Trust Evaluation in Online Social Networks Using Generalized Network Flow

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Appendix

A. Conformity with Basic Axioms

For the self-containment of this paper, we introduce the basic axioms of trust, and analyze why GFTrust coincides with them. Sun et al. developed four axioms that address the basic understanding of trust, and the rules for trust propagation: (1) Uncertainty is a measure of trust. (2) Concatenation propagation of trust does not increase trust. (3) Multi-path propagation of trust does not reduce trust. (4) Trust based on multiple recommendations from a single source should not be higher than that from independent sources. More specifically, the four axioms and the reasons why GFTrust is consistent with them are listed below.

Conformity with Axiom 1. The axiom tells the nature of trust, i.e., subjective, fuzzy, and uncertain. We take the natural definition of trust, and uncertainty is taken as a natural property of trust. Therefore, GFTrust coincides with Axiom 1.

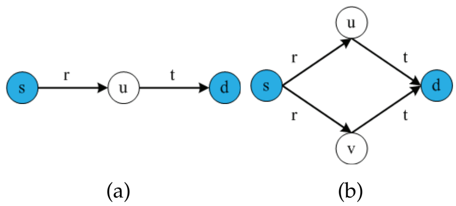

Fig. 1. (a) Trust propagation along a single path. (b) Trust aggregation of multiple paths.

Conformity with Axiom 2. As shown in Fig. 1(a), s trusts u, and u trusts d. Then, Axiom 2 is formally represented as:

$$t(s,d) \leq min\{t(s,u), t(u,d)\}. \tag{1}$$

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Proof: The essence of the axiom is exactly "trust decay through propagation along one path." Consider the trusted path in Fig. 1(a). Under GFTrust, by splitting intermediate node u, path (s, u, d) will be extended to (s, u^+, u^-, d). Capacities of $e(s, u^+)$ and $e(u^-, d)$ will be $c(s, u^+) = t(s, u)$ and $c(u^-, d) = t(u, d)$, respectively. The feasible generalized flow f^Δ through the path should satisfy the capacity constraint. Therefore, $f^\Delta \leq c(s, u^+)$, and $f^\Delta \leq c(u^-, d)$. Under GFTrust, $t(s, d) = f^\Delta$. Therefore, we prove $t(s, d) \leq min\{t(s, u), t(u, d)\}$. □

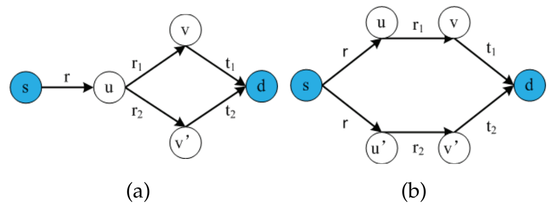


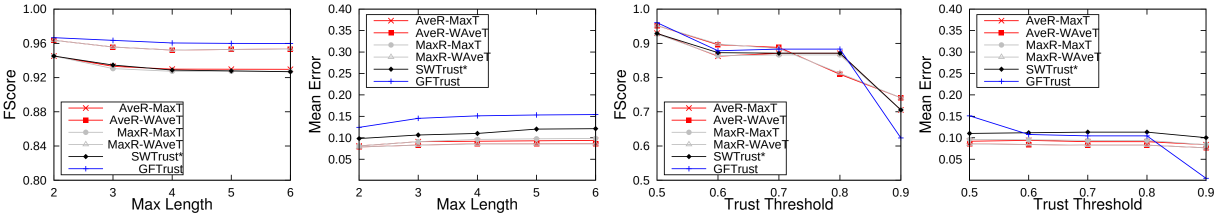
Fig. 2. Trust aggregation of (a) dependent paths, and (b) independent paths.

