

# Balance-aware Cost-efficient Routing in the Payment Channel Network

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# Outline

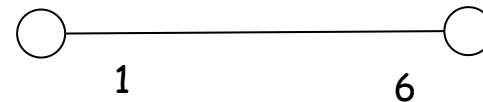
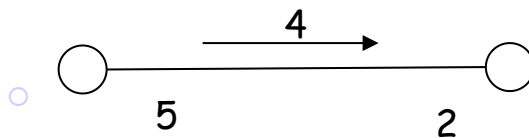
1. Payment Channel Networks
2. Depleted Channels
3. Solution via Game Theory
4. Performance Evaluation
5. Conclusions



# 1. Payment Channel Networks

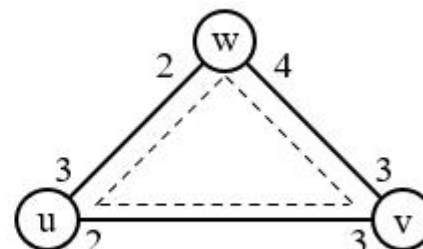
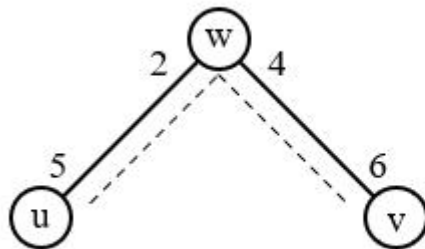


- Micropayment channels
  - Quick transactions between **trusted neighbors**
  - Avoiding block confirmation via **offchain payment**
- Fund allocations
  - Allocation of node funds to channels
- Bidirectional transactions
  - Fund balance in two directions: *channel capacity*



# Payment Path

- Indirect fund transfer
  - Between two untrusted nodes
- Payment path
  - A sequence of non-repeated trusted neighbors (nodes)
- Types of paths
  - Single-path and multi-path
  - e.g., transfer \$4 from u to v



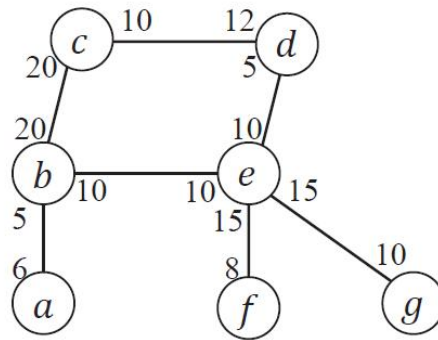
## 2. Depleted Channels



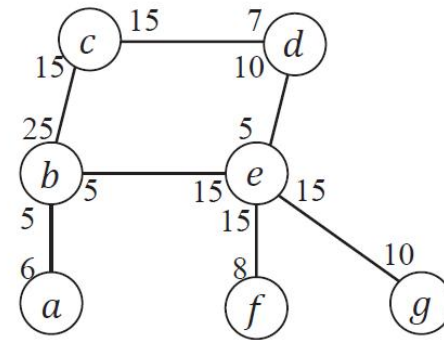
- Depleted Channels
  - Transaction flows from one direction is much heavier
    - Run out of balances and cannot send further payments
    - Reduce network-wide throughput
  - Back to live
    - Balance addition by committing a blockchain transaction
    - Passively wait for a payment from the other direction
    - Perform a cyclic fund rebalancing
    - Actively adjust channel balances

## 2. Depleted Channels (Cont.)

- Active Channel Balance Adjustment
  - Rebalance



(a) Original PCN.



(b) Rebalanced PCN.

Fig. 1: A small PCN where the number associated with each channel close to a node is the deposit allocated by the node to the channel.

- Fee policy
  - Set the higher fees sent from b to than that sent from e to b

# 3. Models and Definitions

- Channel State:  $C_{uv} = (b_{uv}, b_{vu})$ 
  - Channel capacity:  $C_{uv} = b_{uv} + b_{vu}$
- Feasible Transactions:
  - A feasible transaction of  $a$  coins from  $u$  to  $v$  unless  $0 < a \leq b_{uv}$

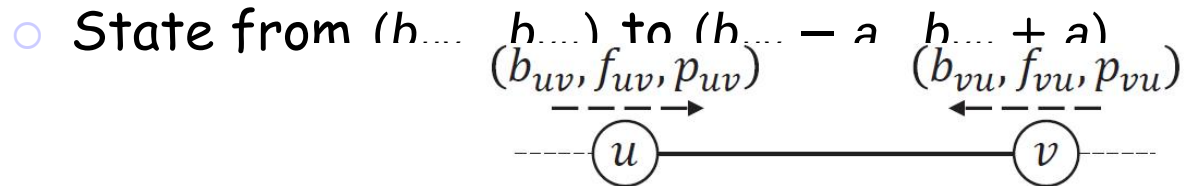


Fig. 2: Both  $u$  and  $v$  only act as intermediate routing nodes.

