practical for small instances of the  $P^3$  problem.

*Fixed Level Algorithm (FLA)*: It consists of two phases. The first phase is to compute the optimal power level of each location irrespective of the other locations, *i.e.*, we set

$$h_i = \arg \max_{h_i \in \{1, \dots, L\}} \frac{\sum_{j=1}^M Q_{\{c_i\}}(s_j)}{p(h_i)}.$$

The second phase is to invoke Alg. 2 to generate a charger placement for  $P^3$  with fixed power levels.

*Random Algorithm (RAN)*: It should generate each possible solution with the same probability. We implement it as follows. Let  $b = 0$  at the beginning. In the  $i$ -th iteration, we uniformly generate a random integer  $x_i$  in the range  $[1, L]$ , let  $b = b + x_i$ ; if  $(B/p_{\min} - b)$  is no less than  $L$ , go to the next iteration, else let  $x_{i+1}$  be  $(B/p_{\min} - b)$ . Suppose the index of the last iteration is  $k$ , for  $k+1 \leq j \leq N$ , we let  $x_j$  be 0. We then sort  $x_1, x_2, \dots, x_N$  randomly to get a random permutation  $(x'_1, \dots, x'_N)$ . For each  $c_i$ , we set  $\mathbf{H}[c_i] = x'_i$  in the random solution.

### B. Experiment Setup

We assume wireless devices and candidate locations are randomly distributed over a  $1,000m \times 1,000m$  two-dimensional square area. The default number of candidate locations is 20. The minimum power  $p_{\min}$  of a charger is 50. By default, the maximum power level  $L$  of a charger is 6. The default number of devices is 200. Following prior works [9, 11, 12], we set  $\alpha = 0.64$  and  $\beta = 30$  in the charging model (Equ. (1)). The threshold  $p_{th}$  of negligible power is 0.01. Therefore, the minimum coverage radius is  $D(1) = \sqrt{0.64 \times 50/0.01} - 30 \approx 27m$  (see Equ. (2)), and by default the maximum coverage radius is  $D(6) = \sqrt{0.64 \times 300/0.01} - 30 \approx 109m$ . The maximum power consumption rate of each device is uniformly generated between 0.02 and 0.03. The default power budget is 3,000.

### C. Performance Comparison

As we have mentioned, it is impractical to run OPT using brute force in general, we thus use the following setting to generate some small instances for comparing TCA with OPT:  $N = 8$ ,  $M = 50$ ,  $B = 800$ ,  $L = 4$ , and the side length of the 2-D plane is 300m. Fig. 6 shows the results of different experiment setups of small instances. We ran each different setup ten times and averaged the results. The max and min values among ten runs are also provided in the figures.

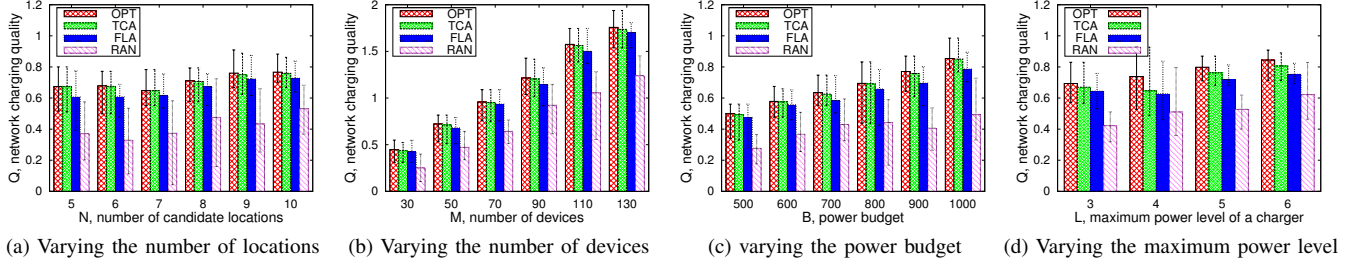


Fig. 6: Evaluation results on small instances (the default setting is  $N = 8$ ,  $M = 50$ ,  $B = 800$ ,  $L = 4$ , and the side length of the 2-D plane is 300m).

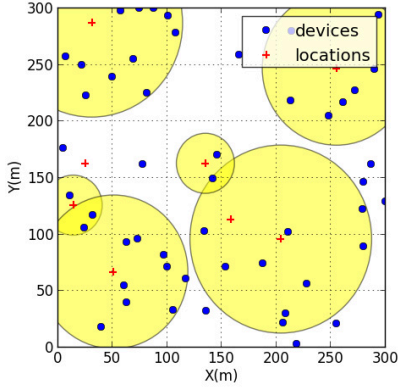


Fig. 7: A small instance visualization example

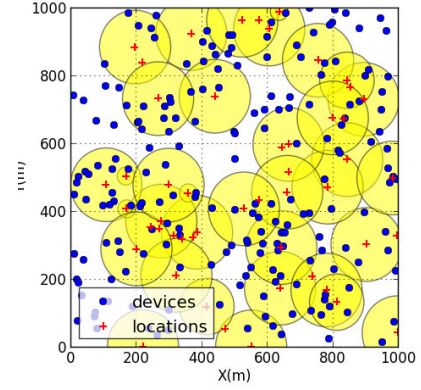


Fig. 8: A large instance visualization example

In general, TCA achieves a near optimal solution and outperforms the other algorithms. Specifically, the gap between TCA and OPT is 4.5% at most and 2.0% on average. This observation validates our theoretical results. FLA uses a similar idea as our algorithm, thus, it performs much better than RAN, which has the worst performance in all setups. On average, the charging quality RAN achieves is roughly 64.4% of that of TCA.