Conformity with Axiom 3. Axiom 3 considers multiple paths; it compares two cases in Fig. 1: (1) s establishes trust with d through one concatenation path (s, u, d); and (2) s establishes trust with d through the two paths (s, u, d) and (s, v, d). We denote the trust value from s to d in case (1) as $t_1(s, d)$, and in case (2) as $t_2(s, d)$. The mathematical representation of Axiom 3 is:

$$t_2(s, d) \geq t_1(s, d). \tag{2}$$

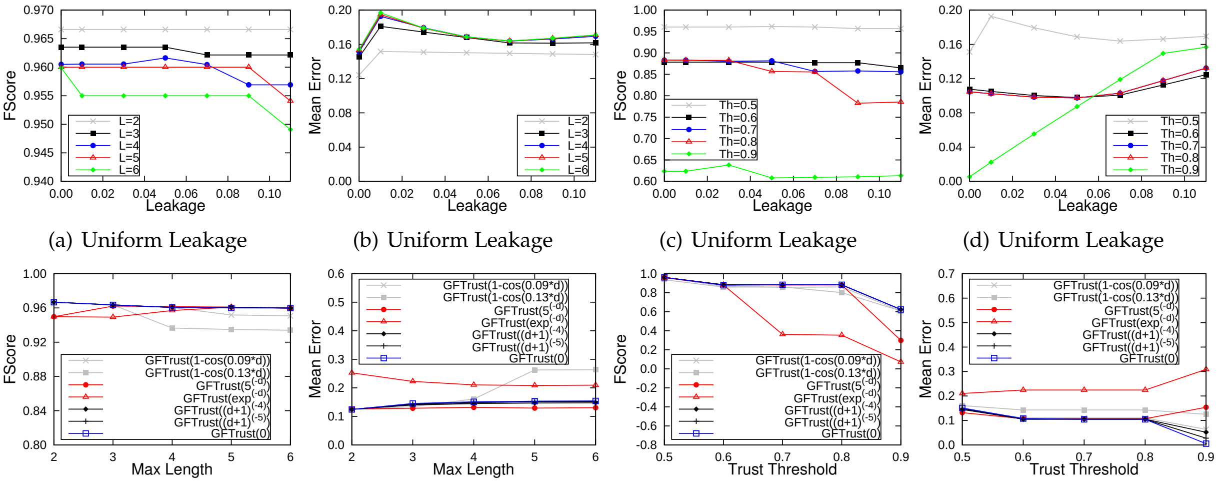
Proof: GFTrust satisfies Axiom 3, since flows are summarized as the trust value. Consider the trusted path in Fig. 1. We represent the feasible flow in case 1 (Fig. 1(a)) as $f^\Delta = f_1$. Then, $t_1(s, d) = f_1$. In case 2 (Fig. 1(b)), the same path is added into the trusted graph. Then, we can send the residual flow through the new path, which will get a flow $f_2 \geq 0$. The final feasible flow will be $f^\Delta = f_1 + f_2$. Then, $t_2(s, d) = f_1 + f_2$. Since $f_2 \geq 0$, we prove $t_2(s, d) \geq t_1(s, d)$. □

Conformity with Axiom 4. Axiom 4 is related to overlapped paths. It compares the two cases in Fig. 2: (1) s establishes trust with d through two overlapped paths (s, u, v, d) and (s, u, v', d), which share edge $e(s, u)$; and (2) s establishes trust with d through two disjoint paths (s, u, v, d) and (s, u', v', d). Again, we denote the trust value



(a) FScore (b) Mean Error (c) FScore (d) Mean Error

Fig. 3. The accuracy in Advogato.



(a) Uniform Leakage (b) Uniform Leakage (c) Uniform Leakage (d) Uniform Leakage

(e) Non-uniform Leakage (f) Non-uniform Leakage (g) Non-uniform Leakage (h) Non-uniform Leakage

Fig. 4. The accuracy of GFTrust with leakage in Advogato.