- Channel Liveness: both side are alive
- Cost of Channel Rebalance:  $\eta_{uv}$

# Payment Channel Lifetime

- $T_x$ : expected lifetime of the channel given initial  $b_{uv} = x$
- Fee-free Symmetric-transaction-rate Payment Channel
  - Optimal initial channel balances:  $b_{uv} = b_{vu} = \frac{C_{uv}}{2}$
- Fee-free Asymmetric-transaction-rate Payment Channel
  - probability  $p \in (0, 0.5)$
  - Optimal initial channel balances by solving a concave function
$$T_x = 1 + (1 - p)T_{x+1} + pT_{x-1}$$
- Payment Channel with Routing Fees
  - Optimal initial channel balances for symmetric transaction rate
  - Optimal initial channel balances for asymmetric transaction rate
$$T_x = 1 + pT_{x-(1+f_{uv})} + (1 - p)T_{x+(1+f_{vu})}$$

# Nodes in Payment Channel Network

- Routing Nodes

- Incentivized to participate in others' transactions by charging fee
- Aim to make profits to compensate for its channel rebalance cost

$$f_{vu} \cdot T_x = \eta_{uv} / 2.$$

- Transaction Senders and Receivers

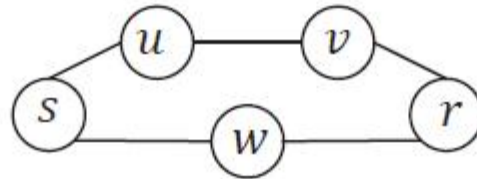


Fig. 3: Sender  $s$  has two feasible path to reach Receiver  $r$ .

- Cost function for sender  $s$  on a feasible path  $P_{s,r,a}$ , simplified as  $P$

$$C_{s,r,a,P} = (1 - w_s) f_{s,r,a,P} + w_s \|P\|$$

- Aim to select a path that minimizes his cost among all feasible paths

# Combination of All Roles

- Node  $u$  always wants to
  - Transfer money in a fast and cheap way
  - Make as many routing fees as possible
- Objective conflict as sender and router

- A discounted factor

$$\Delta_{s,r,a,P} = \left| \frac{b'_{su}}{b'_{us}} - \frac{\lambda_{us}}{\lambda_{su}} \right| - \left| \frac{b_{su}}{b_{us}} - \frac{\lambda_{su}}{\lambda_{us}} \right|$$

- Update path cost function

$$C_{s,r,a,P} = (1 - w_s) f_{s,r,a,P} + w_s \|P\| - \Delta_{s,r,a,P}$$

# Network Analysis



- Uniform transaction flow pattern

**Definition 1.** A PCN is considered to have a uniform transaction flow pattern if any pair of its nodes have the same transaction rate.

- Proportional transaction flow pattern

**Definition 2.** A PCN is considered to have a proportional transaction flow pattern if any pair of its nodes have an identically proportional transaction rate. That is, for  $\forall i, j$ ,  $\lambda_{ij} : \lambda_{ji} = r$ .

# Topology: Ring

- Each node is connected to its two adjacent nodes
  - Fund travels from sender to receiver through intermediate nodes, clockwise or anti-clockwise

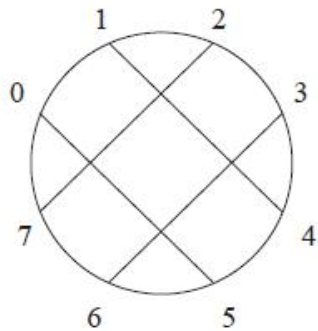
**Theorem 1.** *Given a uniform transaction flow set among all nodes and the same starting budget  $b$ , in a pure equilibrium, all nodes evenly distribute their budgets on two channels, and charge equal routing fees of  $\frac{4\eta}{b^2 - 4\eta}$  in both directions. When facing multiple choices, each transaction sender will choose the shortest path, and otherwise, it picks either path with an equal probability.*