In Figs. 6(a) and 6(d), when the number of candidate locations increases or the maximum power level of a candidate location increases, the chance of having a better solution goes up, so the overall charging quality increases. Since a charger can transfer power to multiple devices simultaneously [17], when the number of energy receiving devices increases, the total power received by all devices would be larger than before, so the charging quality increases as well. This is what we noticed in Fig. 6(b). Fig. 6(c) presents the performance of the four algorithms when the power budget is varying. As we can see, when the budget increases, our objective function increases as expected.

For easy understanding, we visualize two charger placements and the corresponding allocated power levels generated by TCA for two instances of  $P^3$  in Figs. 7 and 8, respectively. In Fig. 7, there are in total 9 candidate locations and 50 devices. TCA picks 6 of them, and the corresponding power levels are 4, 3, 1, 4, 1, and 3. As we mentioned in Equ. (2), the maximum coverage distance is determined by the power level.

In the figure, we use circle radius to indicate the allocated power level of a location. The charging quality (see Def. 1) of this solution is 0.77. In Fig. 8, TCA picks 35 out of 50 candidate locations for charging 200 devices. The allocated power levels consist of four 1's, three 4's, and twenty-eight 6's, yielding a charging quality of 3.34.

Fig. 9 depicts the performance of TCA, FLA, and RAN on large  $P^3$  instances. Most of the findings from Fig. 6 still hold here. We would like to highlight that, in Fig. 9(c), the increasing speed of the charging quality tends to slow down gradually, *e.g.*, the increment of TCA between the first three consecutive groups are 0.21, 0.18, and 0.12. This is in accordance with the submodularity of the objective function.

We also conduct evaluations based on large instances of  $P^3$ . Fig. 10 gives the comparison results on running time of TCA, FLA, and RAN. Remember that the worst case time complexity of TCA is  $O(MN^3L^3)$ , thus, it is expected that the running time of TCA increases as one of  $M$ ,  $N$ , and  $L$  increases. An interesting observation is that, TCA runs averagely faster when  $L$  increases. The main reason behind this phenomenon is that, when  $L$  increases, the number of iterations in each while-loop (*i.e.*, lines 3-7 and 10-14) becomes smaller, which may shorten the running time.

In summary, the proposed algorithm TCA performs very closely to OPT (the gap is no more than 4.5% of OPT), and outperforms the other two baseline algorithms.



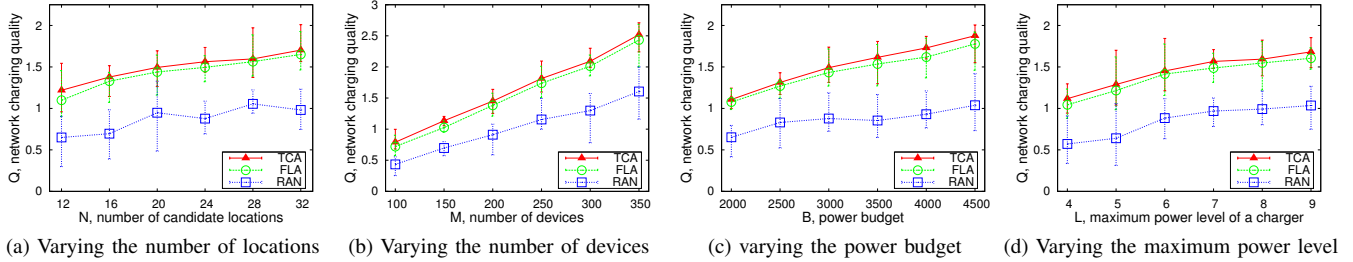


Fig. 9: Evaluation results on large instances (the default setting is  $N = 20$ ,  $M = 200$ ,  $B = 3,000$ ,  $L = 6$ , and the side length of the 2-D plane is 1,000m).

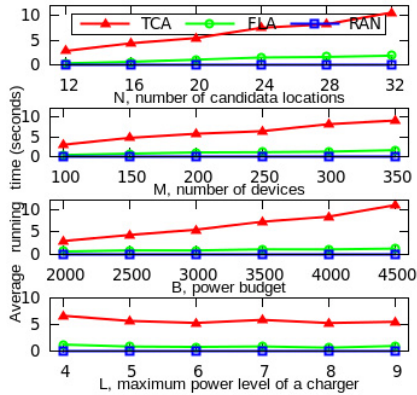


Fig. 10: Running time comparison

## VI. CONCLUSIONS

In this paper, we have studied the joint optimization problem of charger placement and power allocation for wireless power transfer. We proved that this problem is NP-complete by reduction from the set cover problem. We first considered the  $P^3$  problem with fixed power levels, and proposed a  $(1 - 1/e)$ -approximation algorithm. Based on the acquired insights, we then designed TCA for  $P^3$ , which achieves an approximation guarantee of  $\frac{1-1/e}{2L}$ . We also discussed an extension of  $P^3$ , namely  $P^3$  in a cycle. Extensive simulation results confirmed the performance of our algorithm compared with the optimal algorithm and two other heuristic algorithms.

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