from s to d in case (1) as $t_1(s,d)$, and in case (2) as $t_2(s,d)$. The mathematical representation of Axiom 4 is:

$$t_2(s,d) \geq t_1(s,d). \tag{3}$$

Proof: GFTrust satisfies this axiom because flow is limited with capacity constraints. Consider the trusted graph in Fig. 2. Without loss of generality, we suppose that path (s,u,v,d) is the highest-gain path through which we can send flow f_1. We denoted the flow through the second path (s,u,v',d) in case (1) as f_2; and through the second path (s,u',v',d) in case (2) as f_2'. Then, the final trust values will be $t_1(s,d) = f_1 + f_2$, $t_2(s,d) = f_1 + f_2'$, respectively. Now, we only need to compare f_2 and f_2', both of which should satisfy capacity constraints. All corresponding edges in the two cases have equal residual capacities, except $e(s,u)$ in case (1), and $e(s,u')$ in case (2). Therefore, we only need to compare the two edges. For case 1, $f_2 \leq c_f(s,u) < r - f_1$. For case 2, $f_2' \leq c_f(s,u') = r$. Therefore, $f_2' \geq f_2$. We prove $t_2(s,d) \geq t_1(s,d)$. □

B. Experimental Results in Advogato

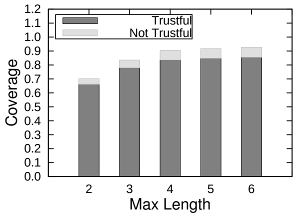
Advogato (advogato.org) is an online social networking site dedicated to free software development. We use the snapshot collected in June 2012. It contains 7,436 users and 56,667 links. On Advogato, users can certify each other on 4 levels: Observer, Apprentice, Journeyer, and Master, which we assign 0.4, 0.6, 0.8, and 1.0, respectively, to numerate the level of trust.

TABLE 1
Trust value distributions.

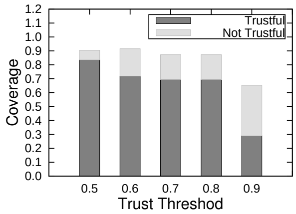
Trust Threshold	Advogato(%)	Epinions(%)
0.5	91.91	49.81
0.6	75.04	39.88
0.7	75.04	29.87
0.8	33.58	20.05
0.9	33.58	10.01

The Difference Between the Two Data Sets. Recall that we transform the trust values in Epinions to be continuous in [0,1]. Different preprocesses of the two data sets make a significant difference in their trust value distributions. (1) The trust value in Advogato is discrete in four levels, with a minimum value of 0.4. We further analyze the data set and find that more than 91.9% of trust values are larger than 0.4. This finding indicates that if we set the trust threshold as $Th = 0.5$, most of the edges will be trustful according to their direct trusts. (2) Quite different from that, the trust values in Epinions are uniformly distributed in the range of $[0,1]$. Only 49.81% of trust values are equal to or larger than 0.5. Table 1 shows the percentage of trust value with respect to trust threshold. In addition, Fig. 5 shows the coverage, which is much higher than that in Epinions.

The Effects of Different Strategies. Table 2 and Fig. 3 show the comparison of accuracy in Advogato. The FScore of using GFTrust is higher than those of other strategies, which shows the advantage of GFTrust in predicting whether to trust or not. However, quite different from that in



(a)



(b)

Fig. 5. The coverage in Advogato.

TABLE 2
Accuracy in Advogato, $L = 4$, $Th = 0.5$, $leakage = 0$.

Method	Mean Error	Recall	Precision	FScore
AveR-MaxT	0.092	0.9343	0.9256	0.9299
AveR-WAveT	0.0858	0.9812	0.9248	0.9522
MaxR-MaxT	0.0962	0.9296	0.9252	0.9274
MaxR-WAveT	0.0854	0.9812	0.9248	0.9522
SWTrust*	0.1102	0.9343	0.9234	0.9288
GFTrust	0.1512	1	0.9241	0.9605

Epinions, the Mean Error of using GFTrust is a little higher than other strategies. The reason lies in two aspects: (1) the trust value distribution: GFTrust takes trust value as continuous, while the trust value in Advogato is discrete; and (2) the summation-like way of GFTrust. In some sense, GFTrust calculates the summation of trust of multiple trusted paths, while the other strategies calculate the weighted average value.

The Effects of Leakage Functions. Fig. 4 shows some representative results with leakage. We only mention the difference here: comparing the Mean Error in Epinions and Advogato, we can say that the leakage has more positive effects in Epinions than in Advogoto. The results indicate that GFTrust fits more in continuous trust value distribution.