**Corollary 1.** *Given that all nodes have the same starting budget while their in and out traffic rate is unbalanced as  $r$ , in an equilibrium, all nodes distribute their budgets and charge an amount of transaction fee with a specific rate (related to  $r$ ) between two directions in an identical way.*

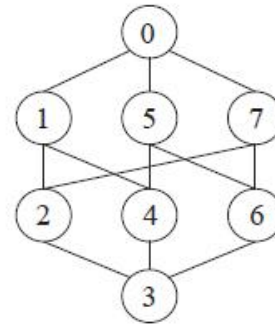
# Topology: Chordal Ring

- Chordal Ring of Degree  $k$

- A ring structured topology in which each node has additional links to other  $k-2$  nodes
- The longest path of this topology is shorter than a ring



(a) Ring structure.



(b) Flatten structure.

Fig. 4: A chordal ring topology of degree 3.

# Topology: Chordal Ring

The title is centered at the top of the slide. It is surrounded by five circles of varying shades of light purple. From left to right: a solid purple circle, a white circle with a purple outline, a solid purple circle, a white circle with a purple outline, and a solid purple circle.

**Theorem 2.** *Given a uniform transaction pattern and the same starting budget, in a pure equilibrium, all nodes evenly distribute their budgets on  $k$  channels, and charge zero fee in both directions. When facing multiple choices, each transaction sender will choose the shortest path, and otherwise, it picks any path with an equal probability.*

# Traffic-aware Balance Redistribution in General PCNs

- Assume each sender-receiver pair in the PCN has its transaction pattern
- Nodes cannot immediately decide their optimal initial channel balances and fee policies
- Adjust initial channel balances and fee policies through gradual learning
- The network can reach a relatively stable state

# 4. Performance Evaluation

- Custom network topology
  - 25 nodes and 51 edges
  - Based on the BA model

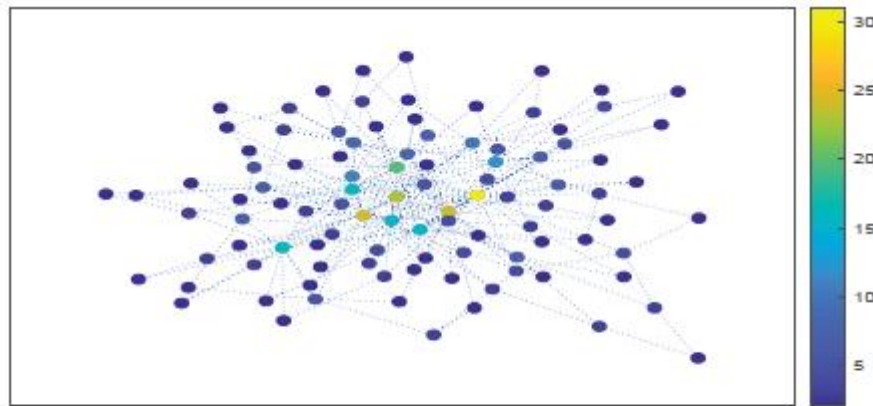


Fig. 5: Topology of the custom network.

# Network Setup



- Channel capacity
  - channel's capacity is set randomly in [50000, 75000)
- Channel initial balance
  - Randomly balanced
- Transaction size
  - Heterogeneous

# Simulation Summary

Round	Traffic-aware rebalancing	Revive rebalancing	
		every 1000 txs	every 2000 txs
1	0.3594	0.3742	0.3594
2	0.4333	0.3789	0.3742
3	0.4545	0.4008	0.4008
4	0.5083	0.4432	0.4225
<b>Improvement</b>	<b>41.43%</b>	<b>18.44%</b>	<b>17.56%</b>

TABLE I: Success ratio updates where each round contains 2000 txs.

Success ratio: the number of completed transactions over the number of total transactions

## 6. Conclusions



- We analyze the factors affect a channel's lifetime and how to distribute its optimal initial balances
- We analyze the roles a PCN node can play and define its objectives under different roles
- We show that nodes will reach a Nash equilibrium in ring and chordal ring structures
- We propose a real-time fee policy to improve nodes' utilities for general PCNs
- Testbed-based evaluation is conducted to show the feasibility of our proposed approach.